

CONTROLLING THE FLOW OF CLASSICAL WAVES

- **INTRODUCTION**
- **GAPS IN CLASSICAL WAVE PROPAGATION**
- **CALCULATIONAL METHODS**
- **DOPING, MINI STOP BANDS, DISORDER**
- **RECENT DEVELOPMENTS (BRIEFLY & SELECTIVELY)**
- **PHOTONIC “CRYSTALS” AS NEGATIVE INDEX MATERIALS**

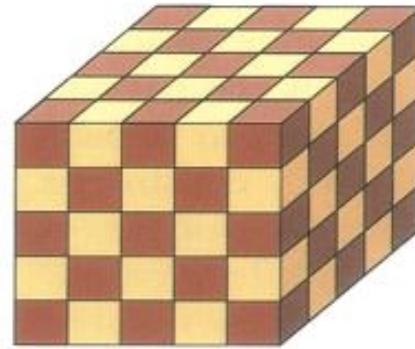
**E.N. ECONOMOU
DEPT. OF PHYSICS, U. of CRETE
FORTH**

PHOTONIC “CRYSTALS”:

**ARTIFICIAL PERIODIC STRUCTURES ($1\mu \sim a \sim 1\text{cm}$)
EXHIBITING SPECTRAL GAPS (or PSEUDOGAPS) IN
THE PHOTON DOS, DUE TO STRONG SCATTERING
AND DESTRUCTIVE INTERFERENCE**

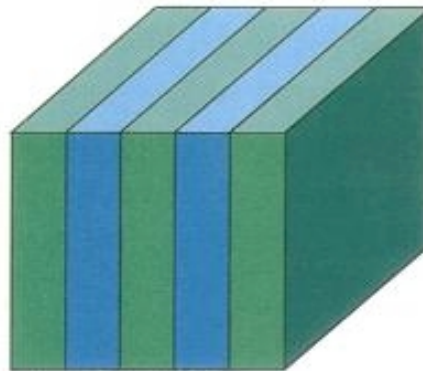
TRUE PC

3-D



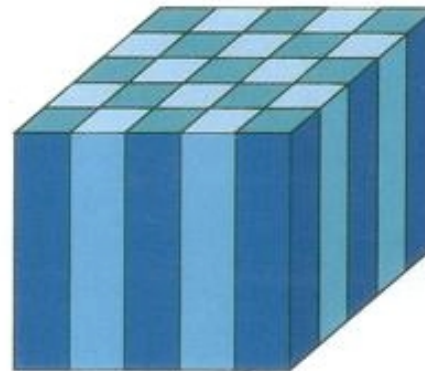
periodic in
three directions

1-D



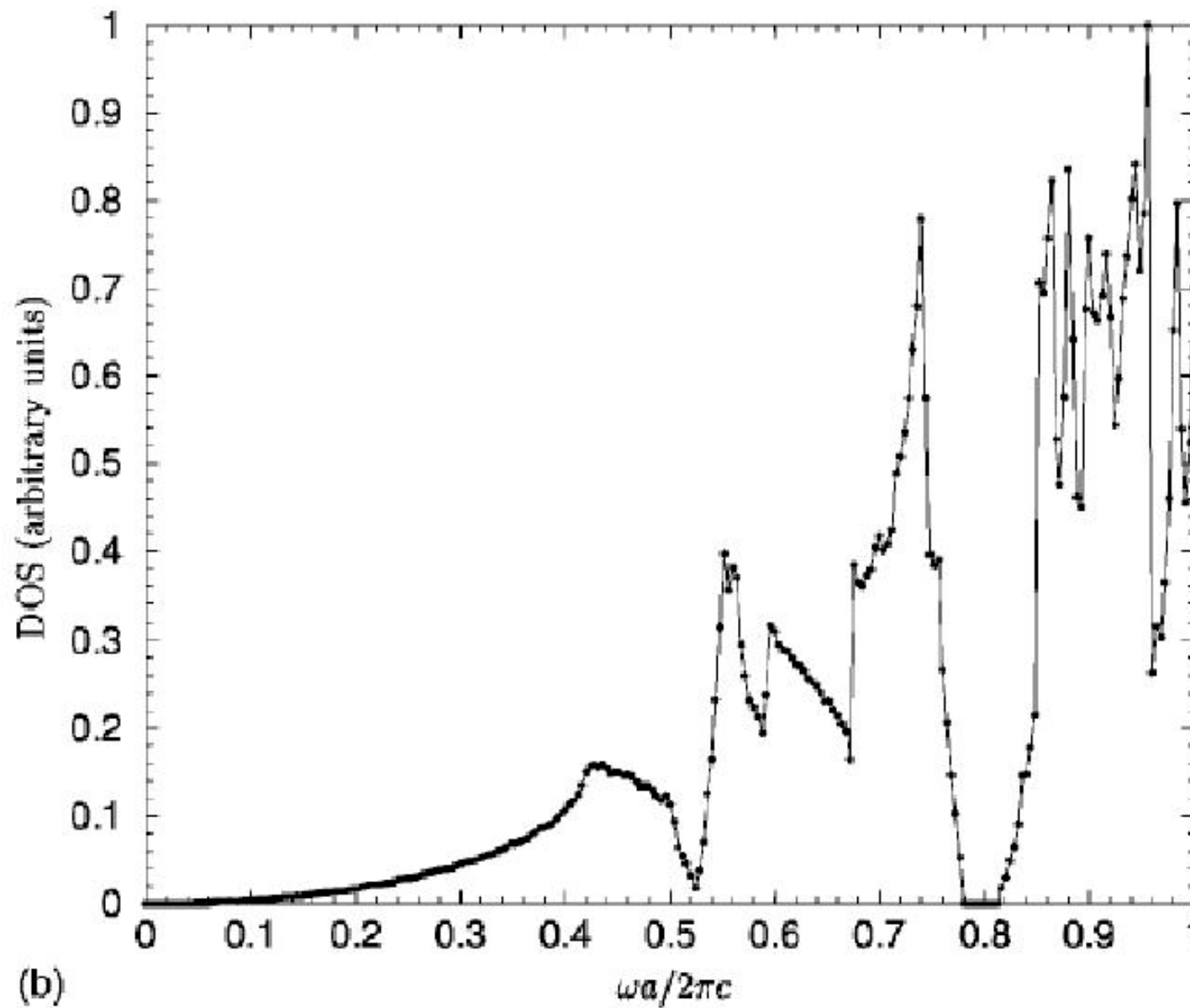
periodic in
one direction

2-D



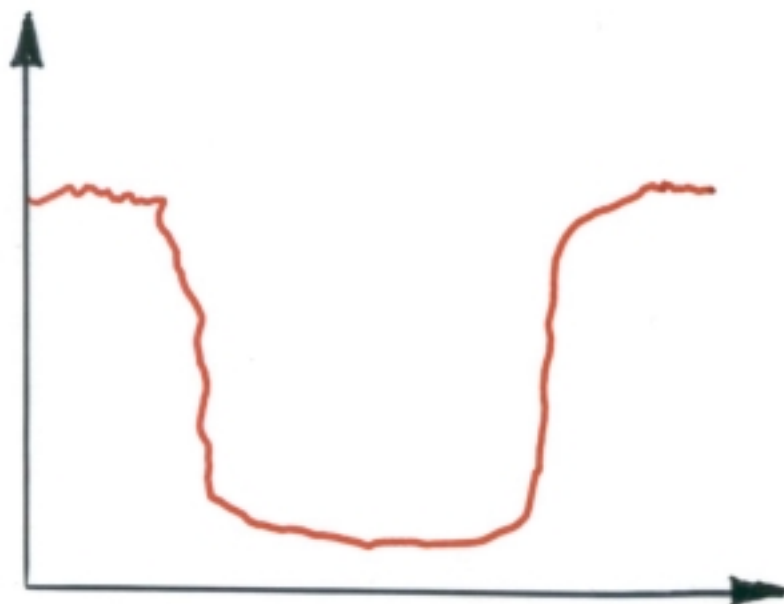
periodic in
two directions

DOS for a close-packed fcc lattice of air spheres in silicon



Busch and John, PRE 58, 3896 (1998)

Transmission



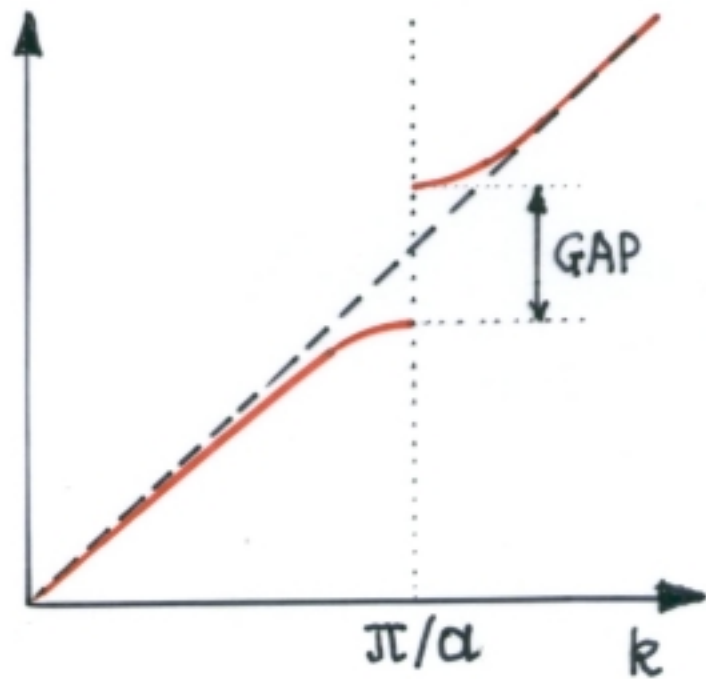
GAP
OR
STOP BAND

Frequency

--- FREE PHOTON

— PC PHOTON

ω



π/a

k

1987

1989

1990

1991

LOCALI-
ZATION

GAPS
(CW)

GAPS (EM)
diamond

BAND
STRUCTURE

OPTIMIZATION

NEW DESIGNS

EFFICIENT
LASERS

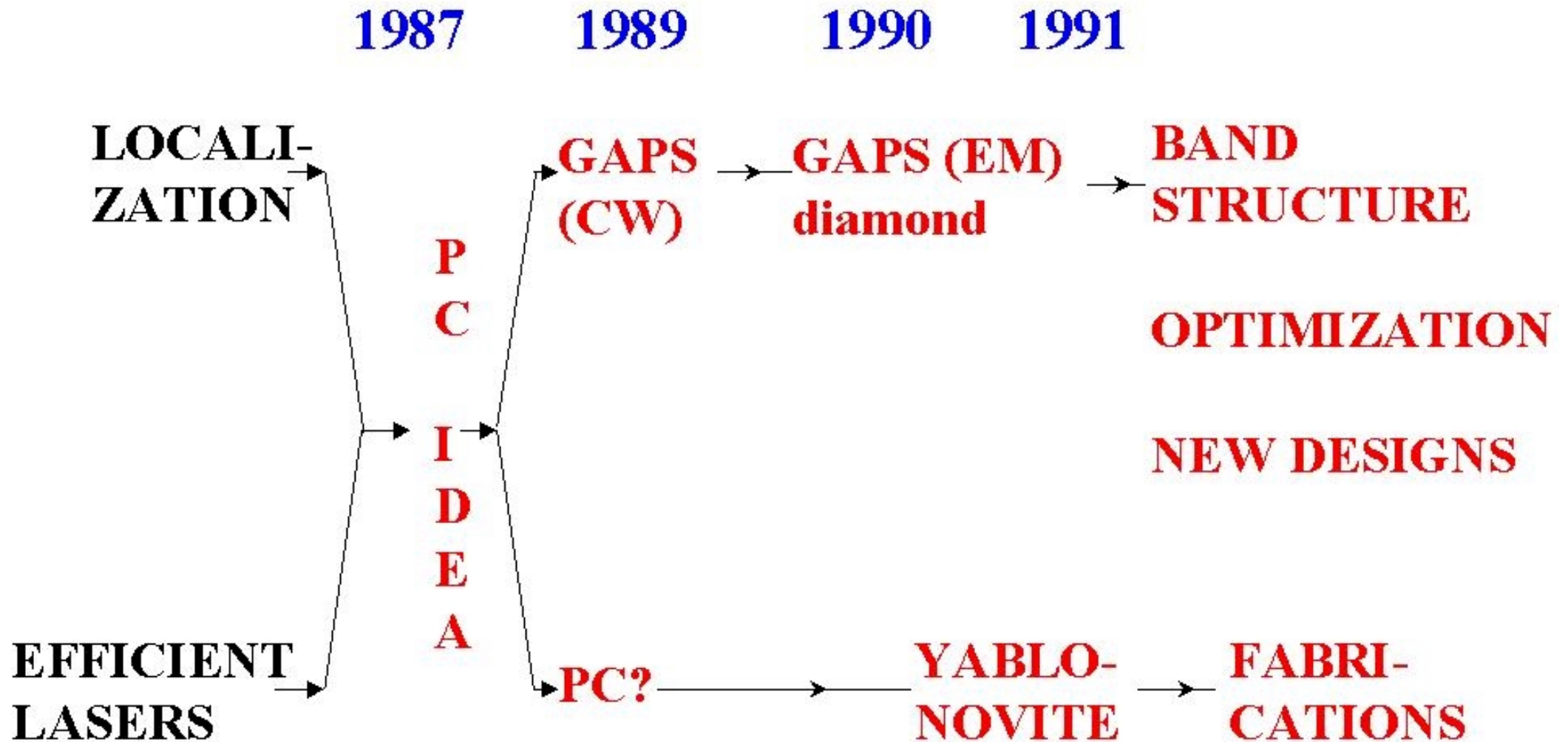
P
C

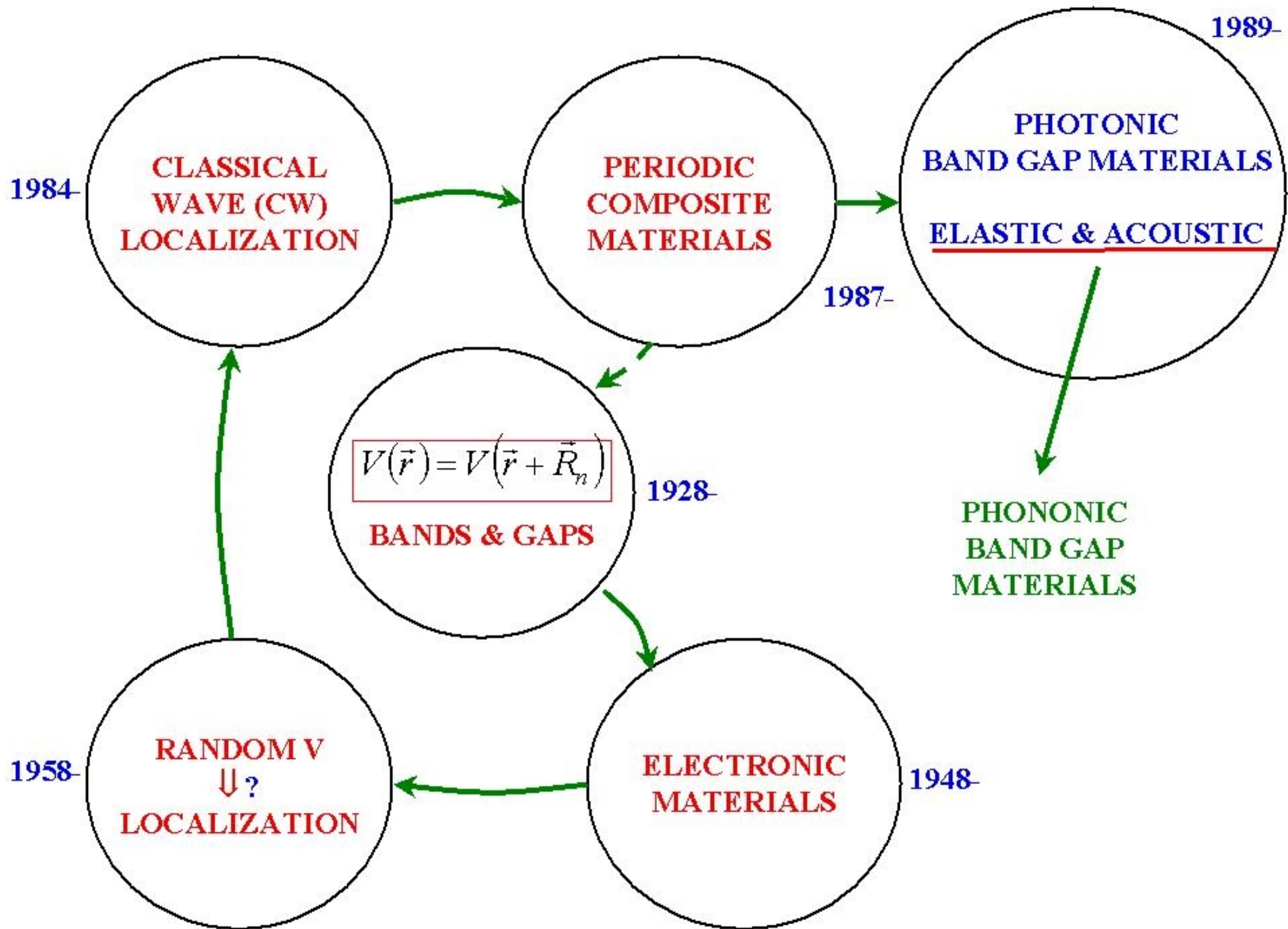
I
D
E
A

PC?

YABLO-
NOVITE

FABRI-
CATIONS





*WE SHALL NOT CEASE FROM EXPLORATION
AND THE END OF OUR EXPLORING
WILL BE TO ARRIVE WHERE WE STARTED
AND KNOW THE PLACE FOR THE FIRST TIME*

T.S. ELIOT

II. GAPS IN CLASSICAL WAVE PROPAGATION

GAP FORMATION MORE DIFFICULT FOR CWs THAN e-Ws

SCHRÖDINGER ($\delta V \equiv V - V_{\max}$):

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V_{\max}) \psi - \frac{2m}{\hbar^2} \delta V \psi = 0$$

ACOUSTIC IN FLUIDS ($\rho = \text{const.}$):

$$\nabla^2 p + \frac{\omega^2}{c_{\max}^2} p - \omega^2 \left(\frac{1}{c_{\max}^2} - \frac{1}{c^2} \right) p = 0$$

$$\frac{\omega^2}{c_{\max}^2} \leftrightarrow \frac{2m}{\hbar^2} (E - V_{\max}) \Rightarrow \boxed{E > V_{\max}}$$

$$\omega^2 \left(\frac{1}{c_{\max}^2} - \frac{1}{c^2} \right) \leftrightarrow \frac{2m}{\hbar^2} \delta V \Rightarrow \boxed{\omega^2 \text{ x fluctuations}^* \leftrightarrow \ddot{V}}$$

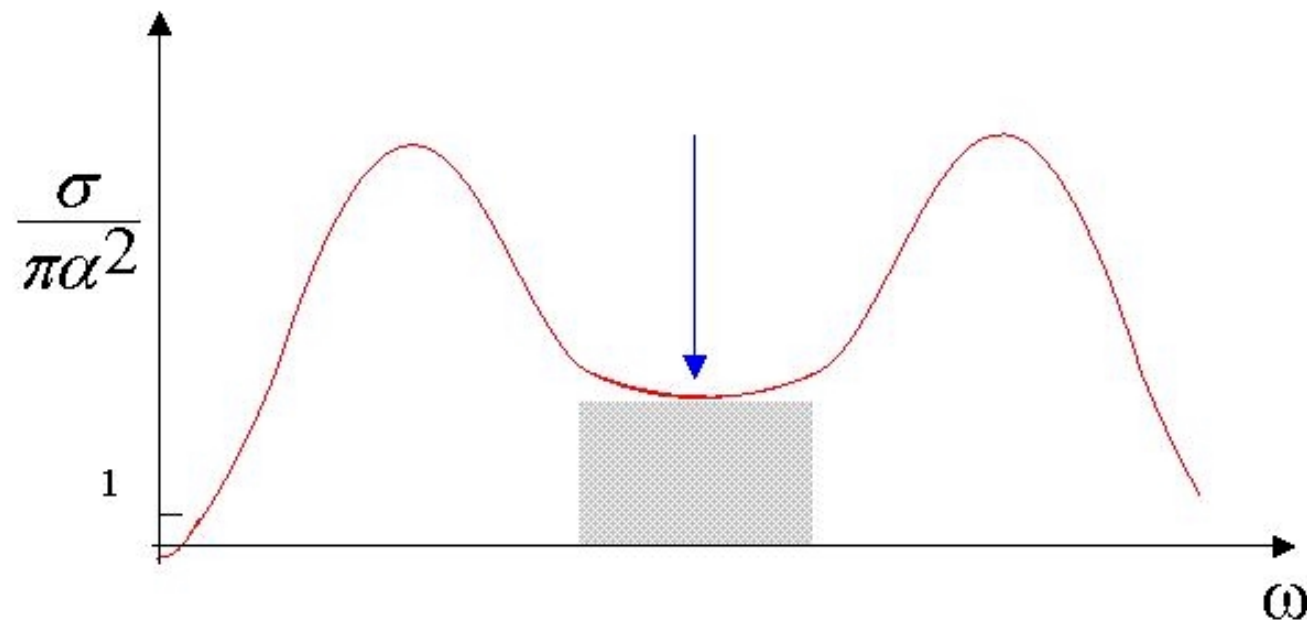
$$\Rightarrow \boxed{\text{low } c \leftrightarrow \text{potential well}}$$

*** $\sigma \sim |\omega^2 \text{ x fluctuations}|^2 \sim \omega^4$, RAYLEIGH**

CONCLUSIONS FOR GAP FORMATION:

- fcc lattice
- LOW ϵ INCLUSIONS IN HIGH ϵ MATRIX
(CERMET TOPOLOGY)
- $\lambda \sim d$ FOR MIE RESONANCES
- $\lambda \sim a$ FOR BRAGG INTERFERENCE
- $d \sim a$ $\Rightarrow f$: VOLUME FRACTION ($f \sim 20\%$)

- **TWO CHANNELS OF PROPAGATION:**
 - ✓ **THROUGH THE MATRIX**
 - ✓ **HOPPING AMONG N.N. M.R. (LCAO-like)**
- **GAPS WHERE BOTH CHANNELS ARE BLOCKED (s vs ?)**



BUT ...

