

Crowd Control:

A physicist's approach to collective
human behaviour

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in collaboration with:

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Group motion of humans (observations)



Pedestrian crossing:
Self-organized lanes



Corridor in stadium:
jamming

Ordered motion (Kaba stone, Mecca)



Collective “action”: Mexican wave





STATISTICAL PHYSICS OF COLLECTIVE BEHAVIOUR

Collective behavior is a typical feature of living systems consisting of many similar units

- We consider systems in which the global behaviour does not depend on the fine details of its units (“particles”)
- Main feature of collective phenomena: the behaviour of the units becomes similar, but very different from what they would exhibit in the absence of the others

Main types: (phase) transitions, pattern/network formation,
synchronization*, group motion*



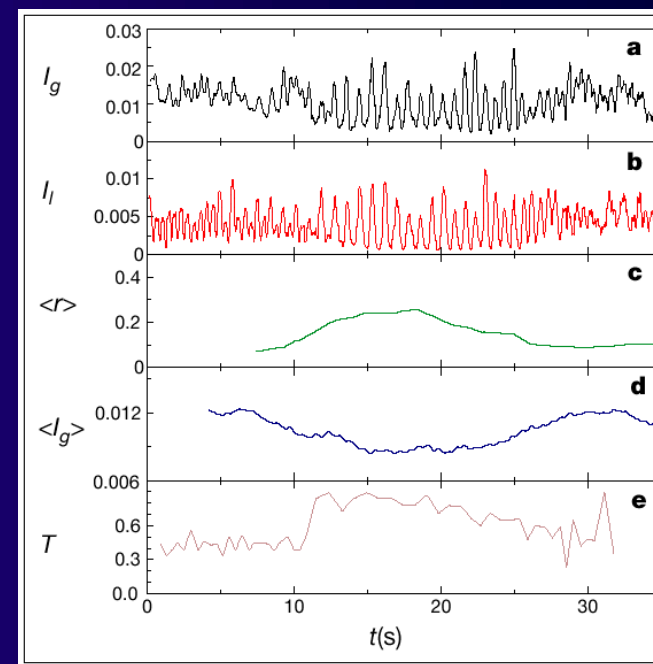
MESSAGES

- Methods of statistical physics can be successfully used to interpret collective behaviour
- The above mentioned behavioural patterns can be observed and quantitatively described/explained for a *wide range of phenomena* starting from the *simplest* manifestations of life (bacteria) up to humans because of the common underlying principles

See, e.g.: *Fluctuations and Scaling in Biology*, T. Vicsek, ed. (Oxford Univ. Press, 2001)

Synchronization

- Examples: (fire flies, cicada, heart, steps, dancing, etc)
- “Iron” clapping: collective human behaviour allowing quantitative analysis



Dependence of sound intensity
On time

Fourier-gram of rhythmic applause

Frequency



Time →

Darkness is proportional to the magnitude of the power spectrum



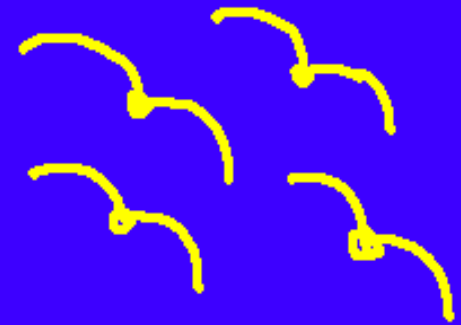
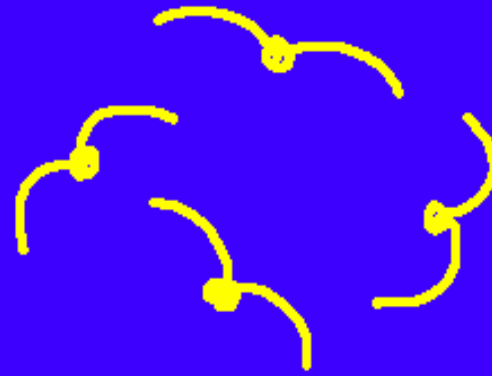
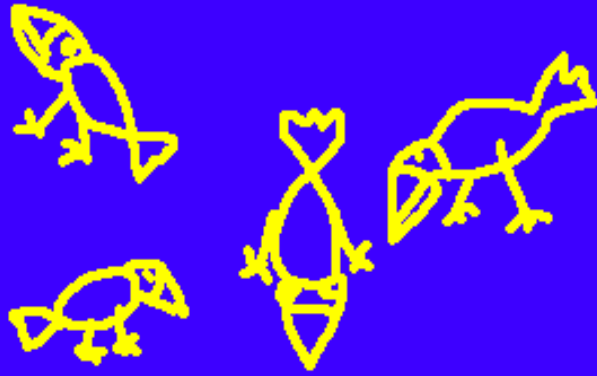
Group motion of humans (theory)

- Model:
 - Newton's equations of motion
 - Forces are of *social*, *psychological* or *physical* origin (herding, avoidance, friction, etc)
- Statement:
 - Realistic models useful for interpretation of practical situations and applications can be constructed

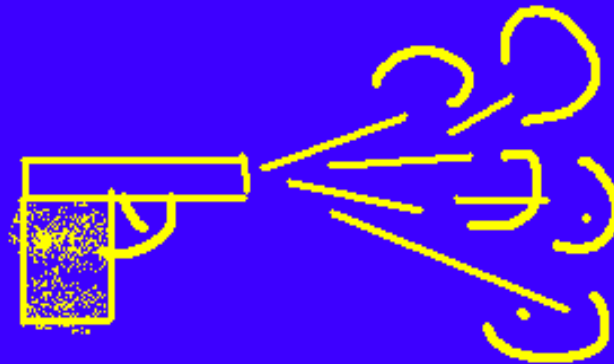
Ordering in velocity space

flocking, herding

2d



1d



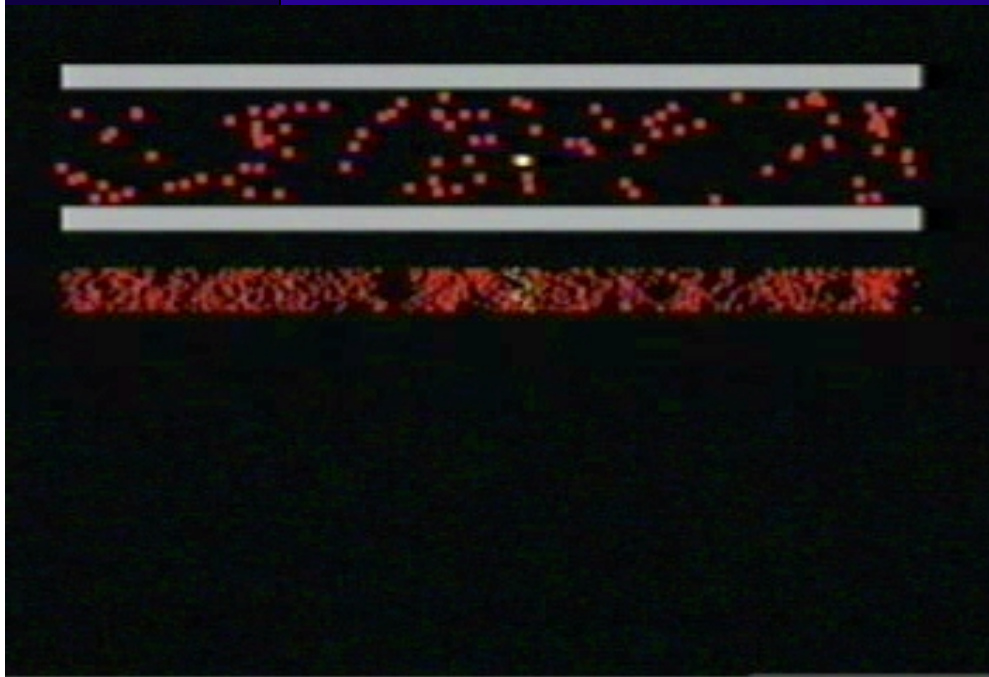
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t+ Δt → → ← ← ← ← ← ← ← ← ← ←

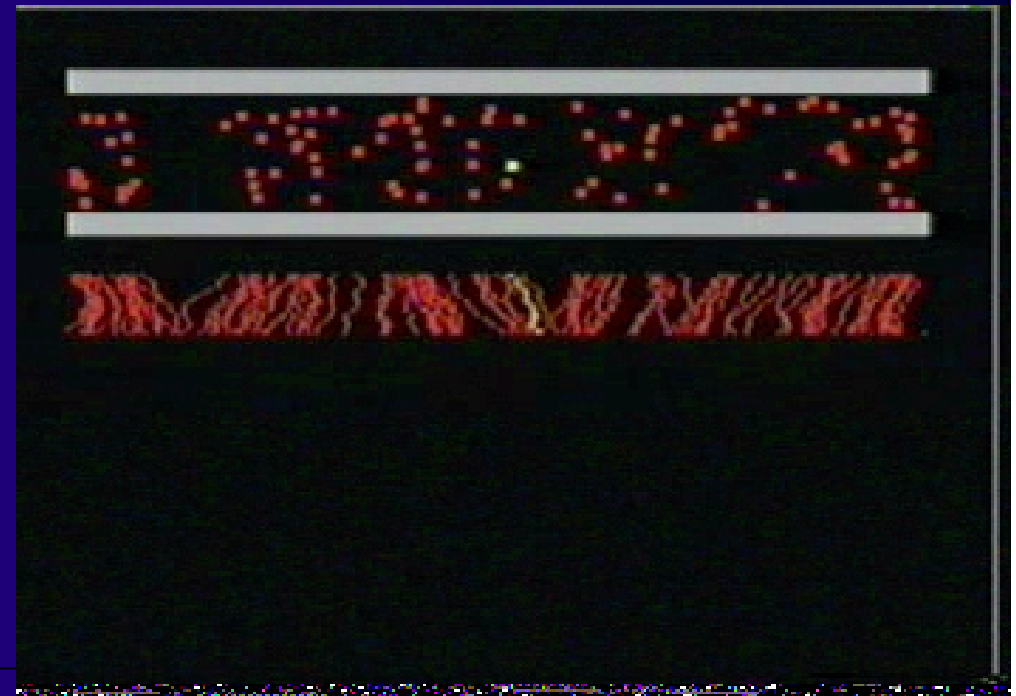
t+2 Δt ← ← ← ← ← ← ← ← ← ← ← ←

Escaping from a narrow corridor

The chance of escaping (ordered motion) depends on the level of “excitement” (on the level of perturbations relative to the “follow the others” rule)