CW EQUATIONS MORE COMPLICATED

- EM:

$$\nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) - \frac{\epsilon \mu}{c^2} \omega^2 \vec{E} + \frac{1}{\mu} (\nabla \mu) \times (\nabla \times \vec{E}) = 0$$
$$\nabla \cdot (\epsilon \vec{E}) = 0$$

- ACOUSTIC IN FLUIDS:

$$\nabla^2 p + \rho \left(\nabla \cdot \frac{1}{\rho} \right) \cdot \nabla p + \frac{\omega^2}{c^2} p = 0$$

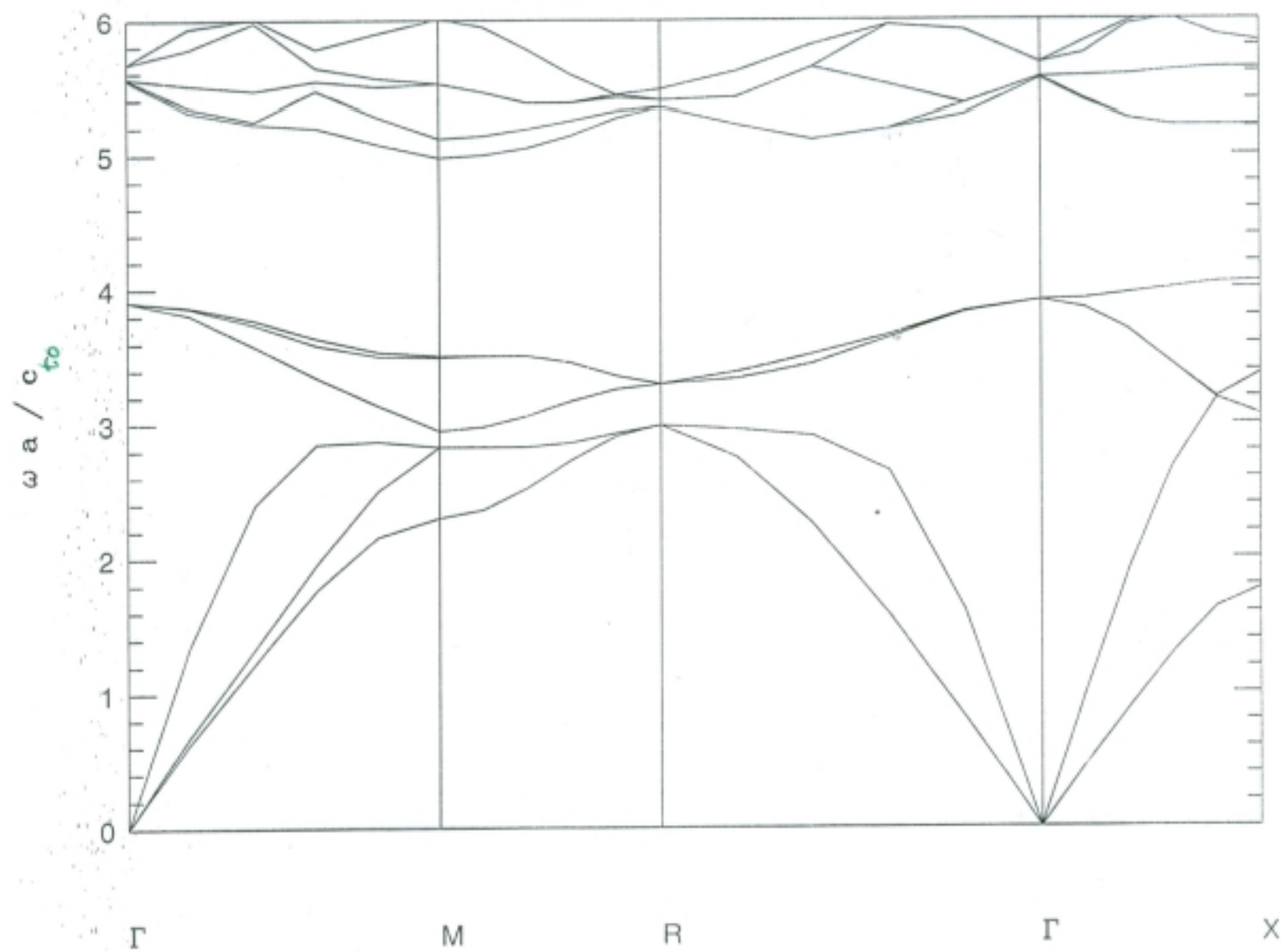
• **ELASTIC:**

$$\frac{1}{\rho} \left\{ \frac{\partial}{\partial x_i} \left(\lambda \frac{\partial u^\ell}{\partial x_\ell} \right) + \frac{\partial}{\partial x_\ell} \left[\mu \left(\frac{\partial u^i}{\partial x_\ell} + \frac{\partial u^\ell}{\partial x_i} \right) \right] \right\} + \omega^2 u^i = 0$$

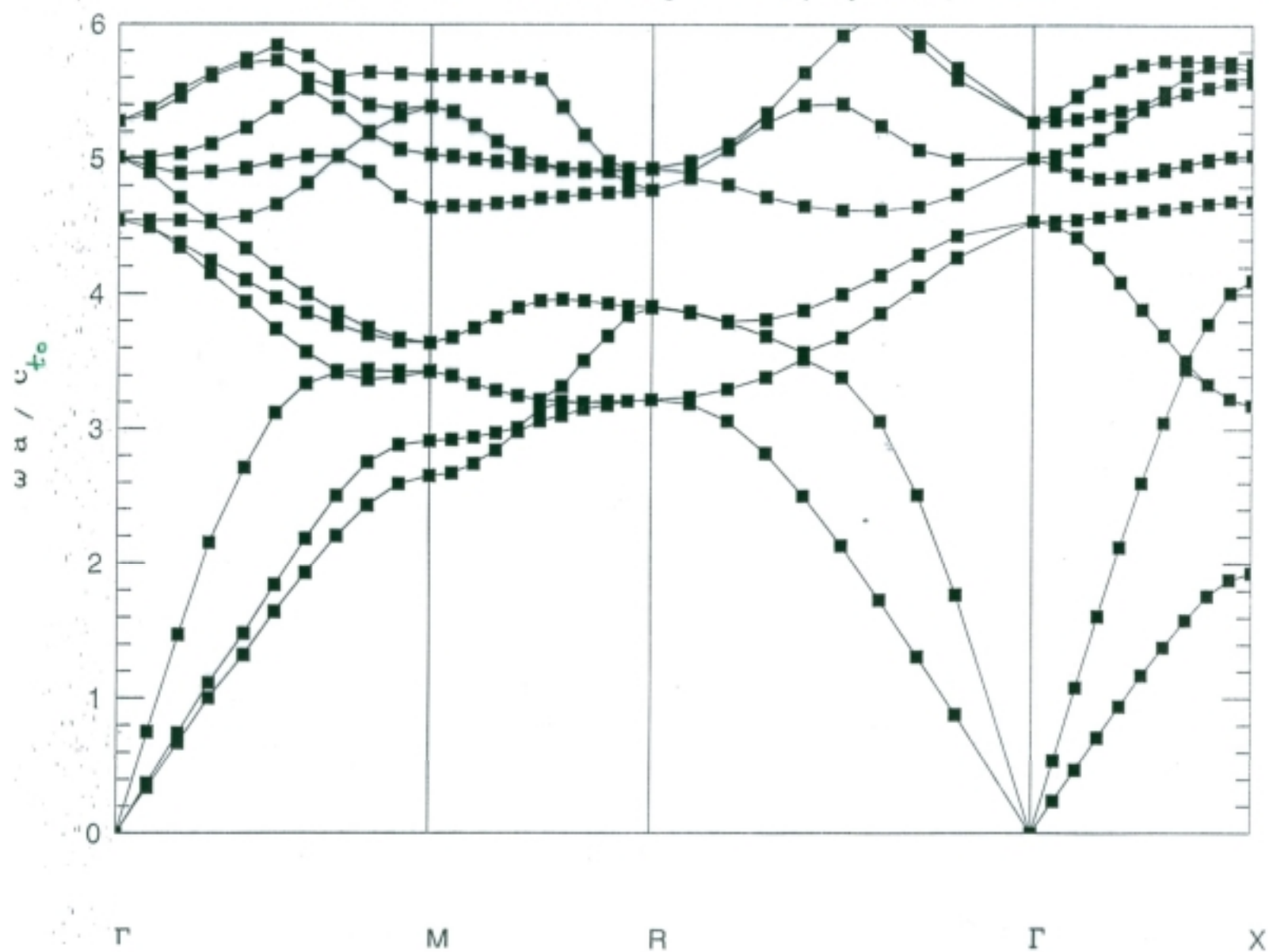
$$\vec{u} \equiv u^i \vec{j}_i, \quad \mu \equiv \rho c_t^2, \quad \lambda \equiv \rho \left(c_\ell^2 - 2c_t^2 \right)$$

**ARE OUR CONCLUSIONS VALID FOR THESE EQUATIONS
AS WELL?**

Pb spheres in epoxy, s.c. lattice, $f=0.268$



Pb tetr. rods conn. near. neighb. in sc,epoxy matrix, $f=0.268$

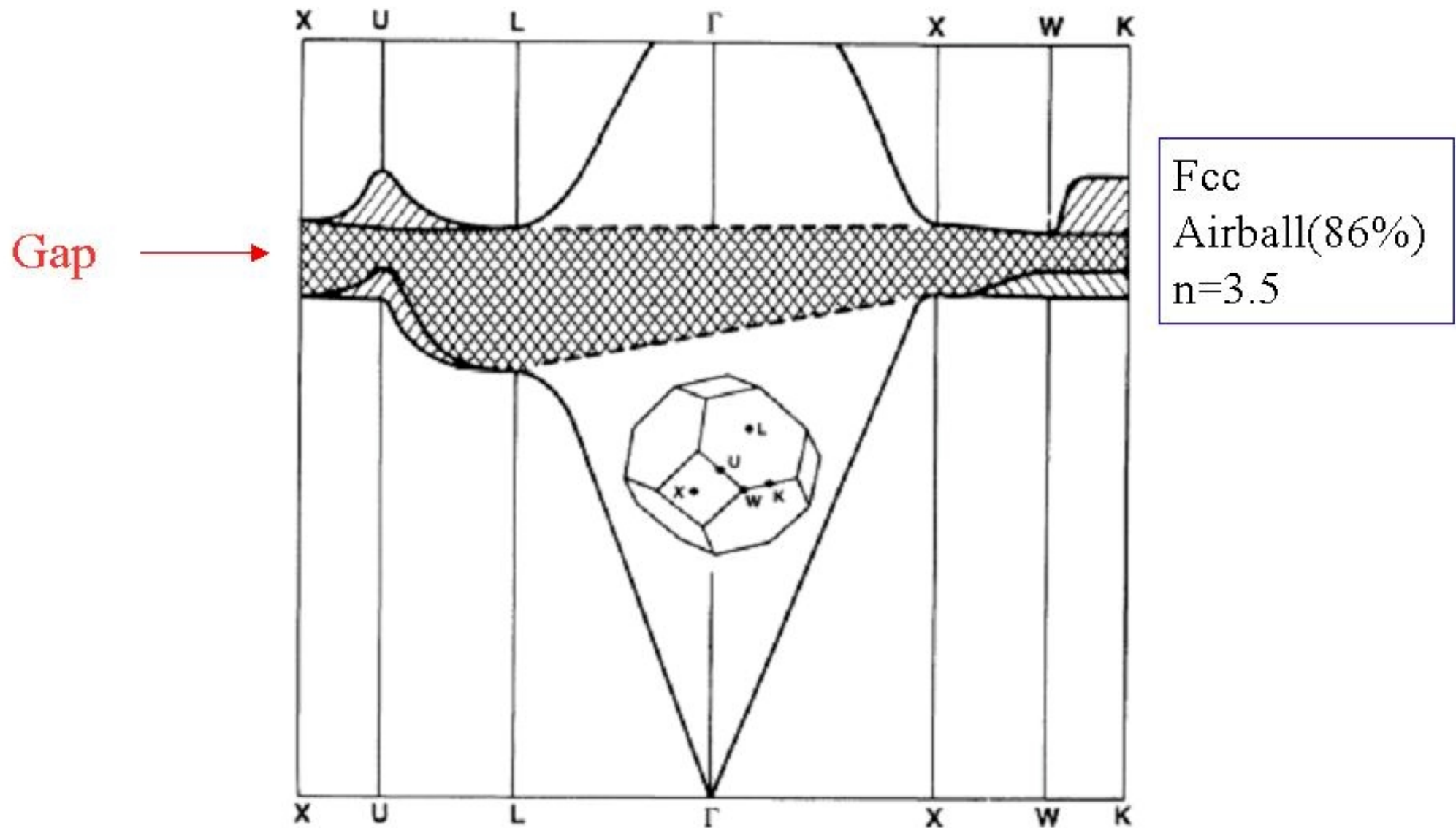


NOT FOR EM WAVES

- **NETWORK TOPOLOGY MORE FAVORABLE**
- **DIAMOND OR DIAMOND-LIKE STRUCTURES
MORE FAVORABLE**

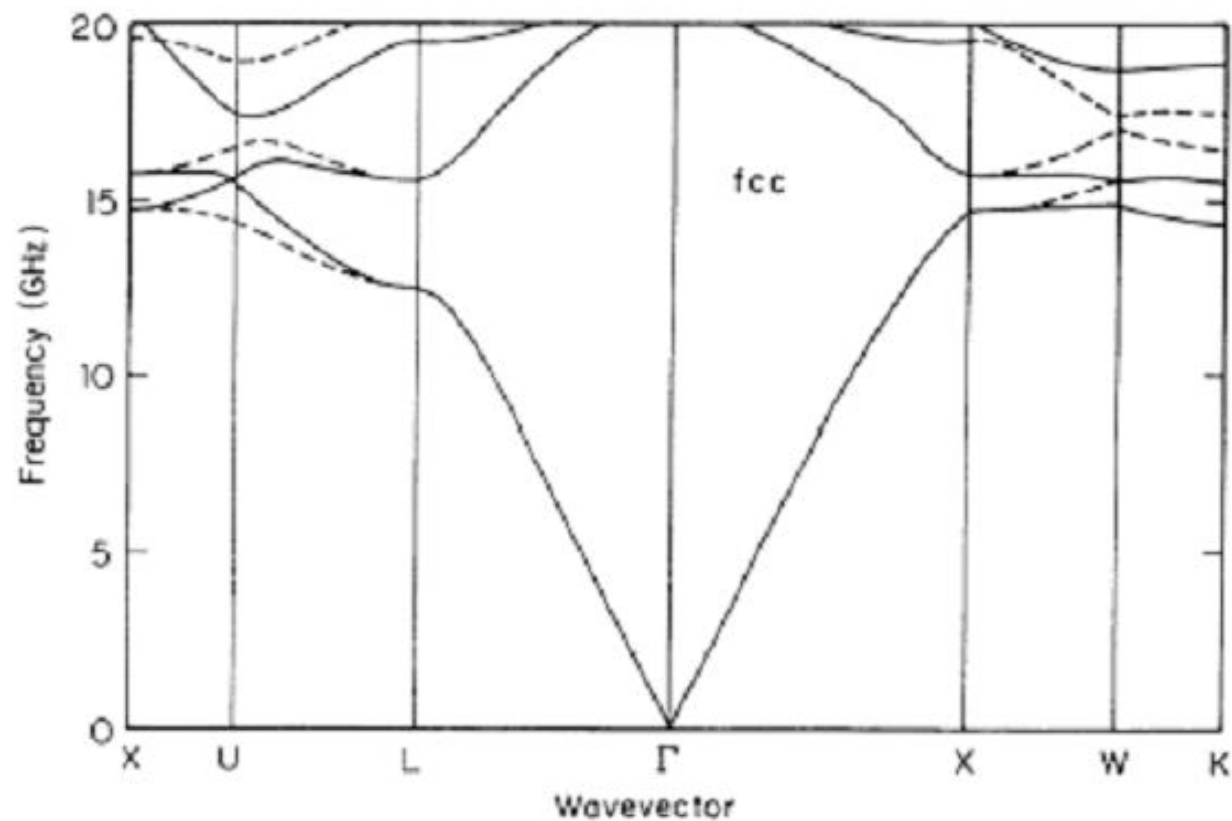
EXPLANATIONS ?