Large perturbatons



Smaller perturbations

EQUATION OF MOTION for the velocity of pedestrian i

$$m_i \frac{dv_i}{dt} = m_i \frac{v_i^0(t) \vec{e}_i^0(t) - \vec{v}_i(t)}{\tau_i} + \sum_{j \neq i} \vec{f}_{ij} + \vec{f}_{iW} \quad ,$$

$$\vec{f}_{ij} = \left[A_i \exp\left[(r_{ij} - d_{ij})/B_i\right] + kg(r_{ij} - d_{ij}) \right] \vec{n}_{ij} + \kappa g(r_{ij} - d_{ij}) \Delta v_{ji}^t \vec{t}_{ij} \quad ,$$

“psychological / social”, elastic repulsion and sliding friction force terms, and $g(x)$ is zero, if $d_{ij} > r_{ij}$, otherwise it is equal to x .

MASS BEHAVIOUR: herding

$$\vec{e}_i^0(t) = N \left[(1 - p_i) \vec{e}_i + p_i \left\langle \vec{e}_j^0(t) \right\rangle_j \right] ,$$

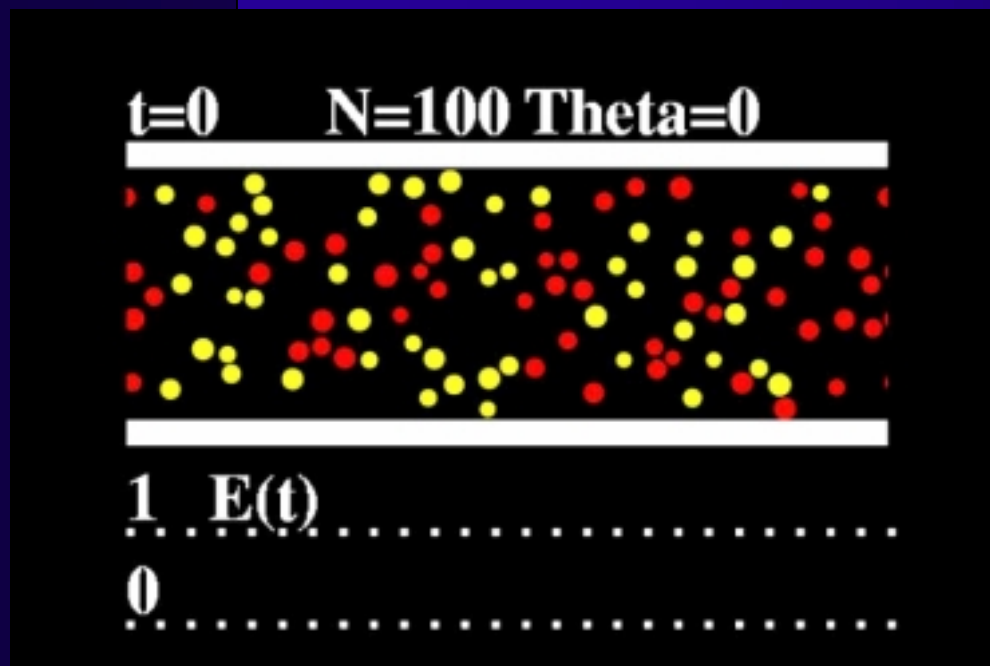
where $N(\vec{z}) = \vec{z} / \|\vec{z}\|$ denotes normalization of $\vec{z}$.