Experimental band structure of a fcc lattice of air spheres



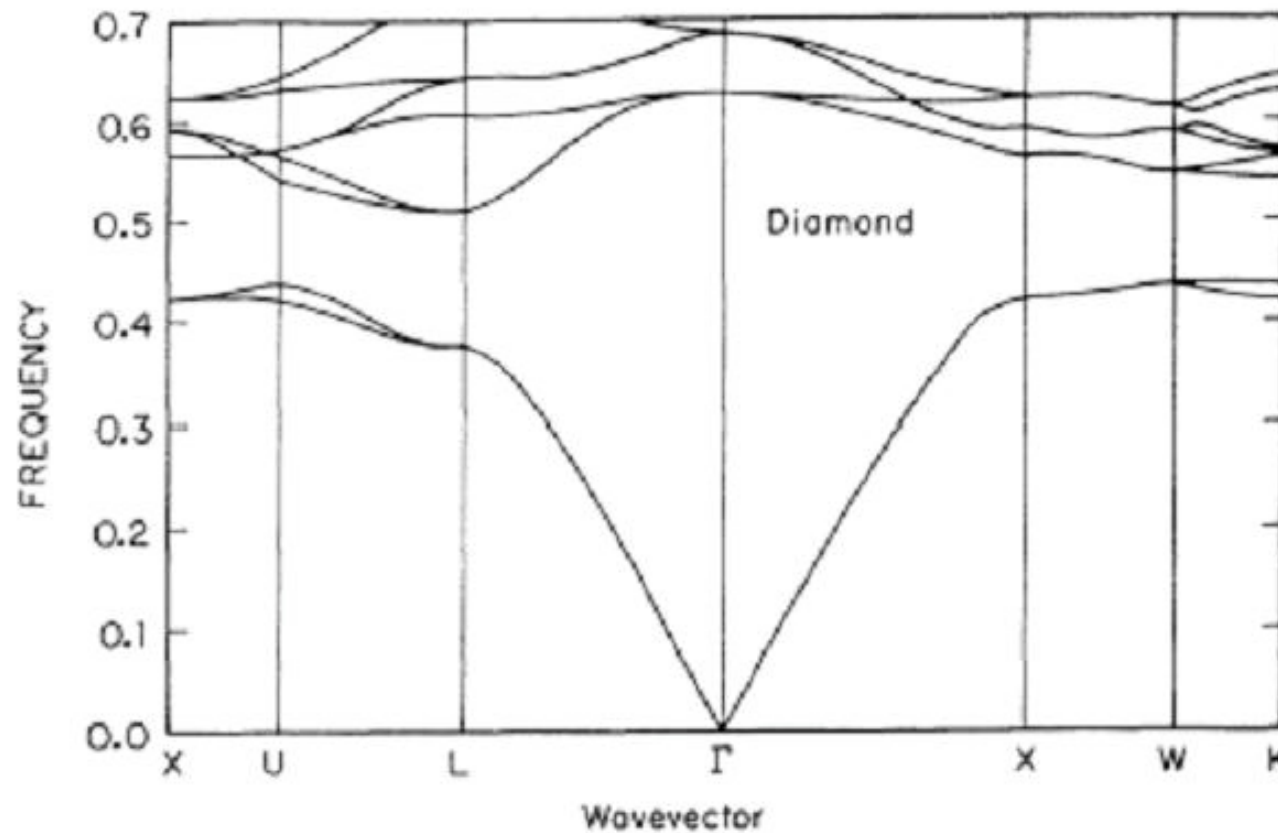
Yablonovitch & Gmitter, PRL 63, 1950 (1989)

FCC lattice has only a pseudogap



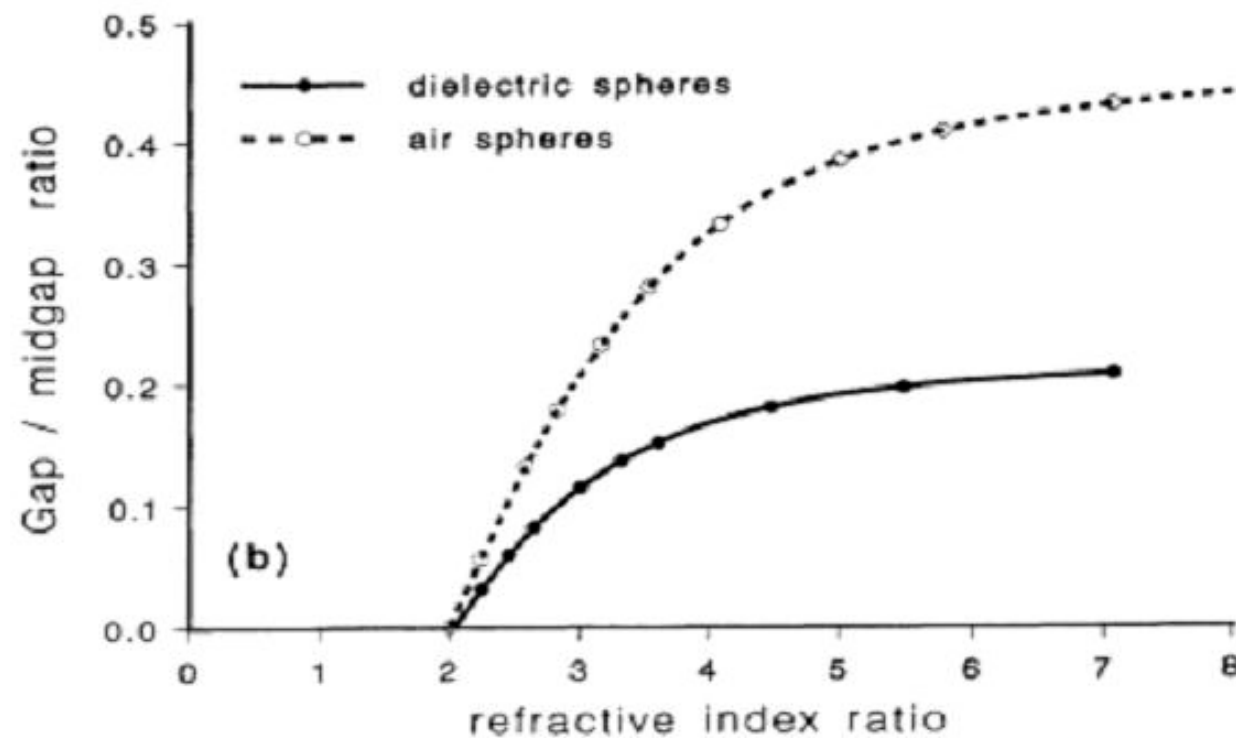
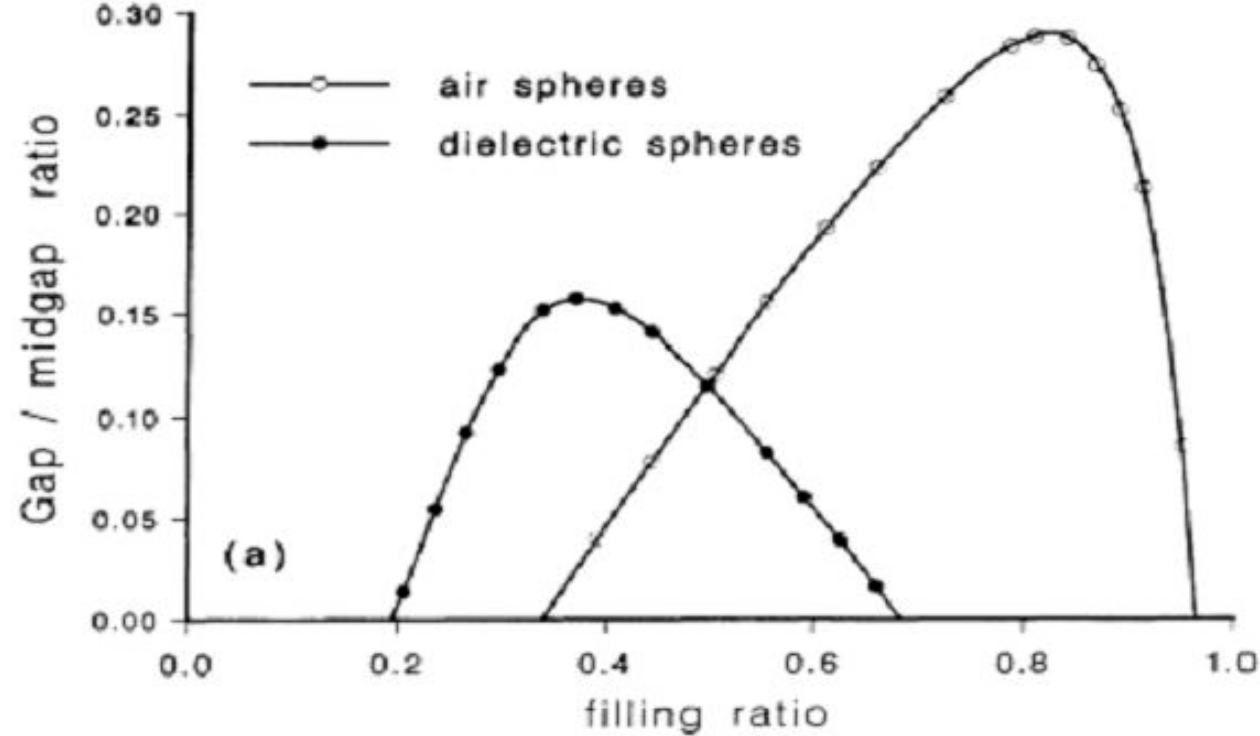
Ho, Chan and Soukoulis, PRL 65, 3152 (1990)

Diamond lattice gives the largest photonic band gap



Ho, Chan and Soukoulis, PRL 65, 3152 (1990)

Diamond lattice



Ho, Chan and Soukoulis, PRL 65, 3152 (1990)

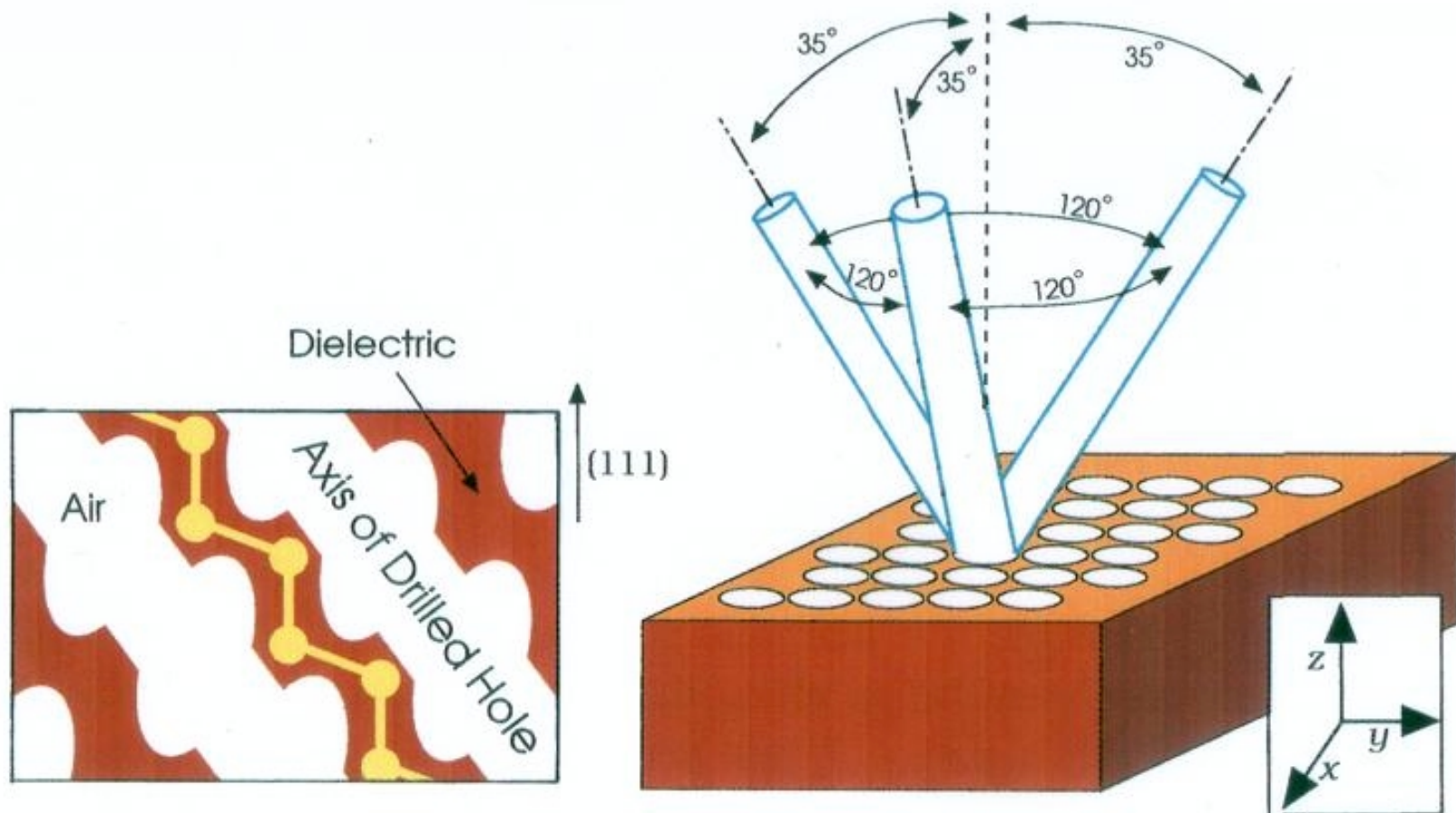


Figure 4 The method for constructing Yablonovite: a slab of dielectric is covered by a mask consisting of a triangular array of holes. Each hole is drilled three times, at an angle of 35.26° away from the normal, and spread out 120° on the azimuth. This results in a three dimensional structure whose $(1\bar{1}0)$ cross section is shown on the left. The dielectric connects the sites of a diamond lattice, shown schematically in yellow. The dielectric veins oriented vertically (111) have greater width than those oriented diagonally $(1\bar{1}\bar{1})$.

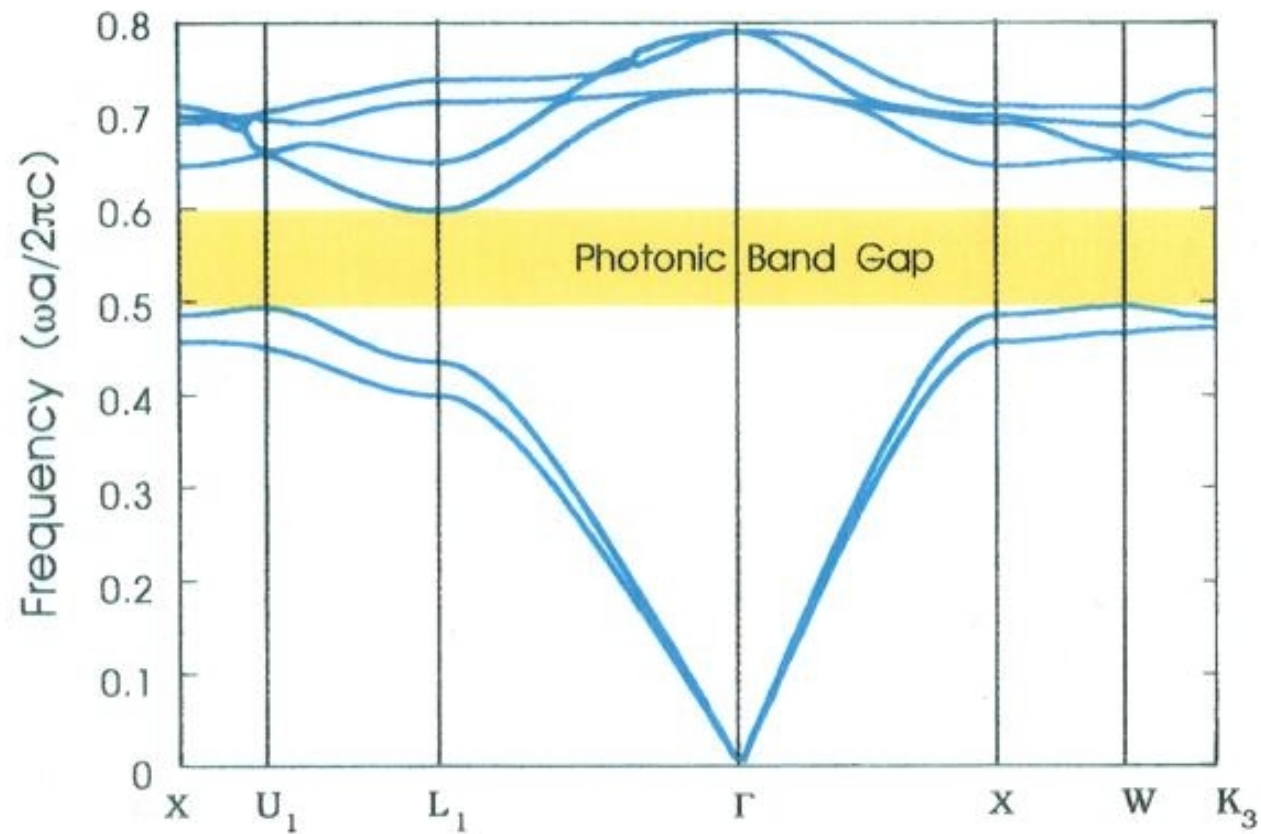


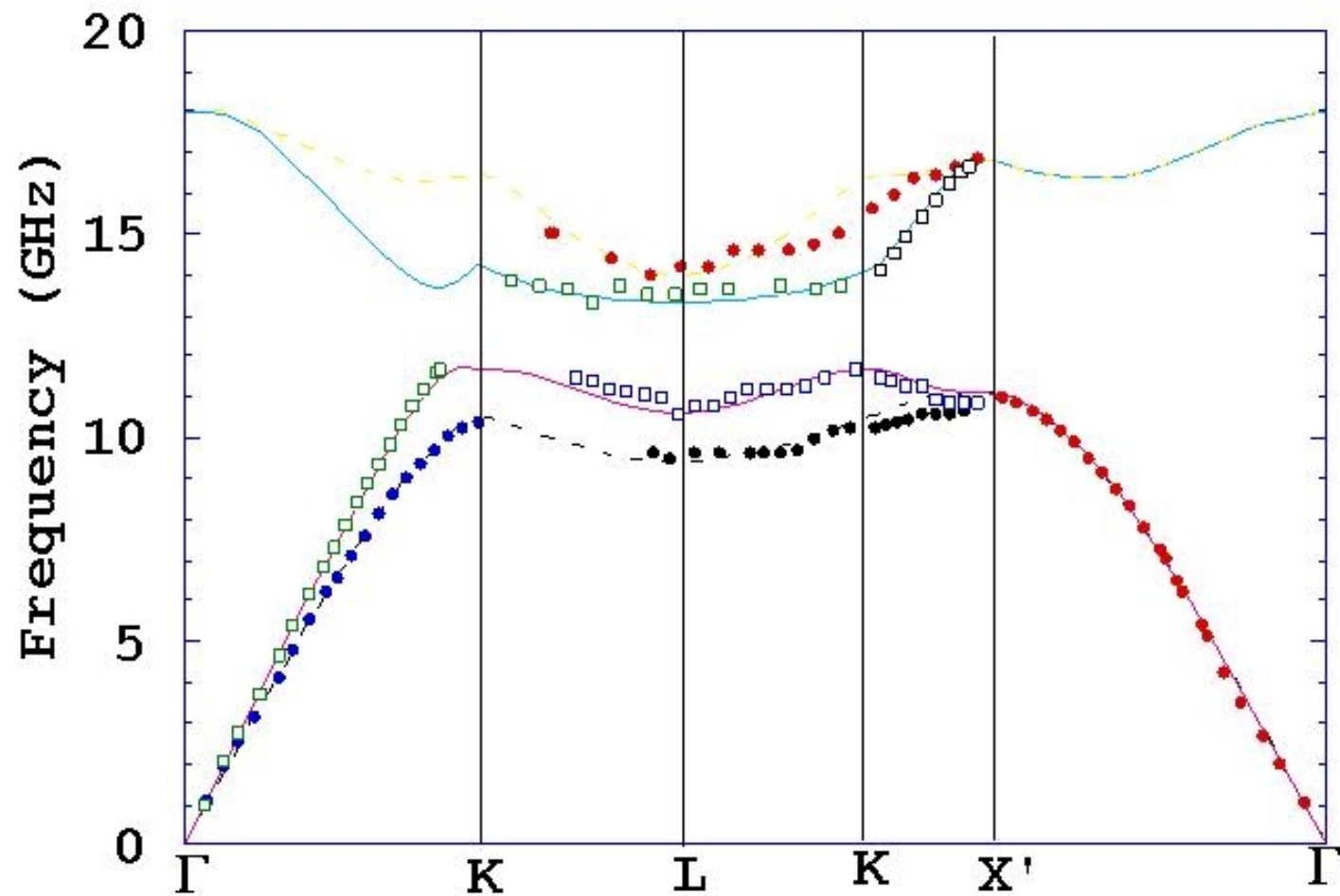
Figure 5 The photonic band structure for the six lowest bands of Yablonovite. A detailed discussion of this band structure can be found in Yablonovitch et al. (1991a).