Social force: "Crystallization" in a pool

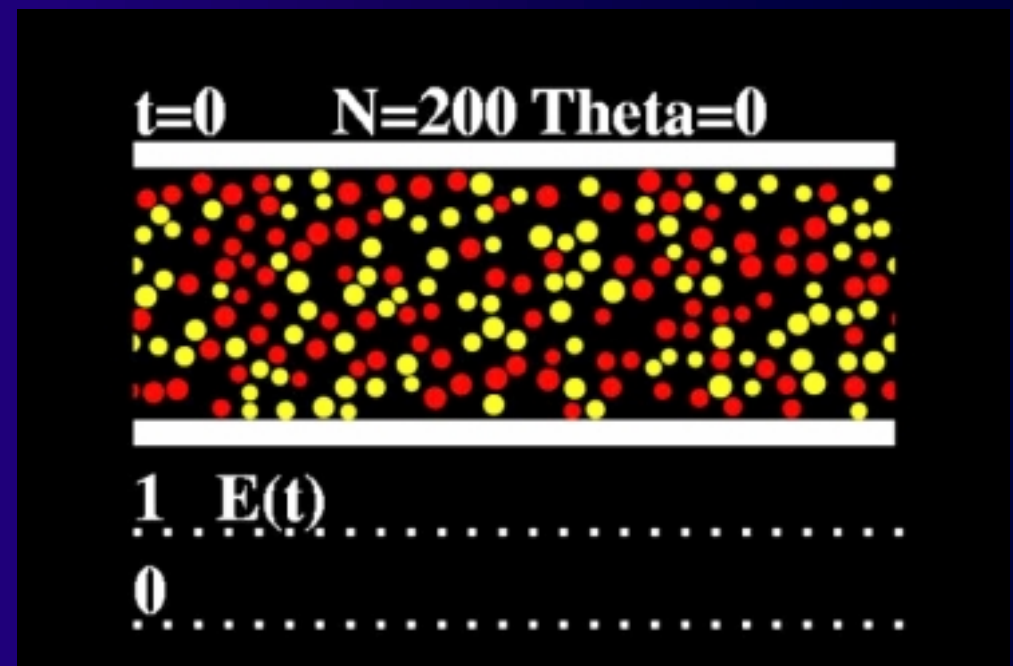


Moving along a wider corridor

- Spontaneous segregation (build up of lanes)
- optimization



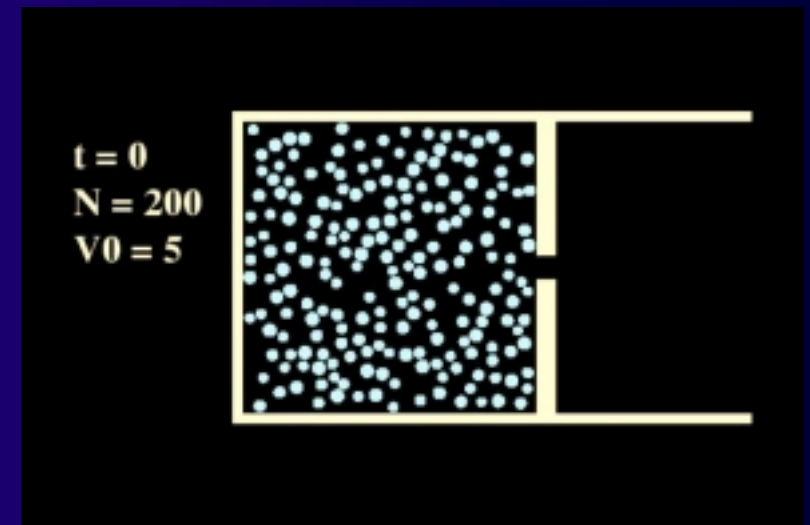
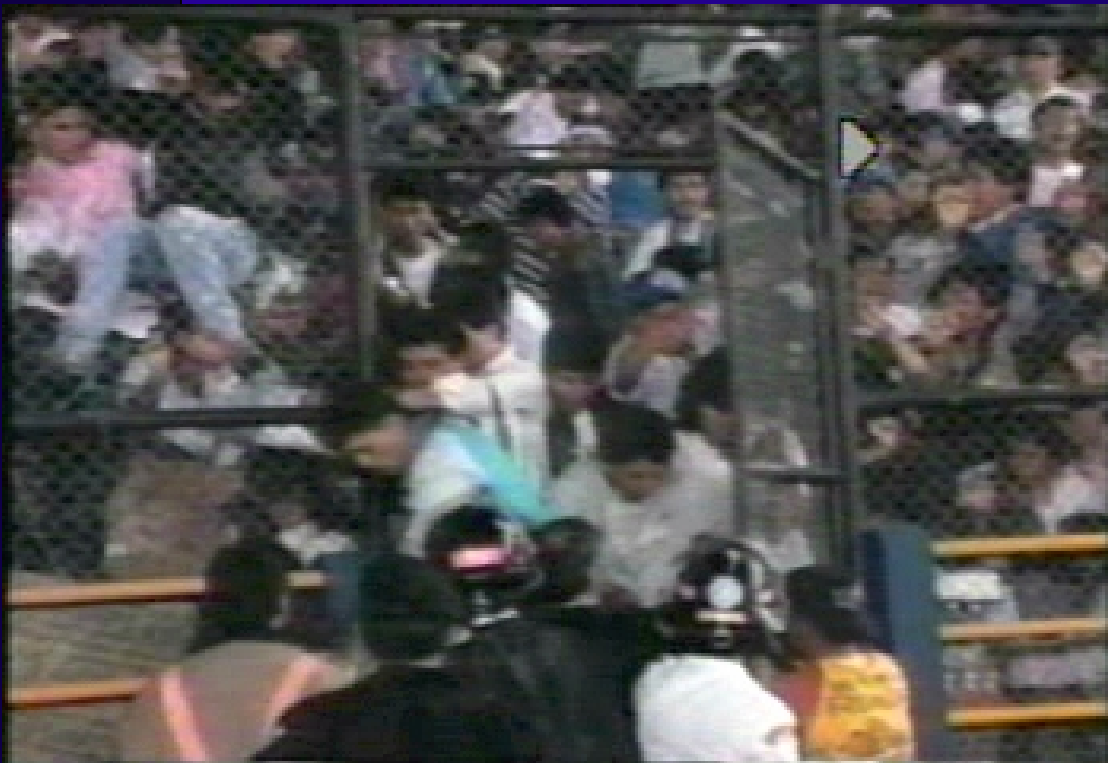
Typical situation