III. CALCULATIONAL TECHNIQUES

- PW (PLANE WAVE) : *PERIODIC, NO LOSSES*
- TM (TRANSFER MATRIX) :
PARTIAL PERIODICITY OR FINITE N° CHAN.
- MS (MULTIPLE SCATTERING): *LOW FREQUENCIES*
- FDTD (FINITE DIFFERENCE TIME DOMAIN) ✓

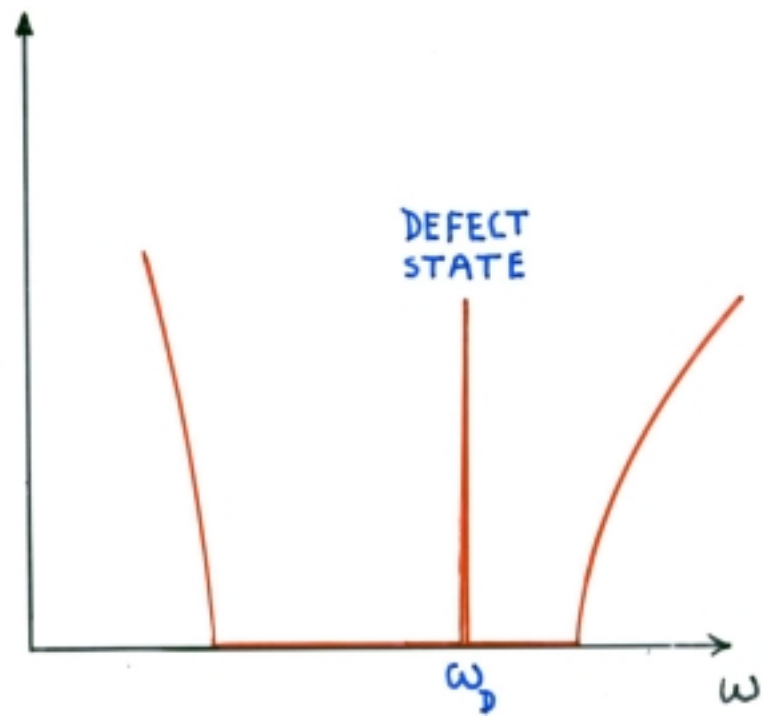
NO UNCONTROLLABLE APPROXIMATIONS
NO ADJUSTABLE PARAMETERS

Theory and experiment are in excellent agreement

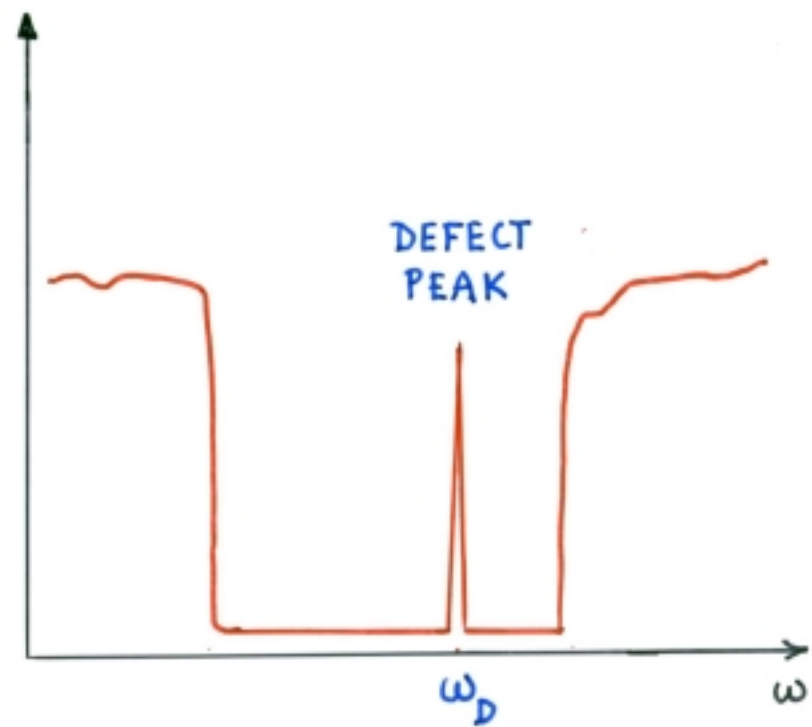


IV. DOPING, MINI STOP BANDS

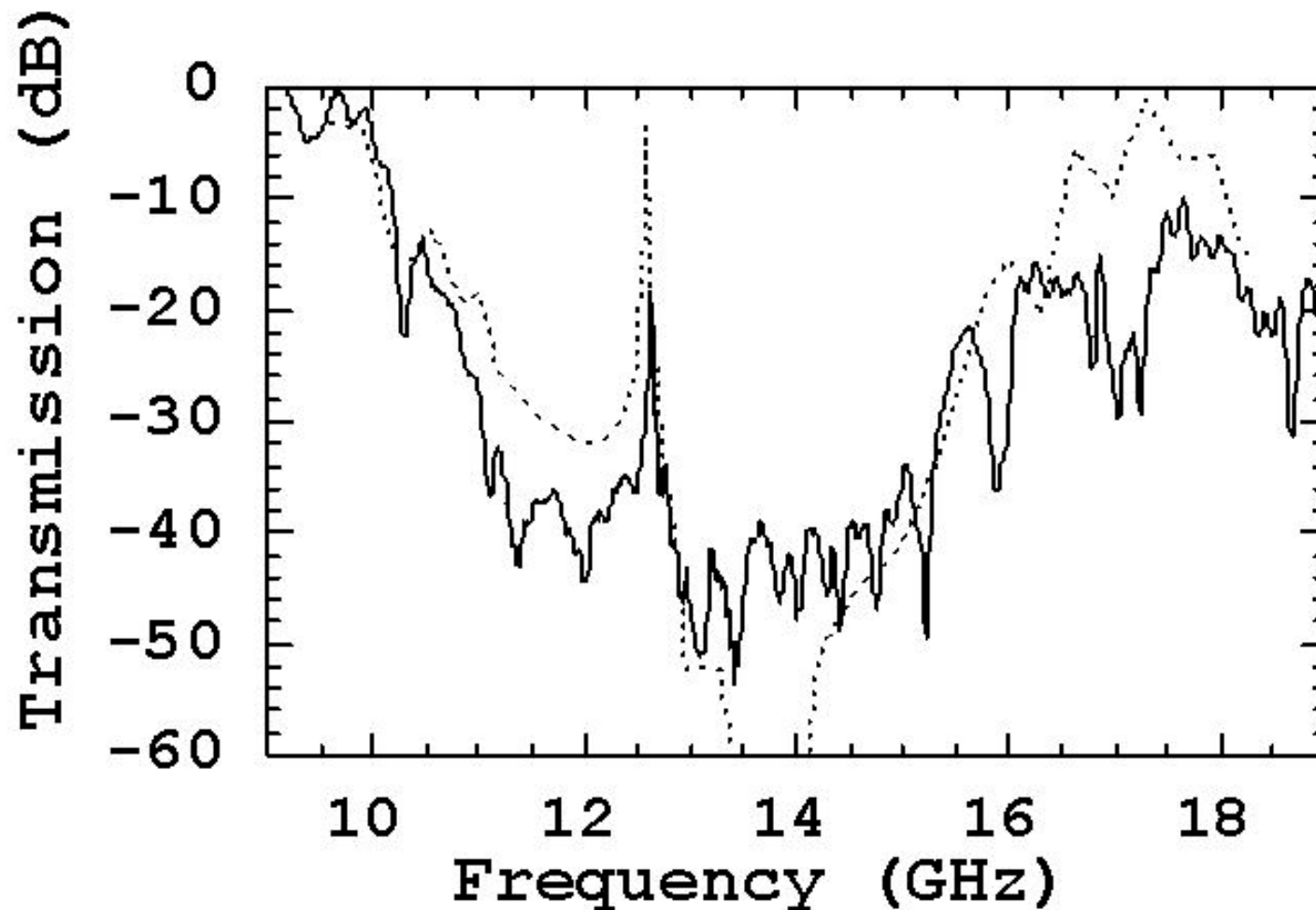
DOS



TRANSMISSION



Theoretical (dashed line) and experimental (solid line) transmission characteristics of the defect structure



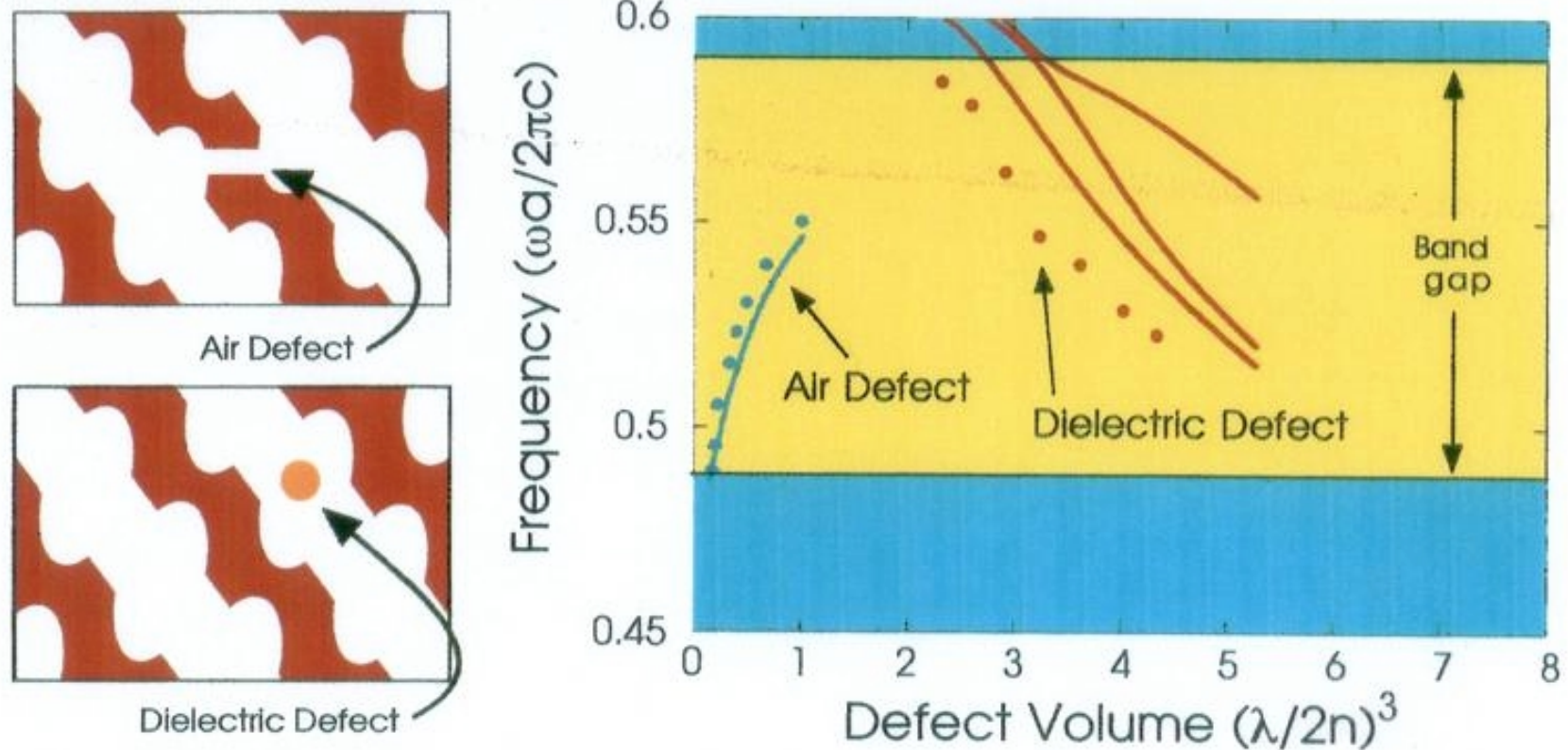
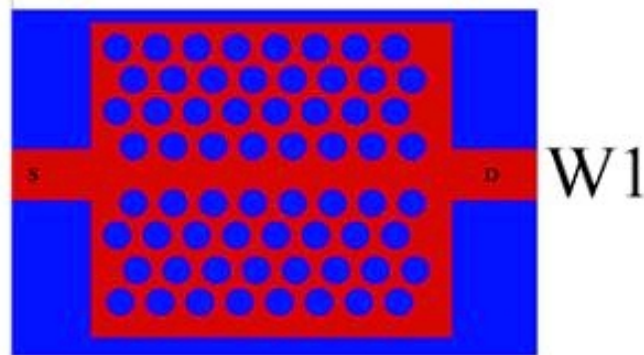


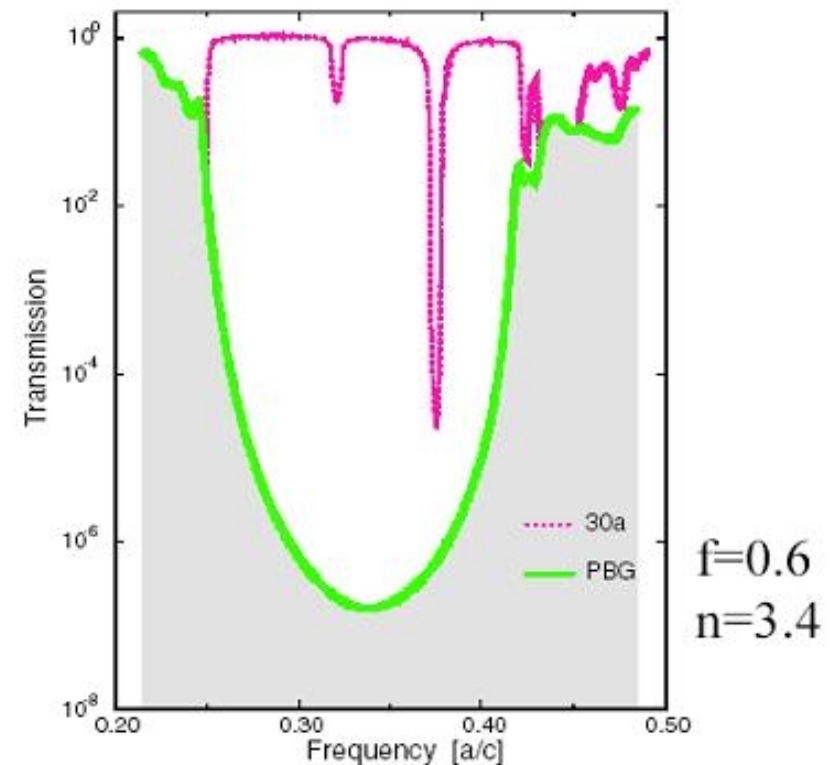
Figure 6 Plotted are the frequencies of the localized modes of Yablonovite as the defect size varies. The dots indicate measured values (Yablonovitch et al. 1991b), the lines indicate computed values (Meade et al. 1993a), and the yellow region is the photonic band gap. The modes on the blue line result from an air defect, while the modes on the red lines result from a dielectric defect. The defect volume is expressed in units of $(\lambda/2n)^3$, where λ is the midgap vacuum wavelength and n is the index of refraction of the dielectric material.

Mini Stop Bands in PC Waveguides:

Physical origin? Dependence on the parameters of the problem (guide length, air filling ratio, losses)?



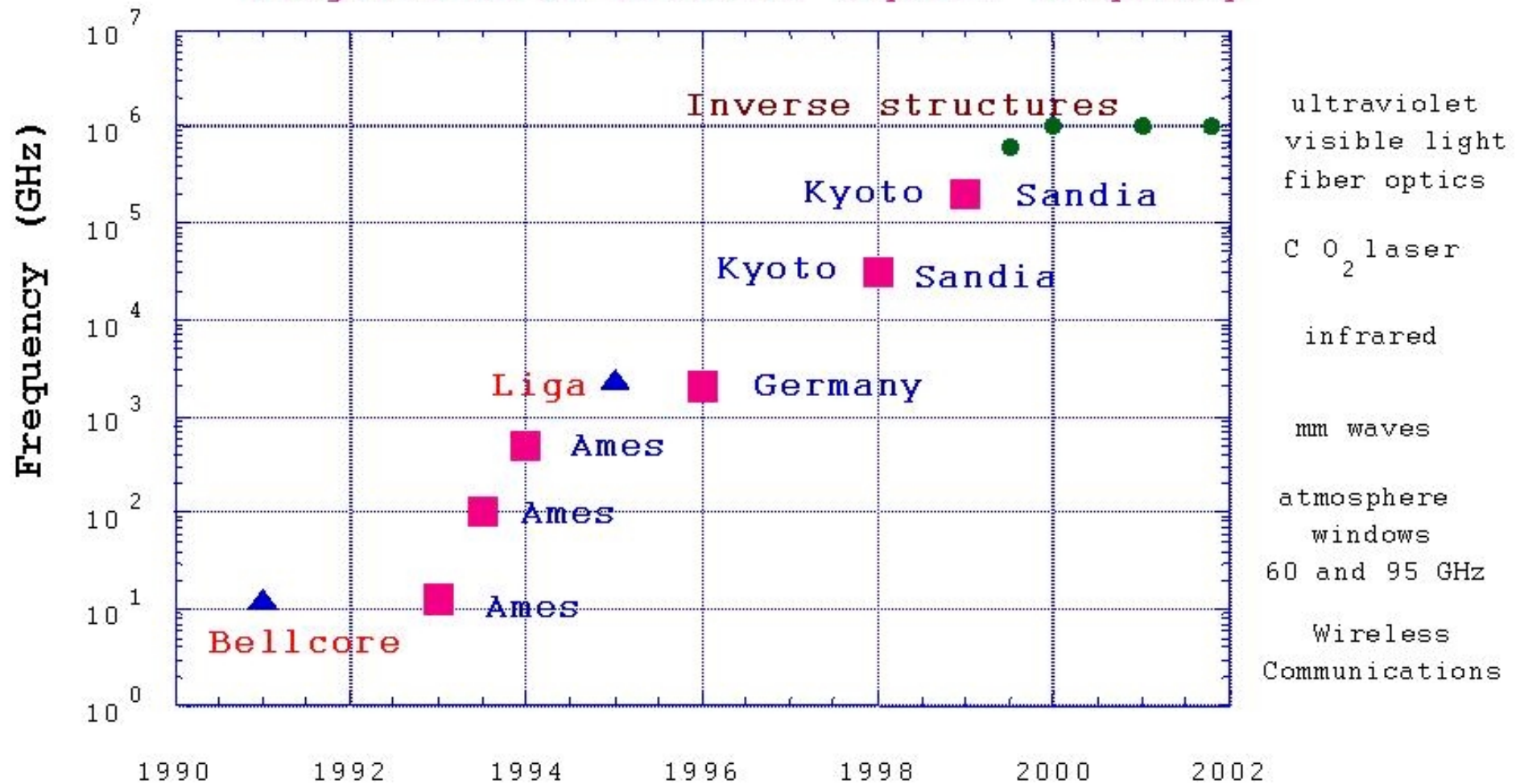
- Origin: Coupling of the guided modes
- For W1 and $n=3.4$ they appear for high f ($f > 40\%$)
- Losses (conductivity) make them broader and less deep



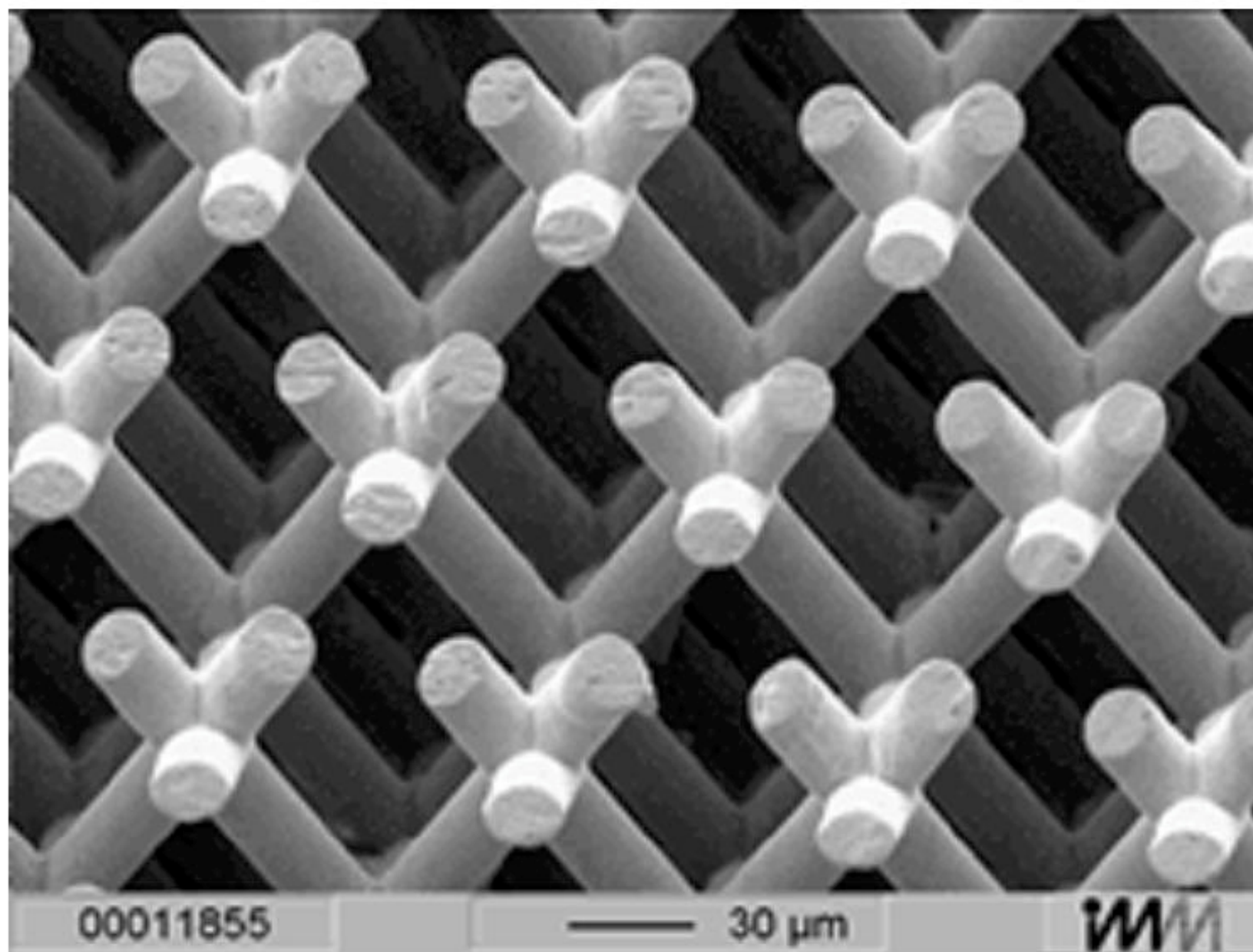
Agio and Soukoulis, PRE, 64, 055603R (2001)