crowd

Panic

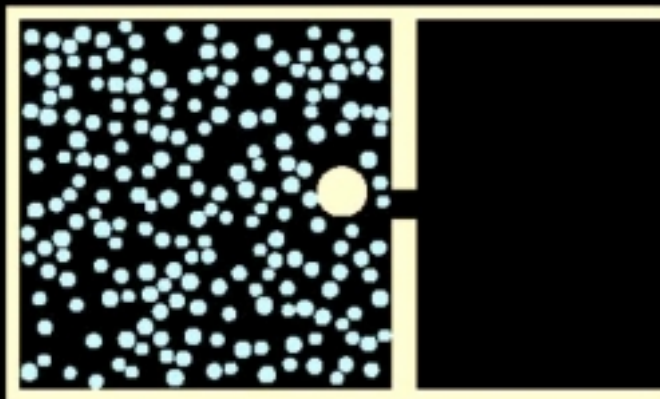
- Escaping from a closed area through a door
- At the exit physical forces are dominant !



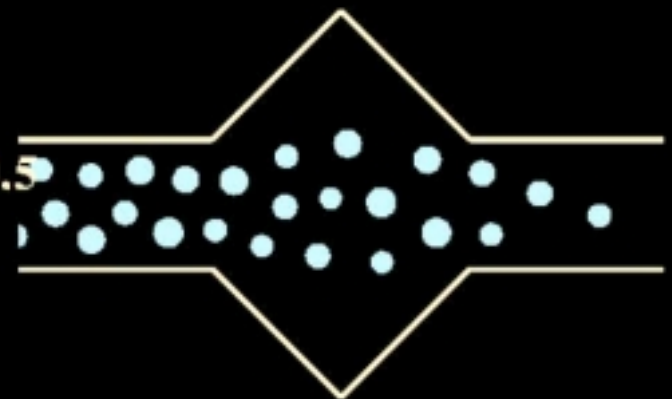
Paradoxical effects

- obstacle: helps (decreases the pressure)
- widening: harms (results in a jamming-like effect)

$t = 0$
 $N = 200$
 $V_0 = 5$
Inj.: 0



$t = 4$
 $J = 15$
 $V_0 = 4 \pm 0.5$

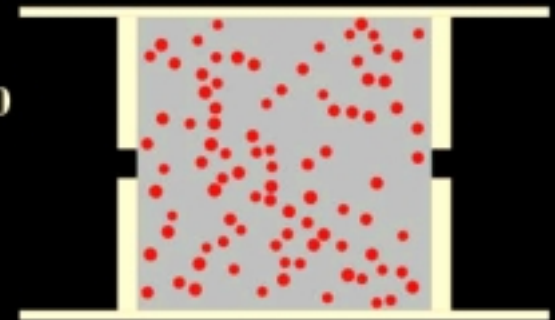


Effects of herding

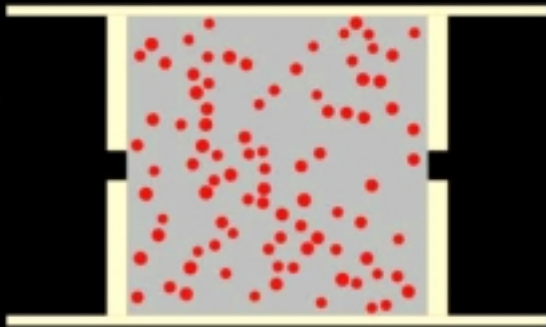
- Dark room with two exits
- Changing the level of herding

medium

$t = 0$
 $N_{in} = 90$
 $V_0 = 5$

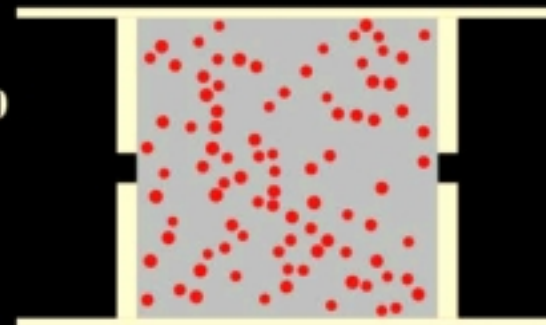


$t = 0$
 $N_{in} = 90$
 $V_0 = 5$



No herding

$t = 0$
 $N_{in} = 90$
 $V_0 = 5$

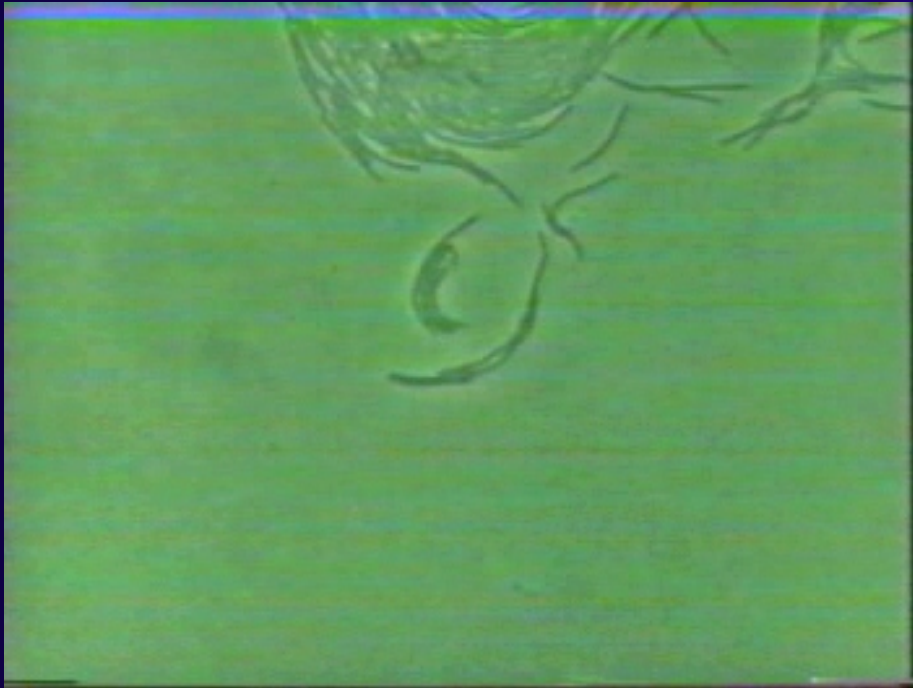


Total
herding



Collective motion

Patterns of motion of similar, interacting organisms



Cells



Flocks, herds, etc



Humans

A simple model: Follow your neighbors !

$$\vec{v}_j(t+1) = v_0 \frac{\langle \vec{v}_i(t) \rangle_R}{|\langle \vec{v}_i(t) \rangle_R|} + \eta_j(t)$$

- absolute value of the velocity is equal to v_0
- new direction is an average of the directions of neighbors
- plus some perturbation $\eta_j(t)$

- Simple to implement
- analogy with ferromagnets, differences:
 - for $v_0 \ll 1$ Heisenberg-model like behavior
 - for $v_0 \gg 1$ mean-field like behavior

in between: new dynamic critical phenomena (ordering, grouping, rotation,...)



Mexican wave (La Ola)

Phenomenon :

- A human wave moving along the stands of a stadium
- One section of spectators stands up, arms lifting, then sits down as the next section does the same.

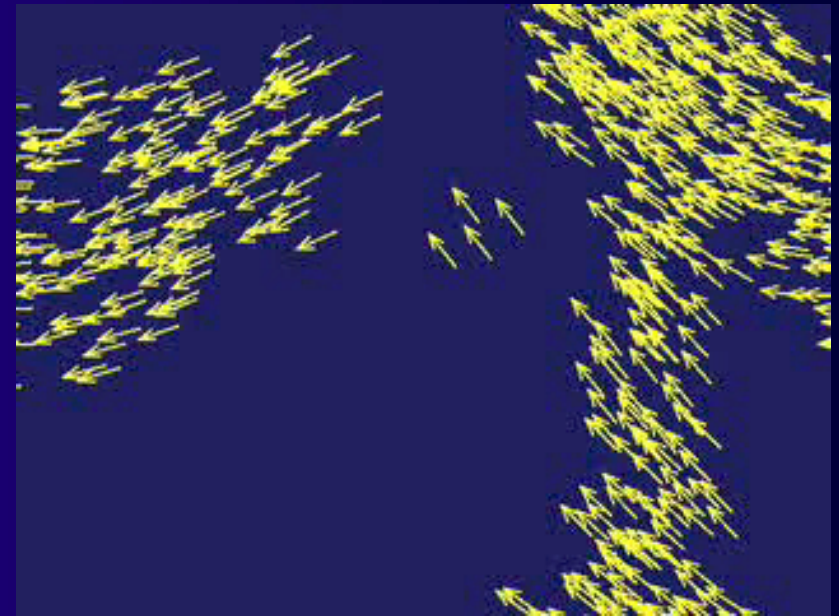
Interpretation: using modified models originally proposed for excitable media such as heart tissue or amoeba colonies

Model:

- three states: excitable, refractory, inactive
- triggering, bias
- realistic parameters lead to agreement with observations

Swarms, flocks and herds

- **Model:** The particles
 - maintain a given velocity
 - follow their neighbours
 - motion is perturbed by fluctuations
- **Result:**
ordering is due to motion





Acknowledgements

Principal collaborators:

Barabási L., Czirók A., Derényi I., Farkas I., Farkas Z.,
Hegedűs B., D. Helbing, Néda Z., Tegzes P.