V. RECENT DEVELOPMENTS

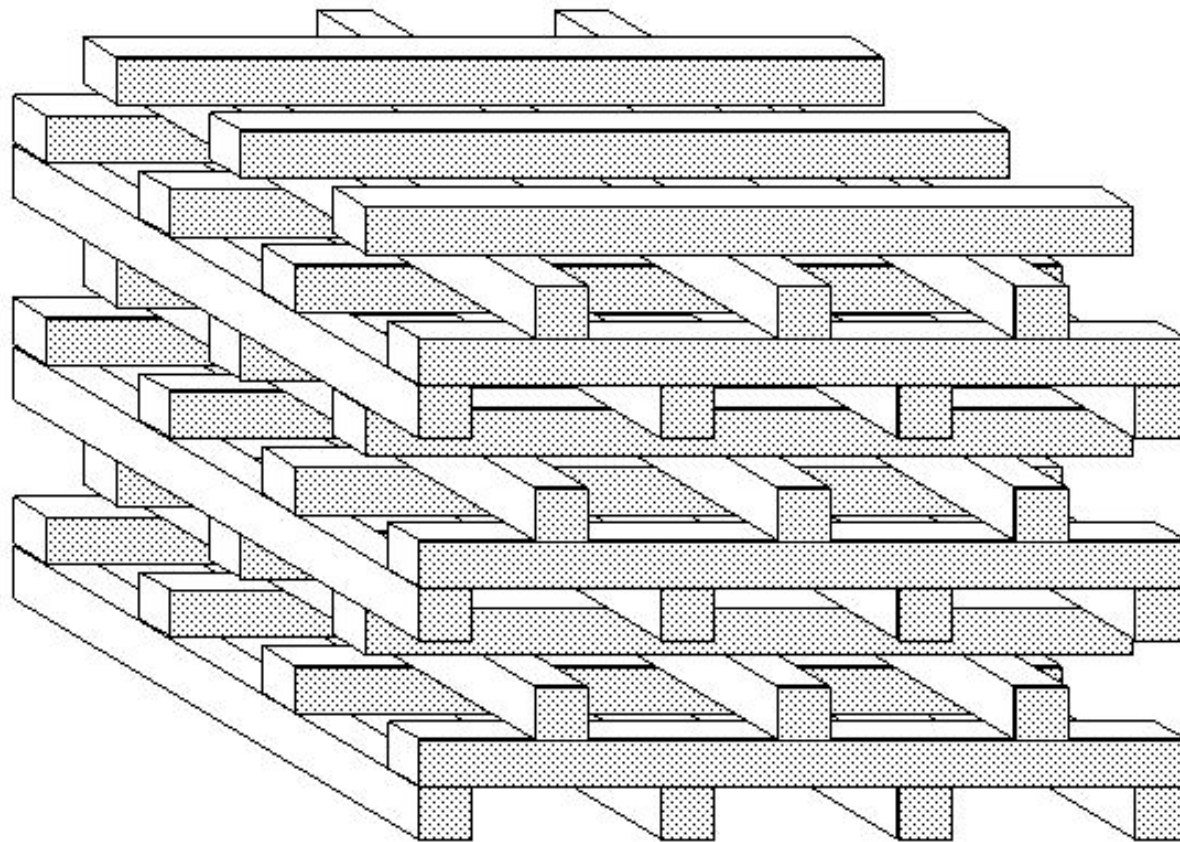
Progress in 3d Photonic crystal frequency



Fabrication of 3-cylinder structure by LIGA technique



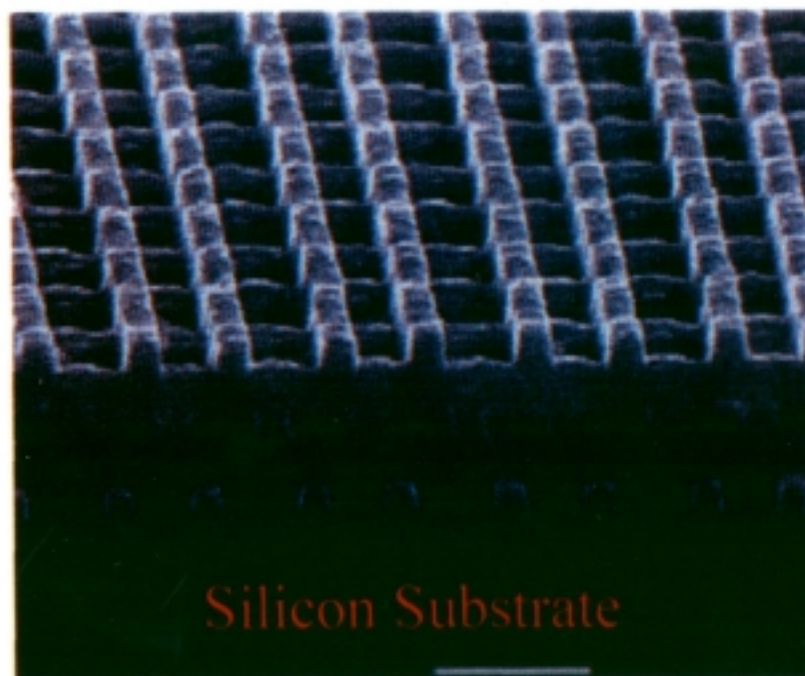
A new easy-to-build structure with a full photonic band gap



Iowa State layer-by-layer structure:

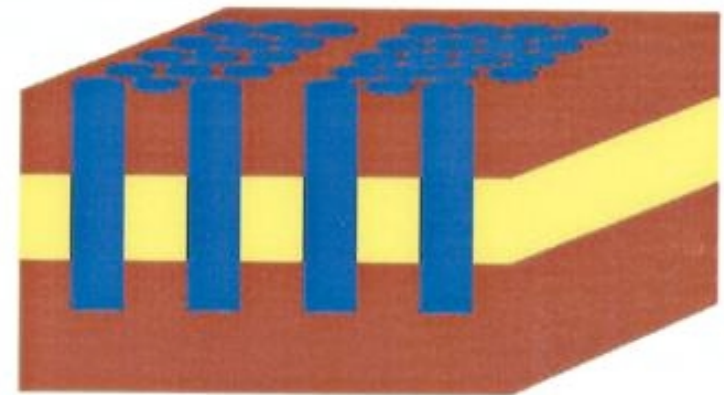
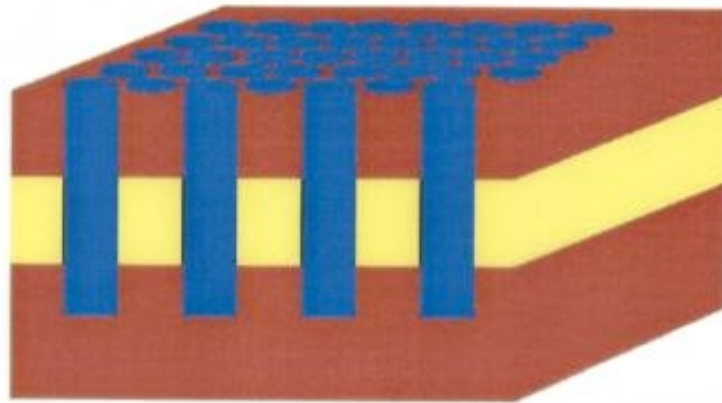
Science News 144, 199 (1993); Solid State Comm. 89, 413 (1994)
Phys. Rev. B 50, 1945 (1994)

A 3-D Silicon Photonic Crystal Operating at $\lambda=1.55\mu\text{m}$

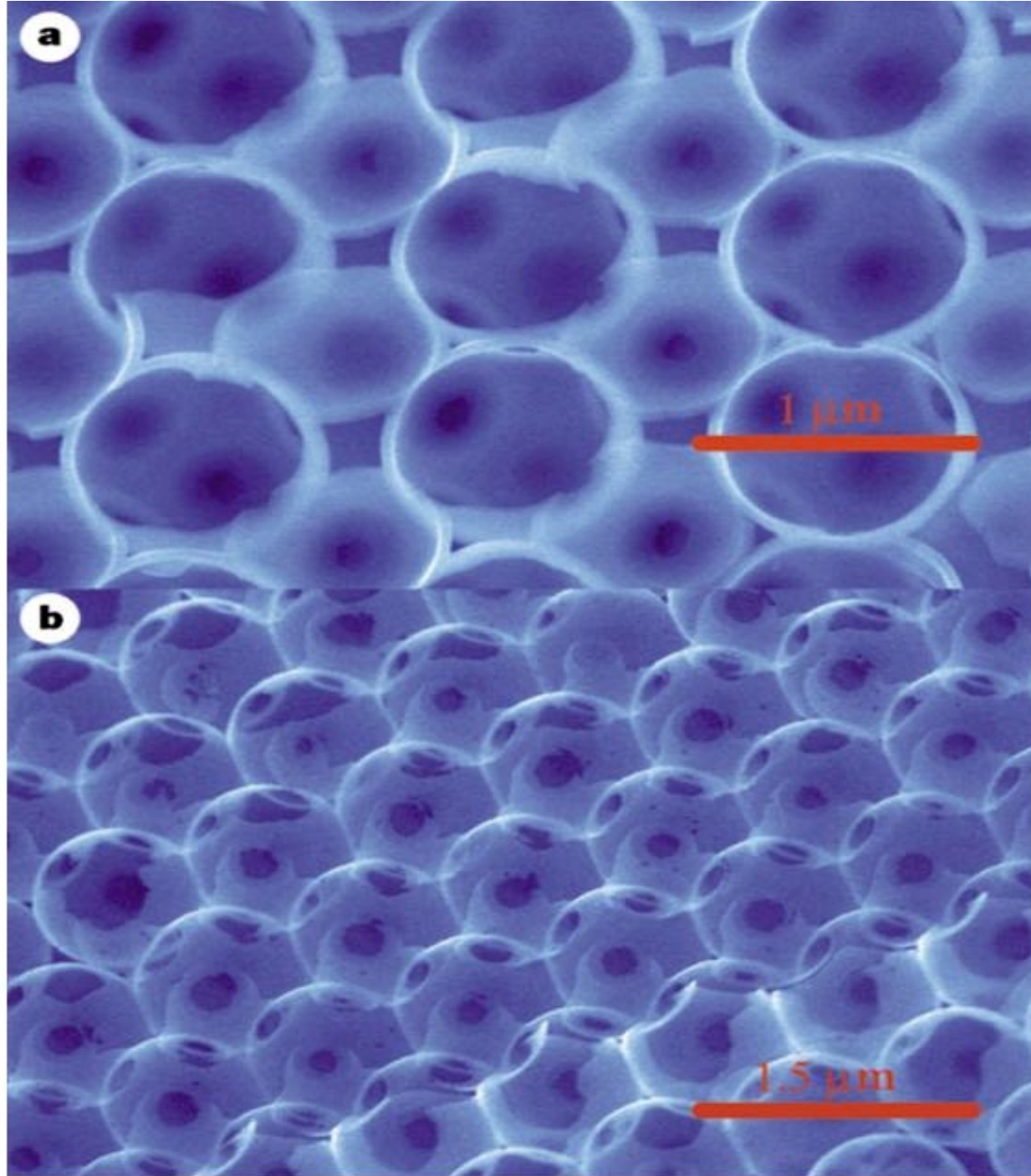


Sandia
National
Laboratories

2D PCs IN LAYERED STRUCTURES **(SANDWICHED OR SUSPENDED)**

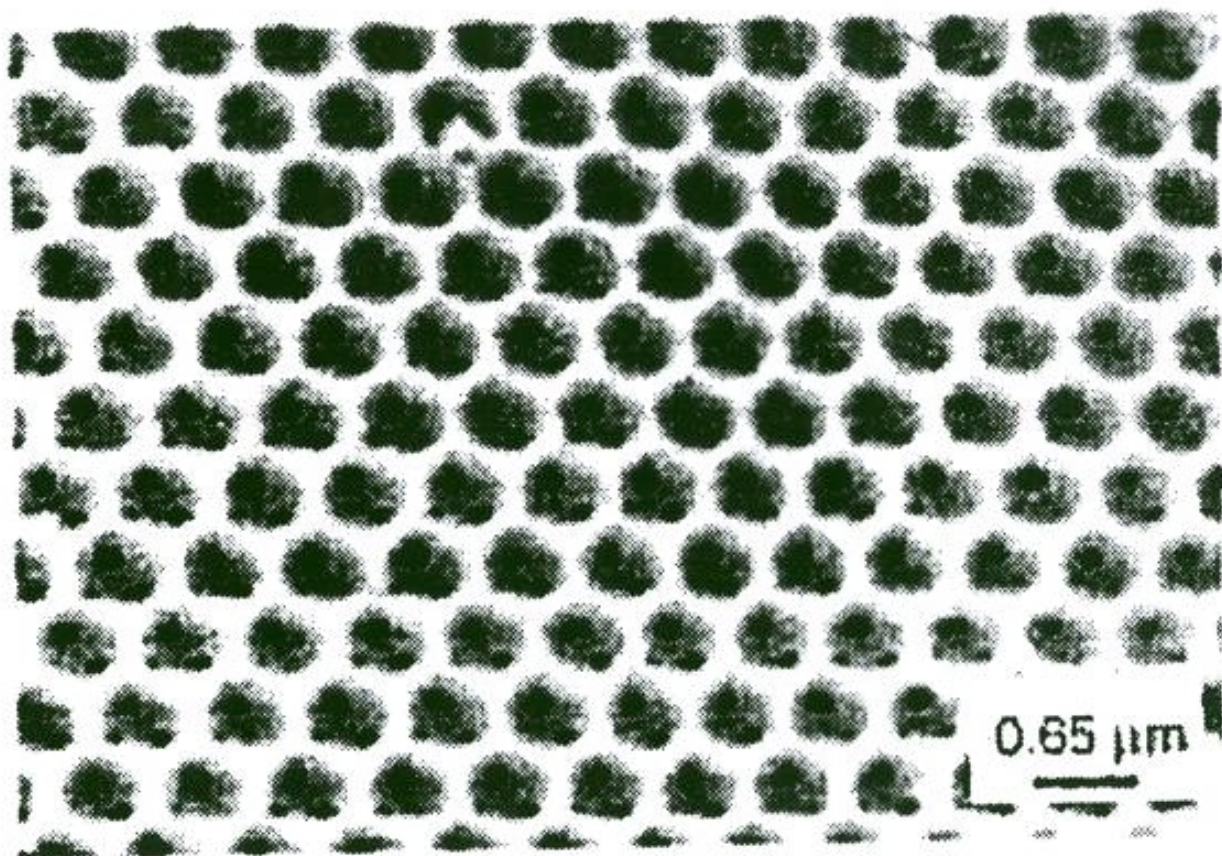


In – plane:	PC
Out of plane:	usual waveguide

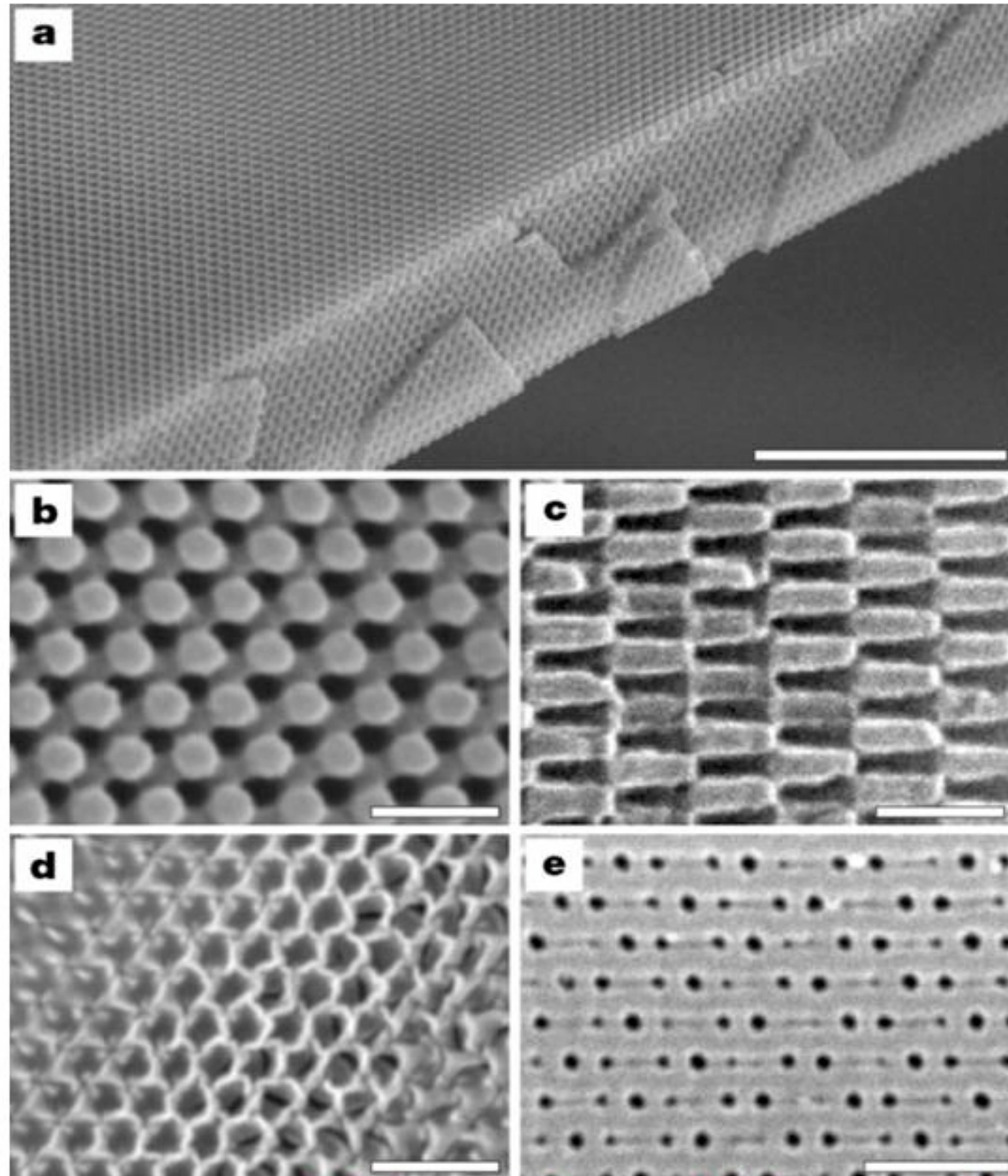


Silicon inverted
opals

A. Blanco et. al. Nature 405, 437 (2002)



Fabrication of photonic crystals by holographic lithography



M.Campbell et. al. Nature, 404, 53 (2000)

MATERIALS*

- **SEMICONDUCTORS**
Si, Ge, GaAs, AlGaAs/GaAs
- **OXIDES**
SiO₂, TiO
- **POLYMERS**
PMMA
- **PROTEIN CRYSTALS**
- **METALS**

FABRICATION TECHNIQUES*

- **LITHOGRAPHIC METHODS**
e-BEAM
ION BEAM (FOCUSED)
DRY ETCHING
fs LASER PULSES

+
- **X-RAY LITHOGRAPHY**
(LIGA)
- **HOLOGRAPHIC LITHOGRAPHY**

- **OTHER**



- **SELF – ASSEMBLED
COLLOIDAL
PARTICLES
(SiO_2 , PMMA) OR
CLUSTER OF PARTICLES AS
TEMPLATES**

*** SEE, e.g.**

**C.M. SOUKOULIS, ed., PHOTONIC CRYSTAL...
...IN THE 21st CENTURY**

**PHOTONIC NANOSTRUCTURES,
SAN DIEGO, Oct. 24-25, 2002**

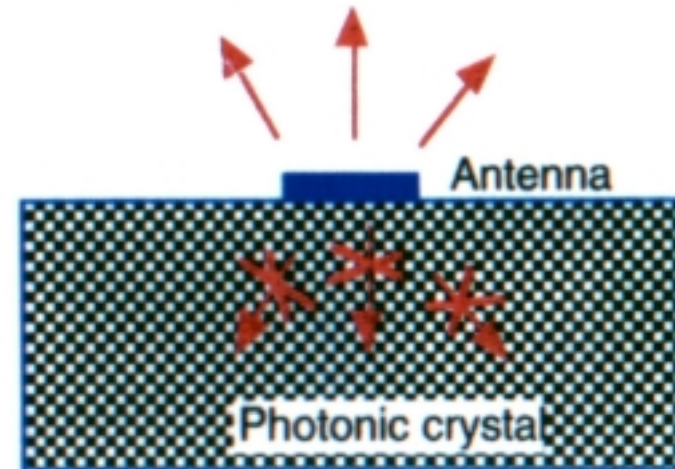
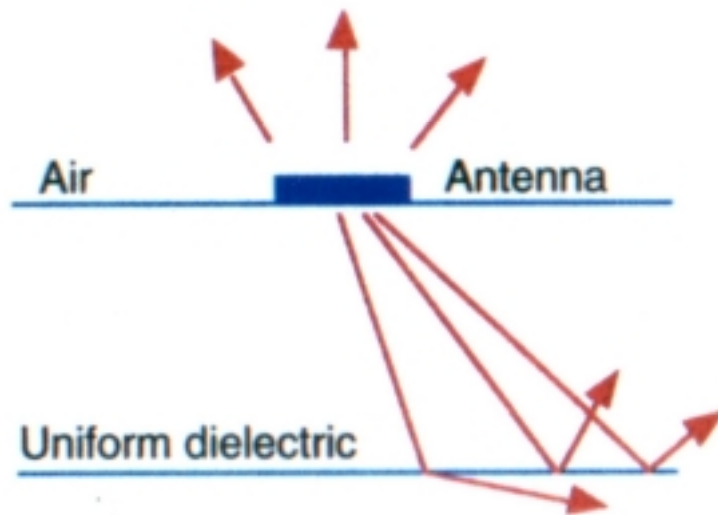
APPLICATIONS FOR PHOTONIC CRYSTALS

DEVICE	DESCRIPTION	STATUS
OPTICAL FIBERS	2-D band-gap material stretched along the third dimension	Early versions already commercialized
NANOSCOPIC LASERS	World's tiniest optical cavities and tiniest lasers; formed in a thin-film 2-D band-gap material	Demonstrated in the lab
ULTRAWHITE PIGMENT	Incomplete 3-D band-gap material, usually patterned as opal structure	Demonstrated; low-cost manufacturing methods under development
RADIO-FREQUENCY ANTENNAS, REFLECTORS	Uses inductors and capacitors in place of ordinary dielectric materials	Demonstrated for magnetic resonance imaging and antennas
LIGHT-EMITTING DIODES	Photonic band-gap structure can extract light very efficiently (better than 50%)	Demonstrated, but must compete with other methods of achieving the same goal
PHOTONIC INTEGRATED CIRCUITS	2-D thin films can be patterned like conventional integrated circuits to make channel filters, modulators, couplers and so on	Under development

E. YABLONOVITCH, SCIENTIFIC AMERICAN, Dec. 2001

Applications: Microwaves

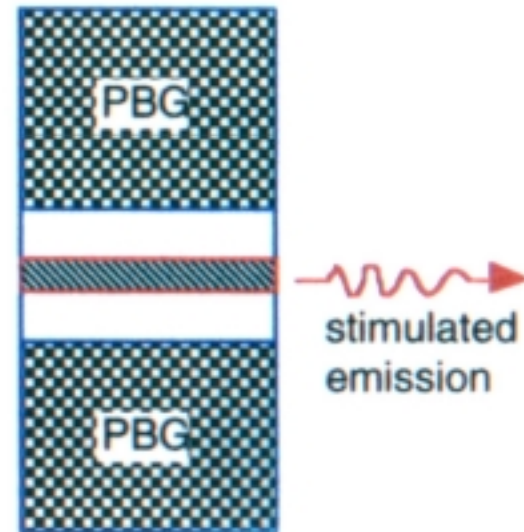
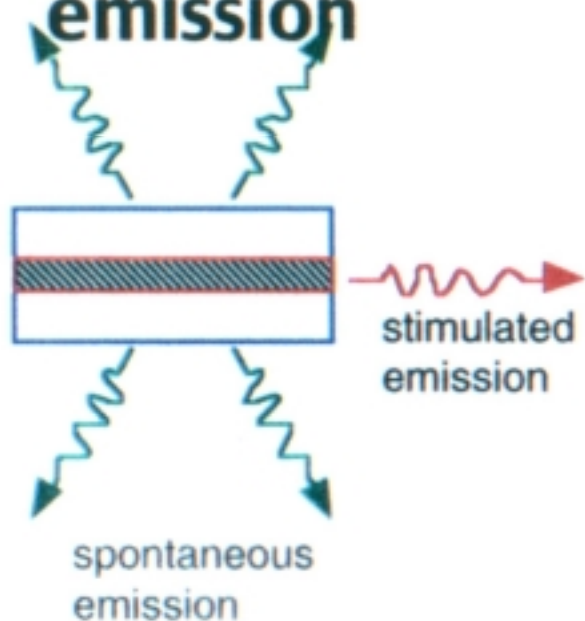
- **Efficient antennas, waveguide, isolators etc.**



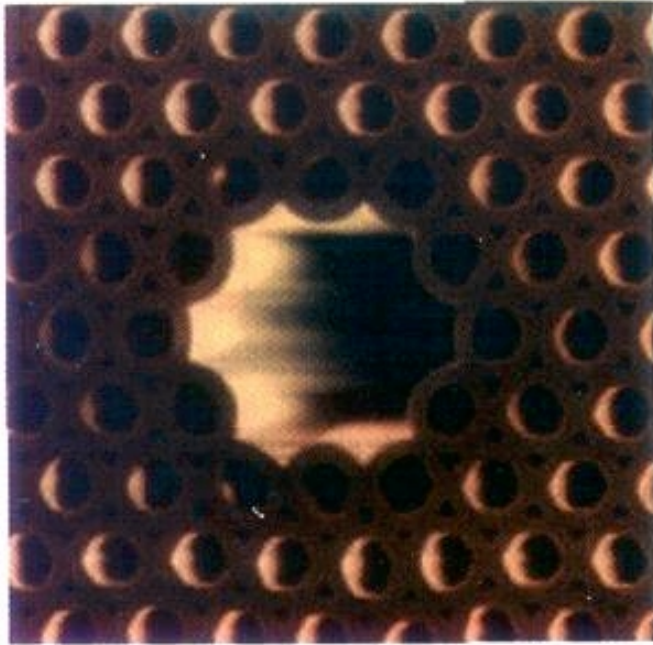
Efficient planar antenna

Applications: Optical range

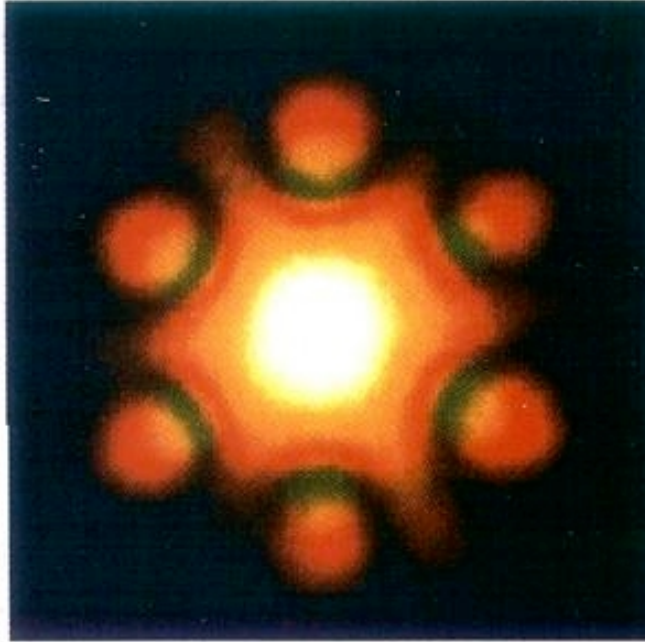
■ Suppression of spontaneous emission



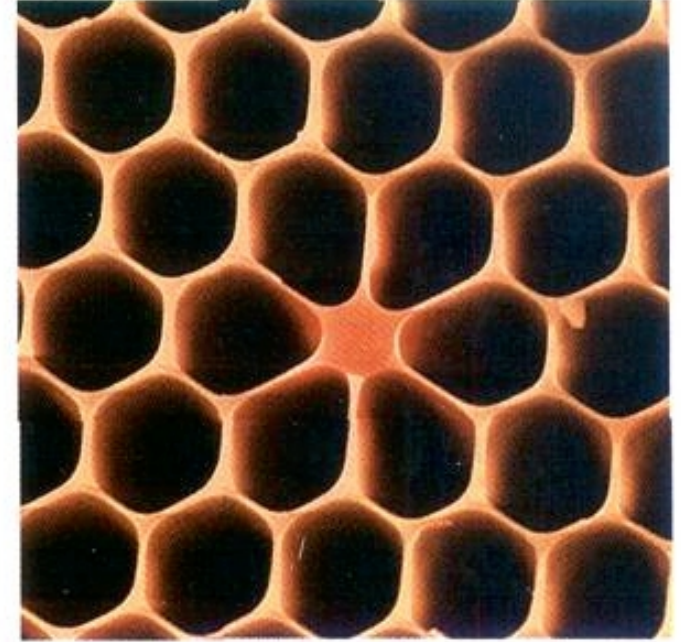
Low-threshold lasers, single-mode LEDs, mirrors, optical filters and waveguides



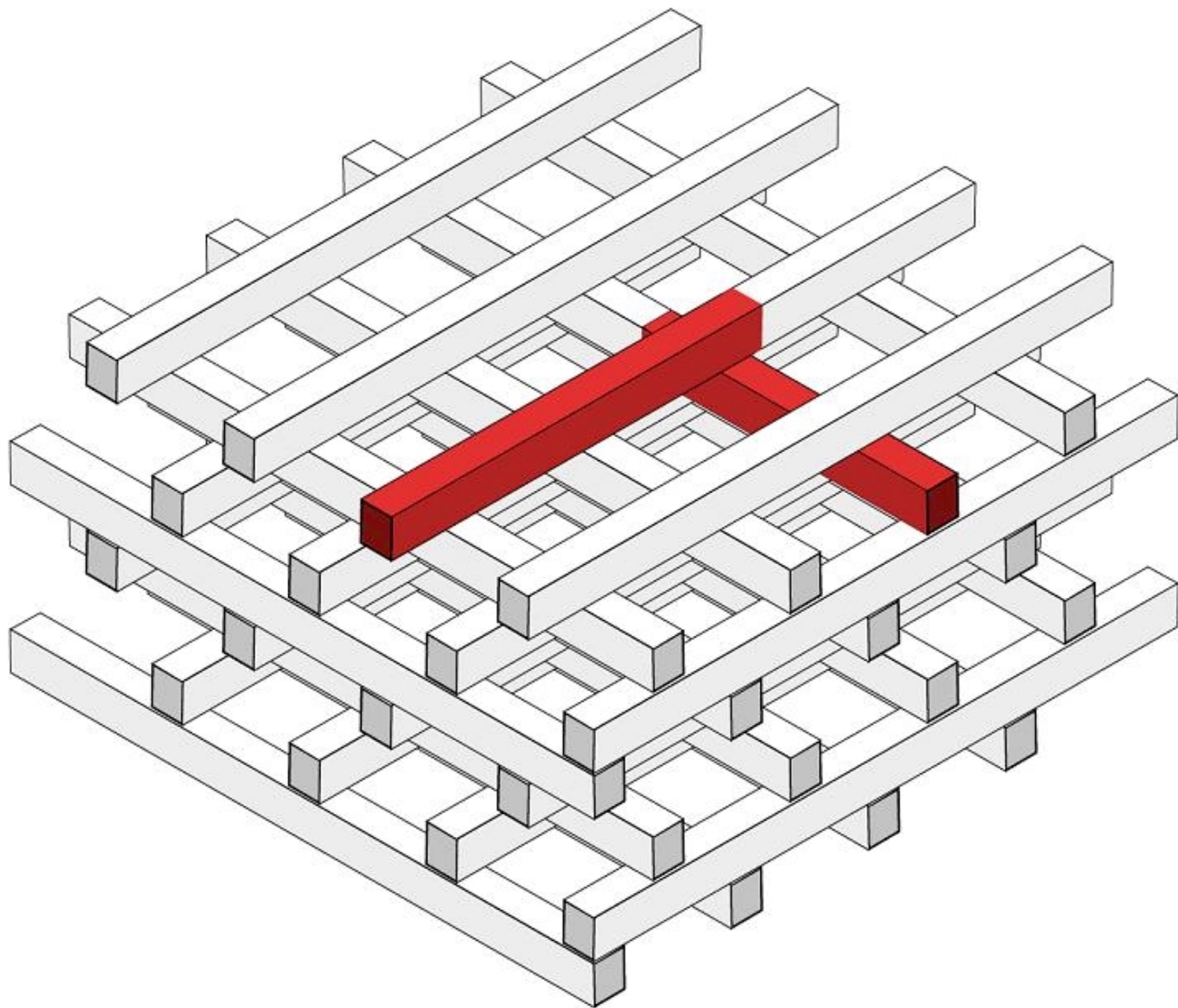
OPTICAL FIBERS can use the photonic band-gap principle to guide light. The cladding of several hundred silica capillary tubes forms an optical band-gap material that confines light to the central hole, which is about 15 microns in diameter (*left*). In the design at the right, in which the light is



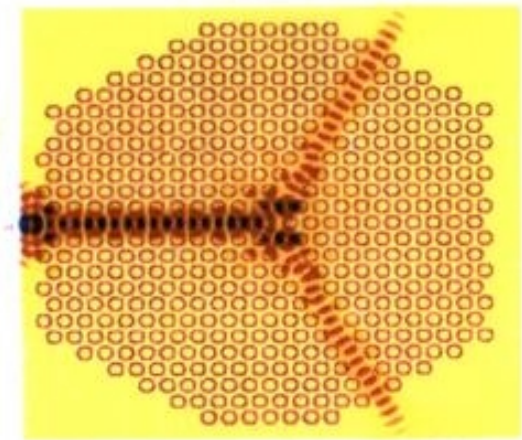
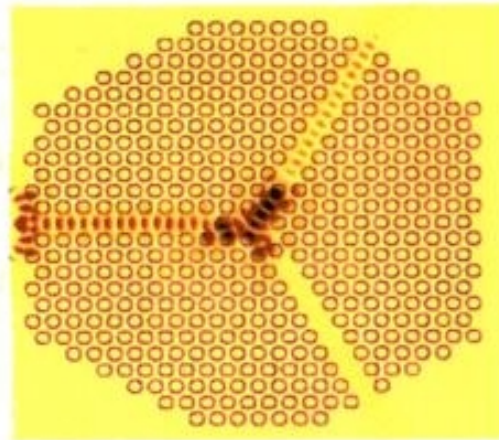
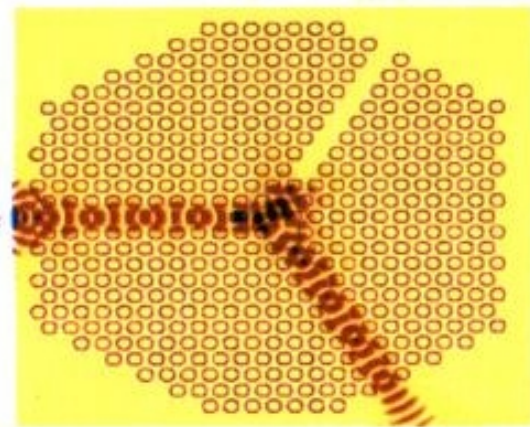
confined to the two-micron solid core, the fiber is highly nonlinear, which can be useful for switching and shaping light pulses. In the center, a pattern of colors illustrates how the confinement property of a band-gap fiber varies for different wavelengths of light.



E. YABLONOVITCH



Design of Y-splitters and Y-switches

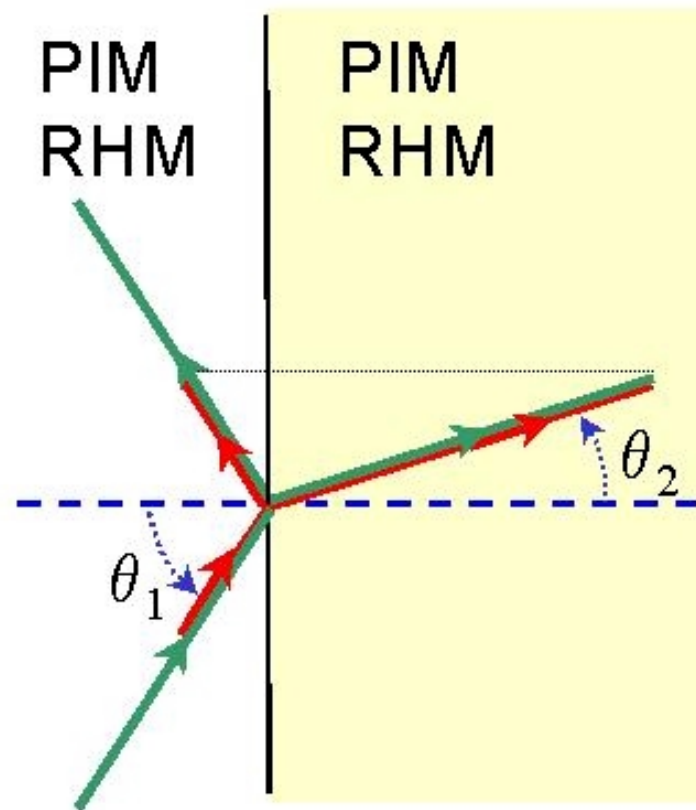


VI. PCs AS NEGATIVE INDEX MATERIALS (NIM)

If $\epsilon < 0$ AND $\mu < 0$ (VESELAGO, 1968)

- $\epsilon\mu > 0 \Rightarrow$ PROPAGATION
- $\vec{k}, \vec{E}, \vec{H}$ Left Handed (LHM) $\Rightarrow \mathbf{S} = c(\mathbf{E} \times \mathbf{H})/4\pi$
opposite to \vec{k}
- Snell's law with $n \equiv -\sqrt{\epsilon\mu} < 0$ (NIM)
- \vec{v}_g opposite to \vec{k}
- Flat lenses
- Super lenses

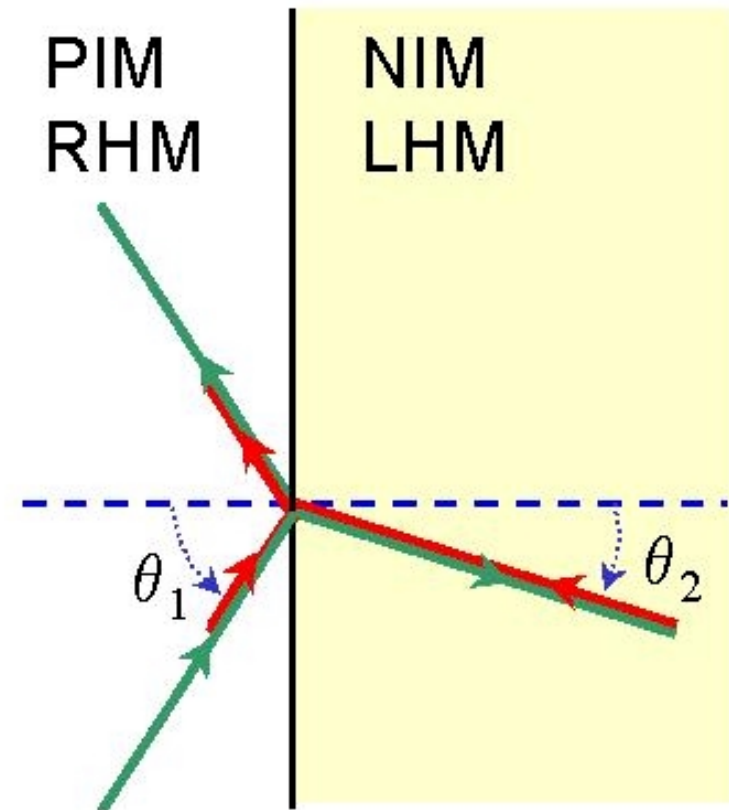
“Reversal” of Snell’s Law



(1)

PIM
RHM

(2)