Grants from: OTKA, MKM FKFP/SZPÖ, MTA, NSF

Social force: "Crystallization" in a pool





Major manifestations

Pattern/network formation :

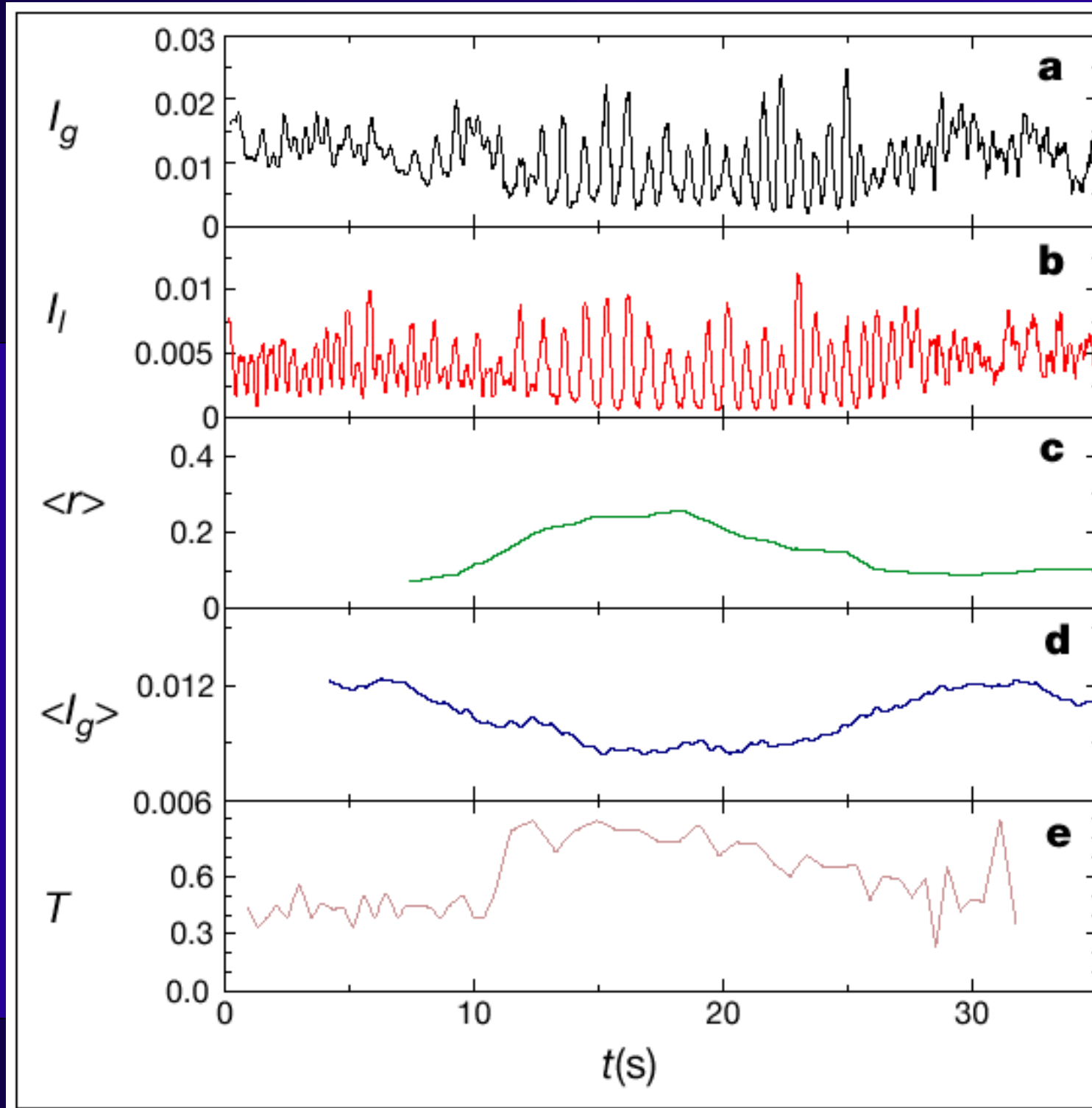
- Patterns: Stripes, morphologies, fractals, etc
- Networks: Food chains, protein/gene interactions, social connections, etc