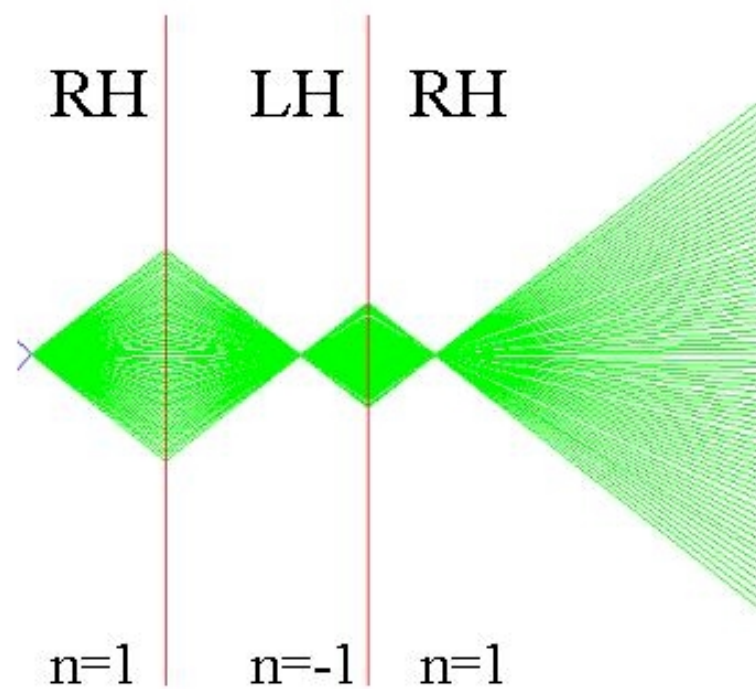
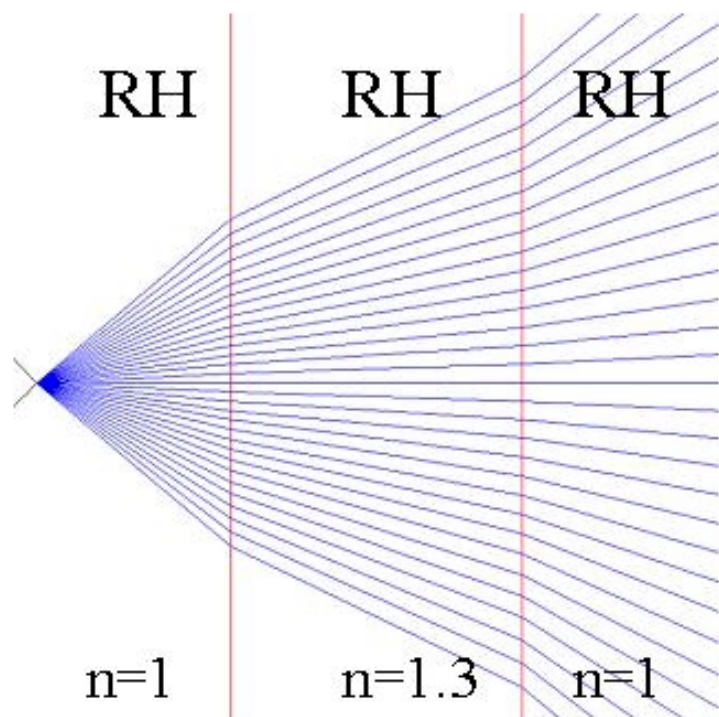
(1)

NIM
LHM

(2)



Focusing in a Left-Handed Medium



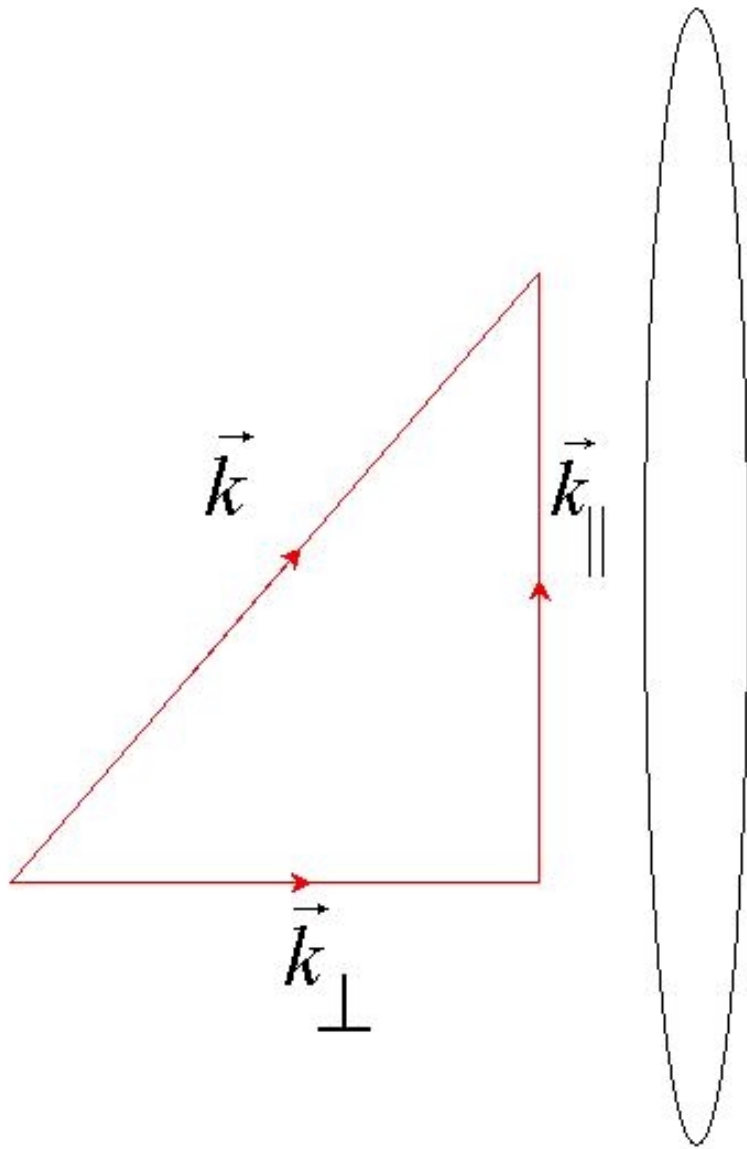
Super lenses

$$\omega^2 = c^2 (k_{\parallel}^2 + k_{\perp}^2) \Rightarrow \text{IF } k_{\parallel} > \omega/c \Rightarrow$$

$$k_{\perp} \text{ IMAGINARY } e^{ik_{\perp} r_{\perp}} \sim e^{-|k_{\perp}| r_{\perp}}$$

\Rightarrow Wave components with decay, i.e. are lost ,

then $\Delta_{\max} \approx \lambda$



IF $n < 0$, PHASE CHANGES SIGN

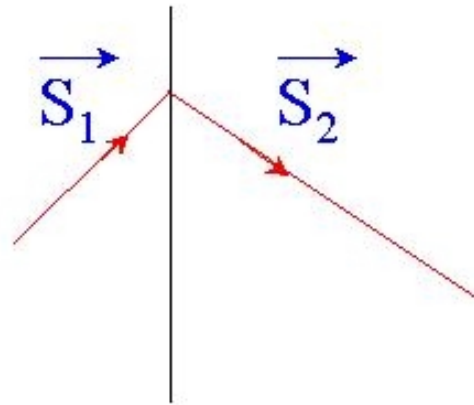
$$\underline{e^{ik_{\perp}} e^{ik_{\parallel}} \sim e^{|k_{\perp}|} e^{ir_{\perp}}}$$

IF $\underline{k_{\perp}}$ IMAGINARY

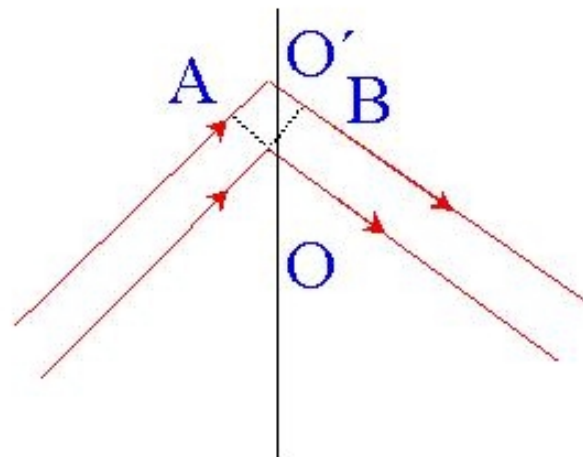
THUS $\underline{k_{\parallel} > \omega / c}$ ARE NOT LOST !!!

OBJECTIONS

- **ABSORPTION INVALIDATES CONCLUSIONS**
- **PARALLEL MOMENTUM IS NOT CONSERVED**



- **CAUSALITY IS VIOLATED**



- **FERMAT's PRINCIPLE**

$$\int n dl \text{ minimum (?)}$$

- **SUPERLENSING IS NOT POSSIBLE**

Materials with $\epsilon < 0$ and $\mu < 0$

Photonic Crystals

$$\vec{v}_g \text{ opposite to } \vec{k}$$

$$\langle \vec{S} \rangle = \langle u \rangle \vec{v}_g$$

$$\langle \vec{S} \rangle \text{ opposite to } \vec{k}$$

$$\alpha \equiv 1 + \frac{d \ell n |n|}{d \omega}$$

$$n \alpha > 0$$

$$\vec{v}_g = \frac{\vec{v}_p}{\alpha}, \vec{v}_p = \frac{c}{|n|} \vec{k}_0$$

$$\vec{v}_g \text{ opposite to } \vec{k}$$

$$\langle \langle \vec{S} \rangle \rangle = \langle \langle u \rangle \rangle \vec{v}_g$$

$$\langle \langle \vec{S} \rangle \rangle \text{ opposite to } \vec{k}$$

$$\alpha \equiv 1 + \frac{d \ell n |n|}{d \omega}, |n| = \frac{c |\vec{k}|}{\omega}$$

$$n \alpha > 0 \Rightarrow n = \pm |n|, \begin{matrix} +, \alpha > 0 \\ -, \alpha < 0 \end{matrix}$$

$$\vec{v}_g = \frac{\vec{v}_p}{\alpha}, \vec{v}_p = \frac{c}{|n|} \vec{k}_0$$

$$\langle \vec{p} \rangle = \frac{\epsilon \mu}{c^2} \langle \vec{S} \rangle + \frac{\vec{k}}{8\pi} \left[\frac{\partial \epsilon}{\partial \omega} \langle \vec{E}^2 \rangle + \frac{\partial \mu}{\partial \omega} \langle \vec{H}^2 \rangle \right] \quad \langle \vec{p} \rangle = \frac{\langle u \rangle}{\omega} \vec{k}$$

REPLY TO THE OBJECTIONS

- PCs HAVE PRACTICALLY ZERO ABSORPTION
- MOMENTUM CONSERVATION IS NOT VIOLATED
- FERMAT's PRINCIPLE is OK

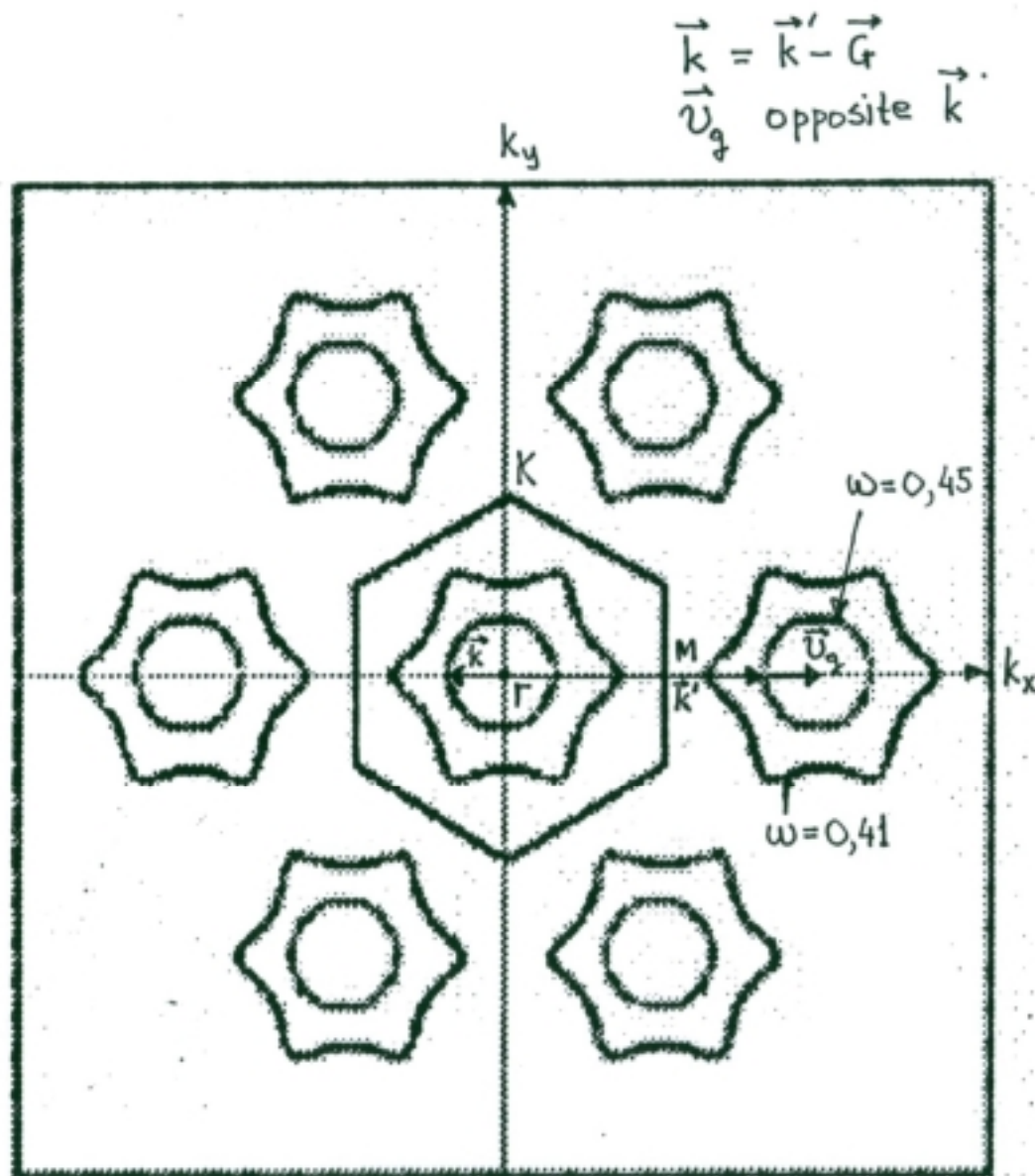
$\int n dl$ extremum

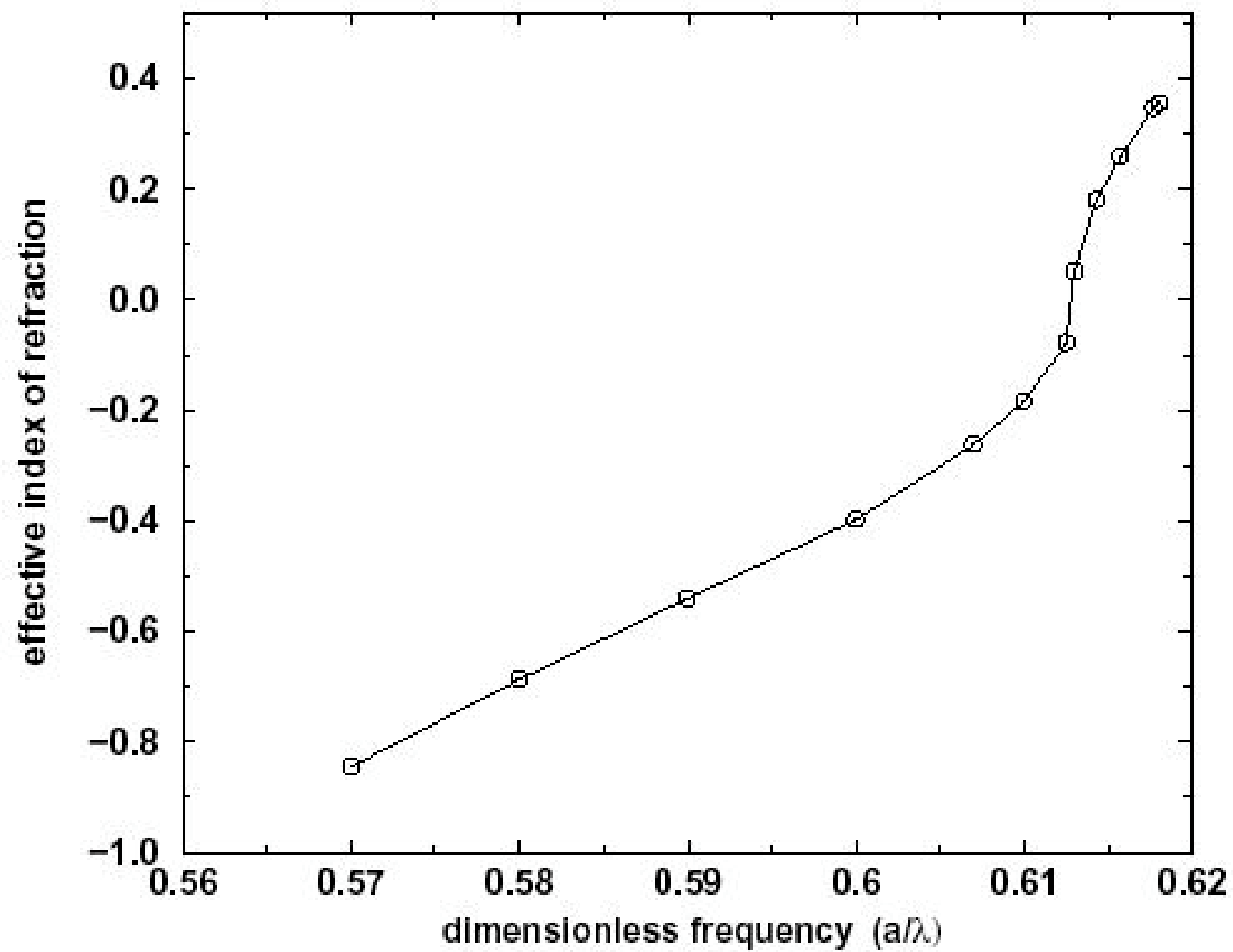
- CAUSALITY IS NOT VIOLATED
- SUPERLENSING POSSIBLE BUT LIMITED TO A CUTOFF k_c OR $1/L$

M. Notomi, PR B 62, 10696 (2000)

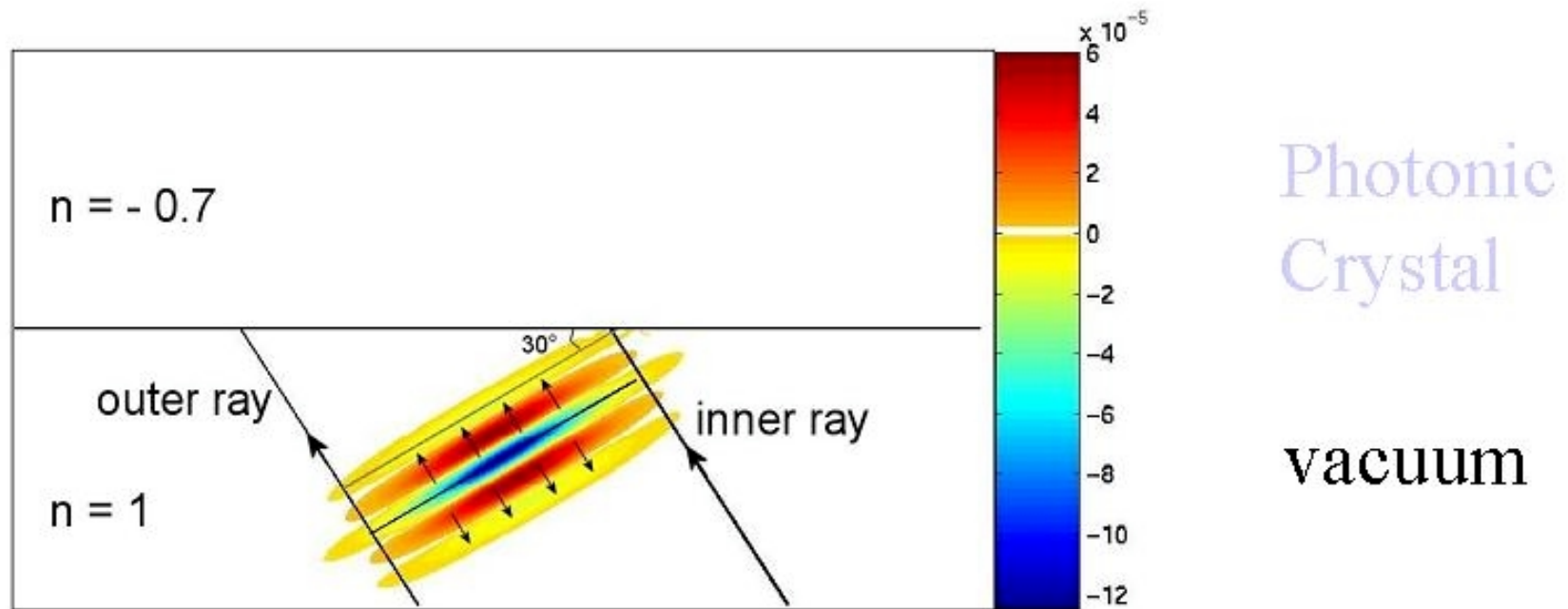
2D GaAs air-hole
($d=0,7a$)

ω in units of
 $\omega a/2\pi c$



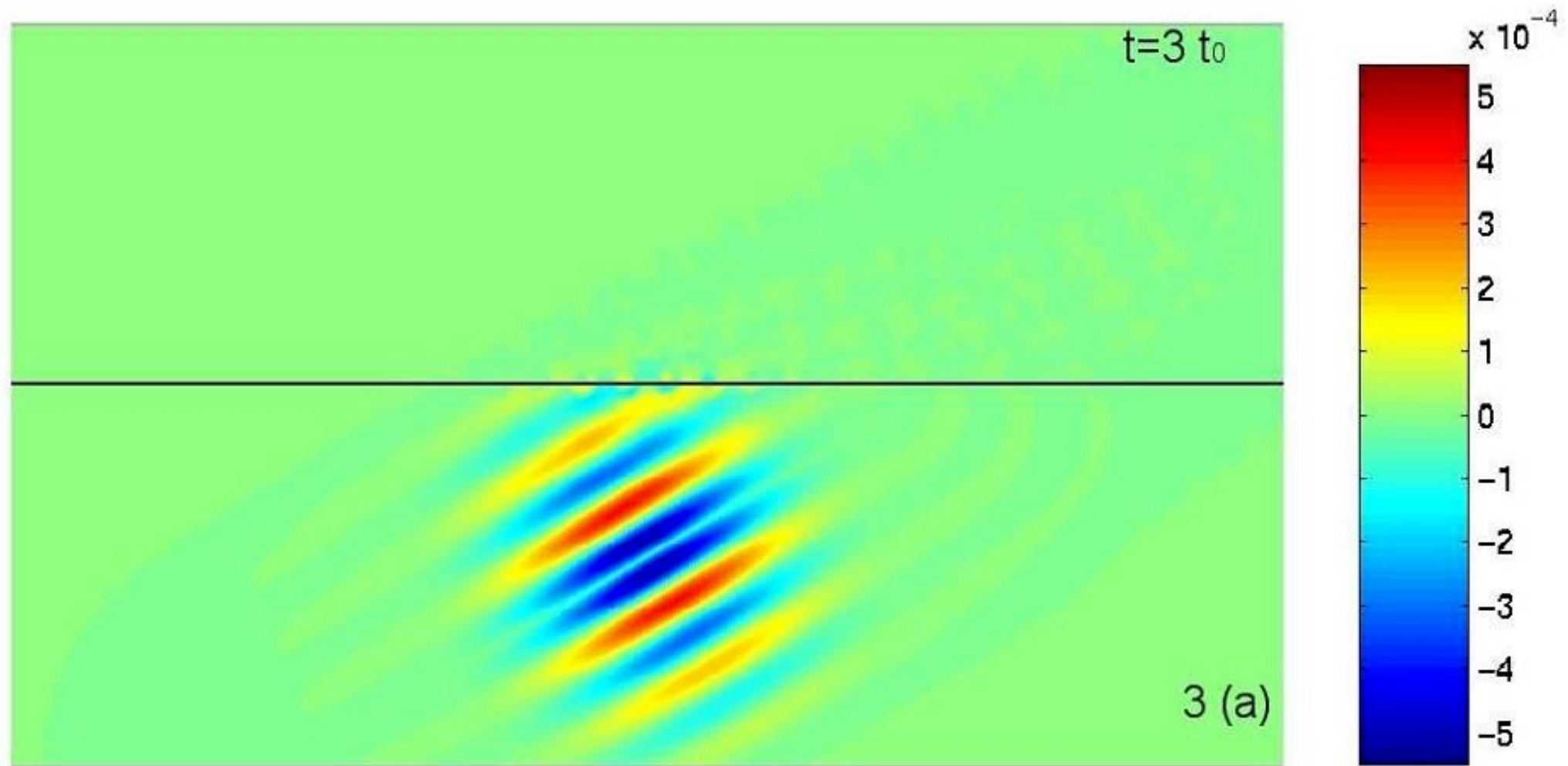


Photonic Crystals with negative refraction.

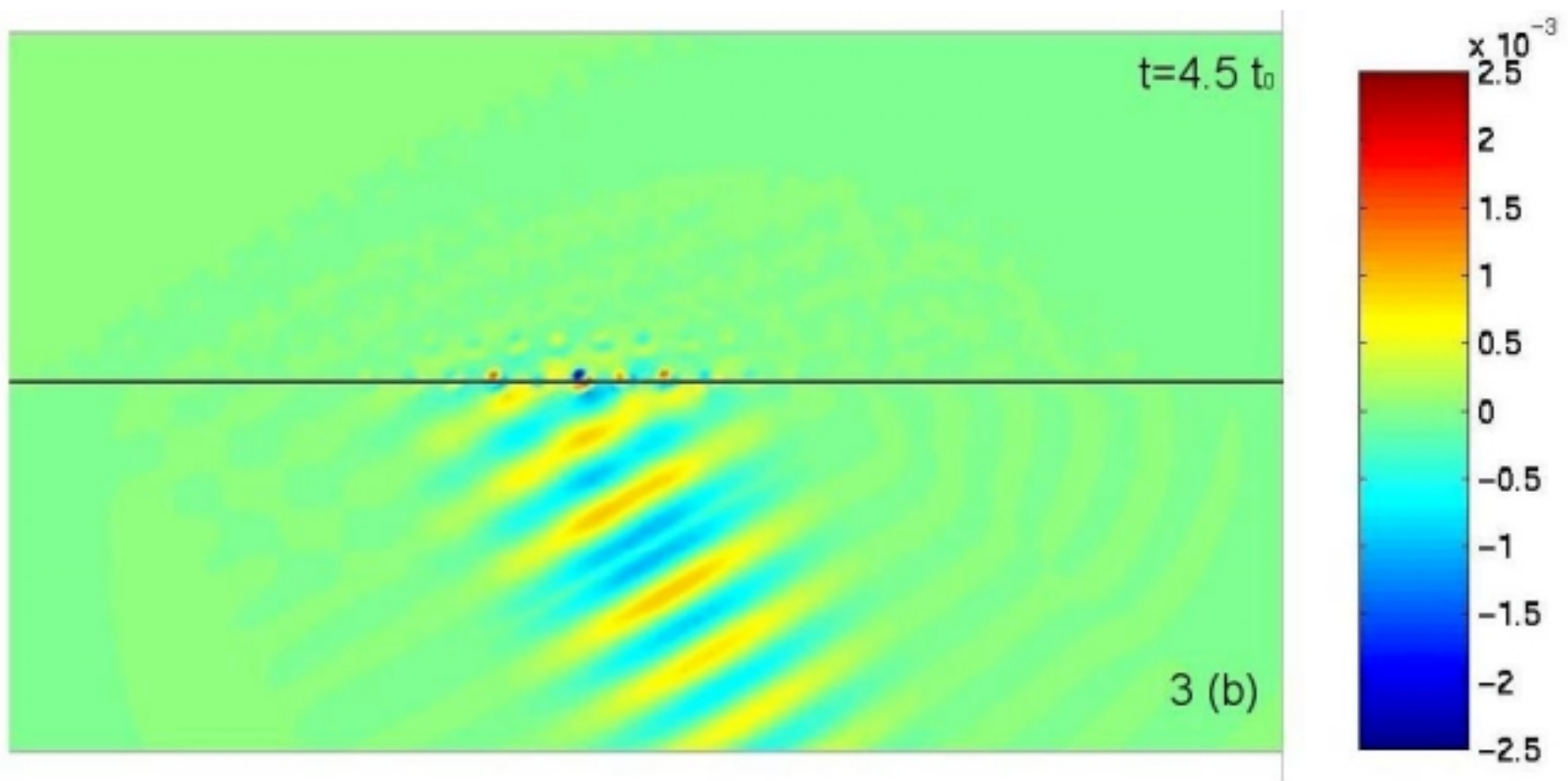


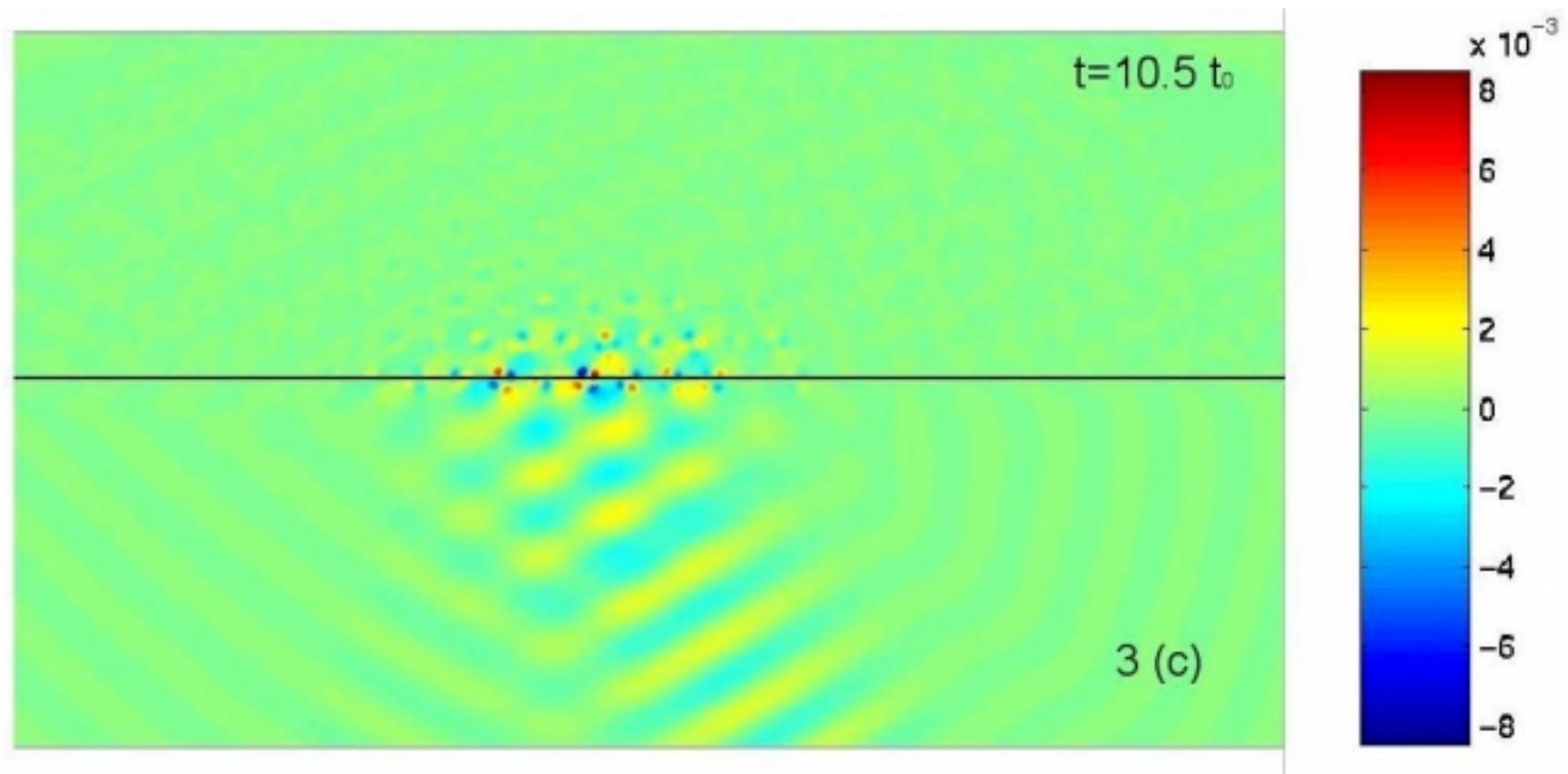
FDTD simulations were used to study the time evolution of an EM wave as it hits the **interface vacuum/photonic crystal**. Photonic crystal consists of an hexagonal lattice of dielectric rods with $\epsilon=12.96$. The radius of rods is $r=0.35a$. a is the lattice constant.

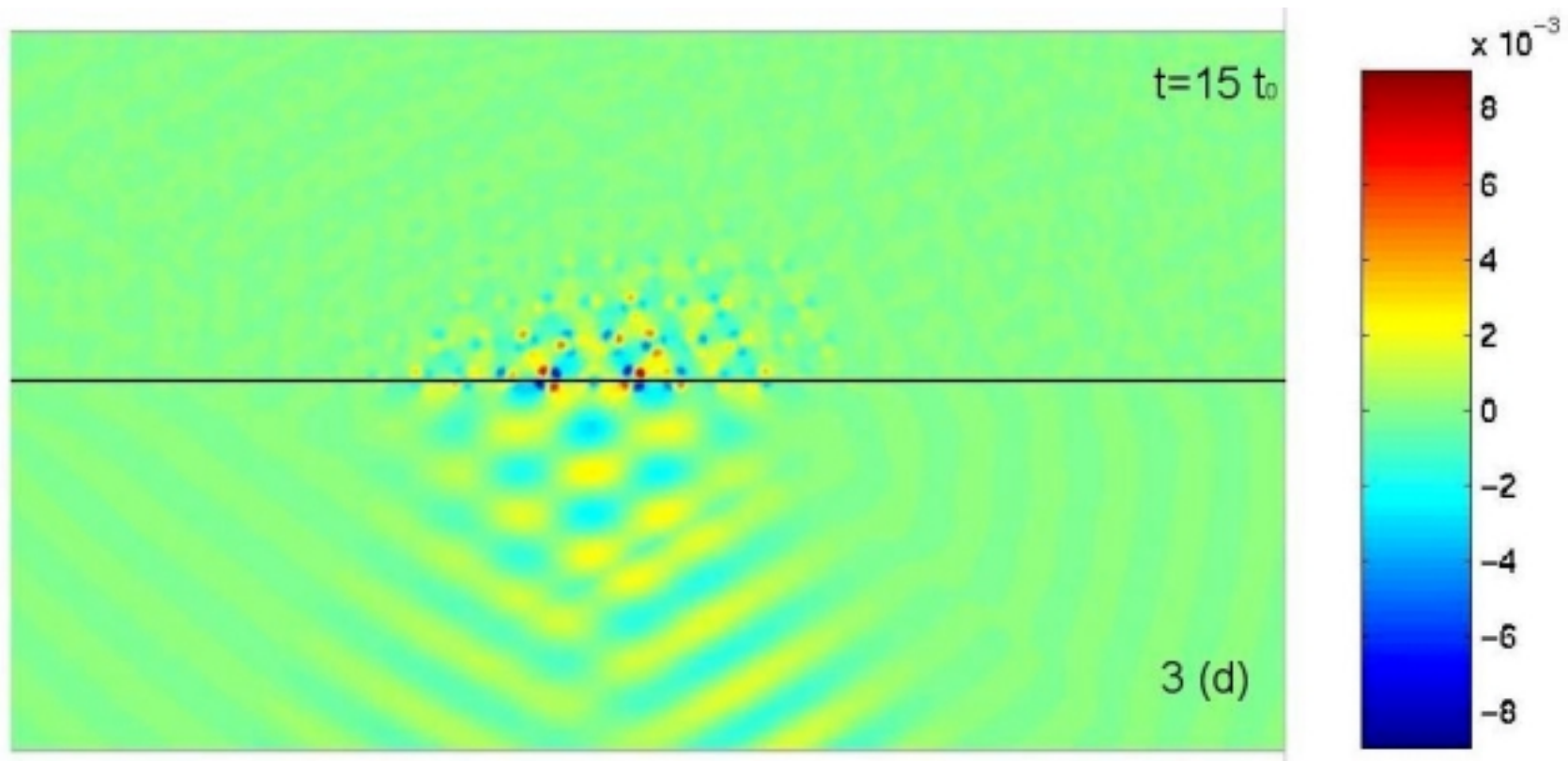
Photonic Crystals with negative refraction.



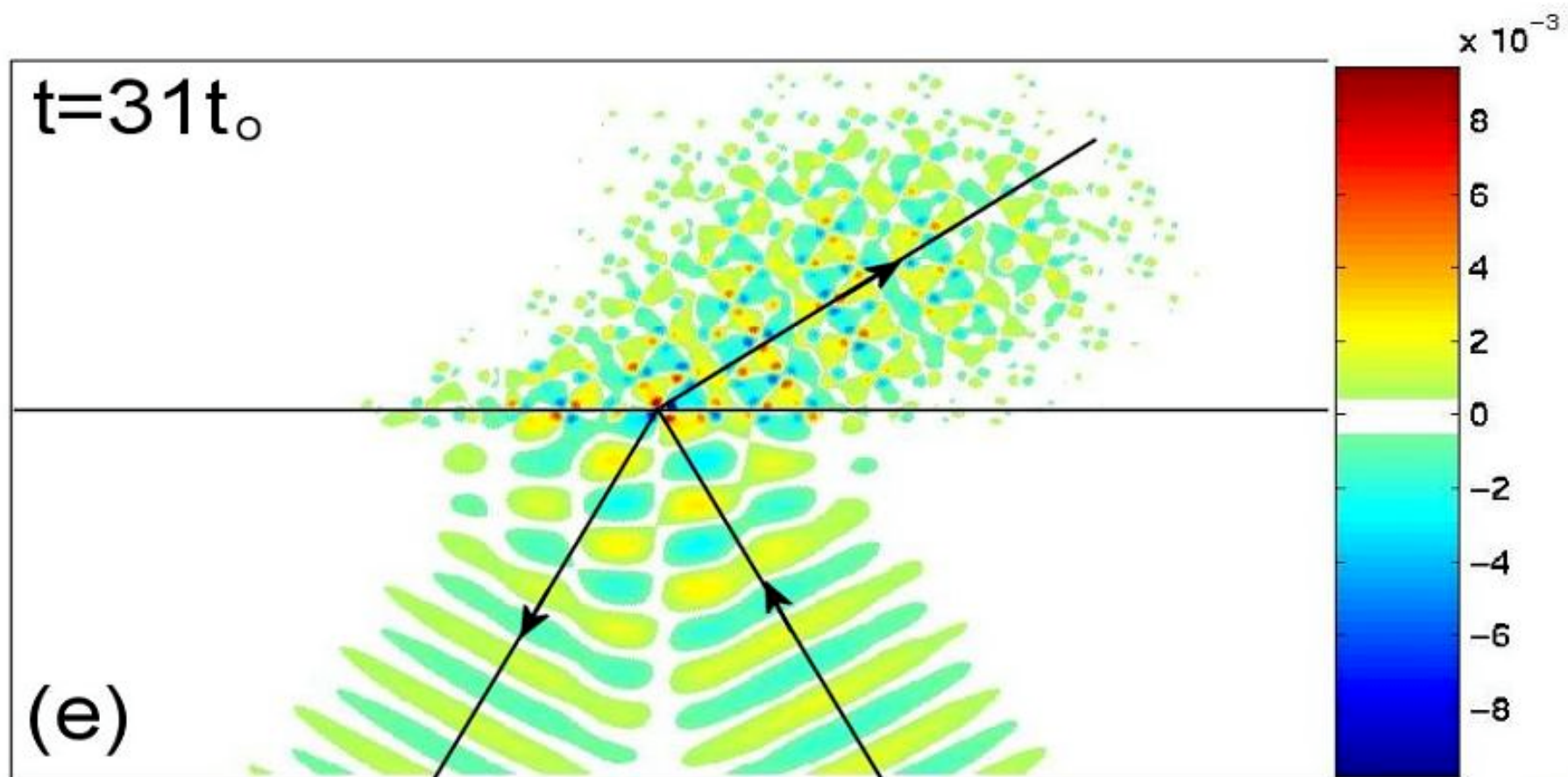
$$t_0 = 1.5T$$
$$T = \lambda/c$$







Photonic Crystals: negative refraction



The EM wave is trapped temporarily at the interface and after a long time, the wave front moves eventually in the negative direction.

Negative refraction was observed for wavelength of the EM wave $\lambda = 1.64 - 1.75 a$ (a is the lattice constant of PC)