Synchronization: adaptation of a common phase during periodic behavior

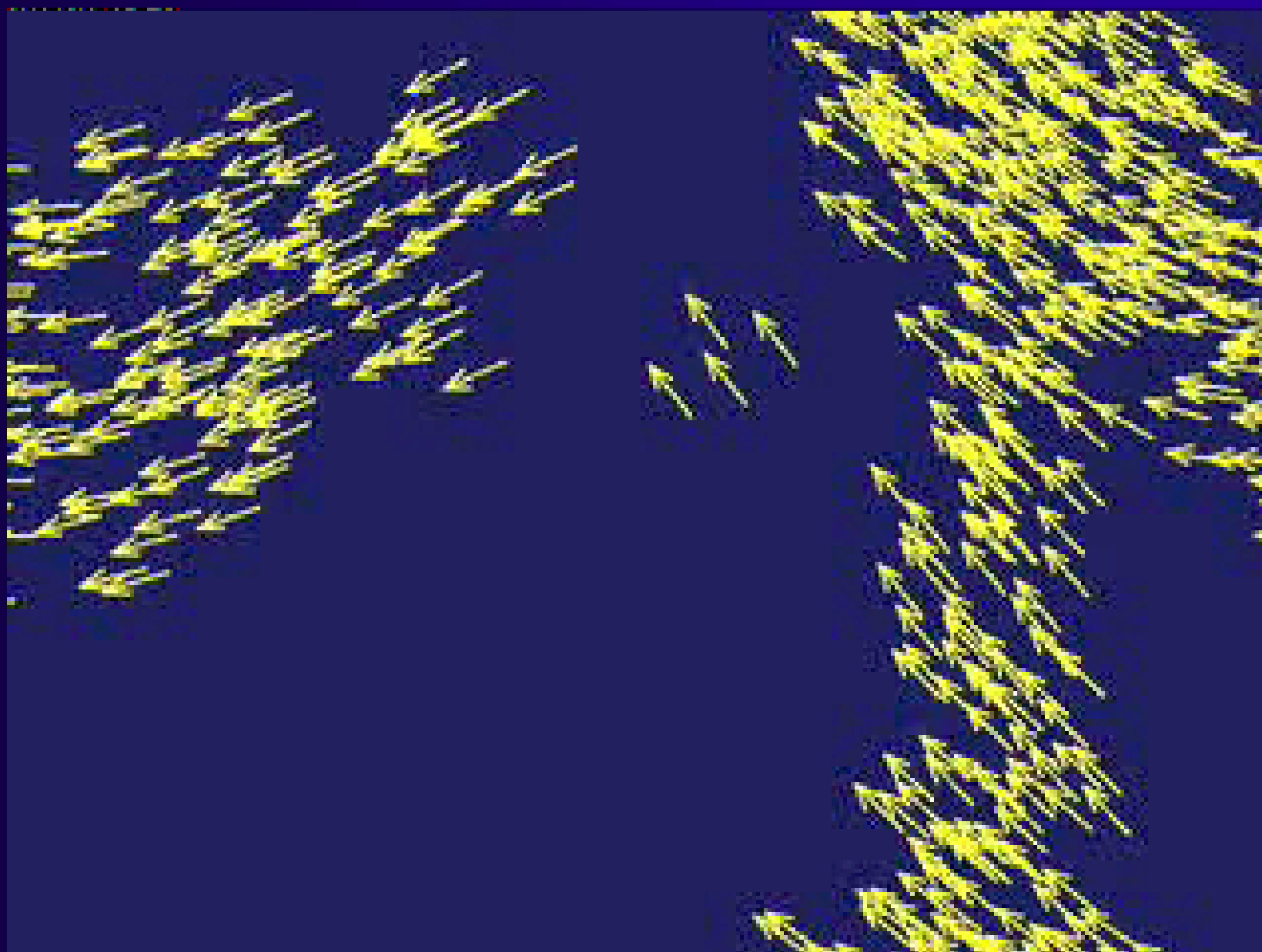
Collective motion:

- phase transition from disordered to ordered
- applications: swarming (cells, organisms), segregation, panic

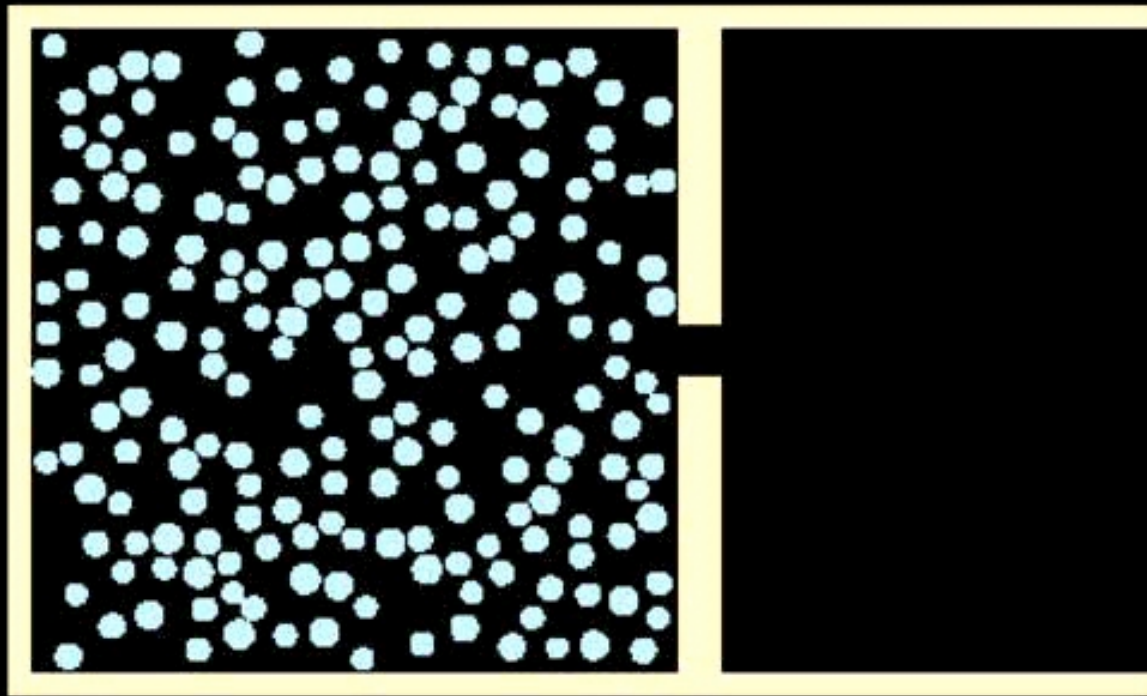




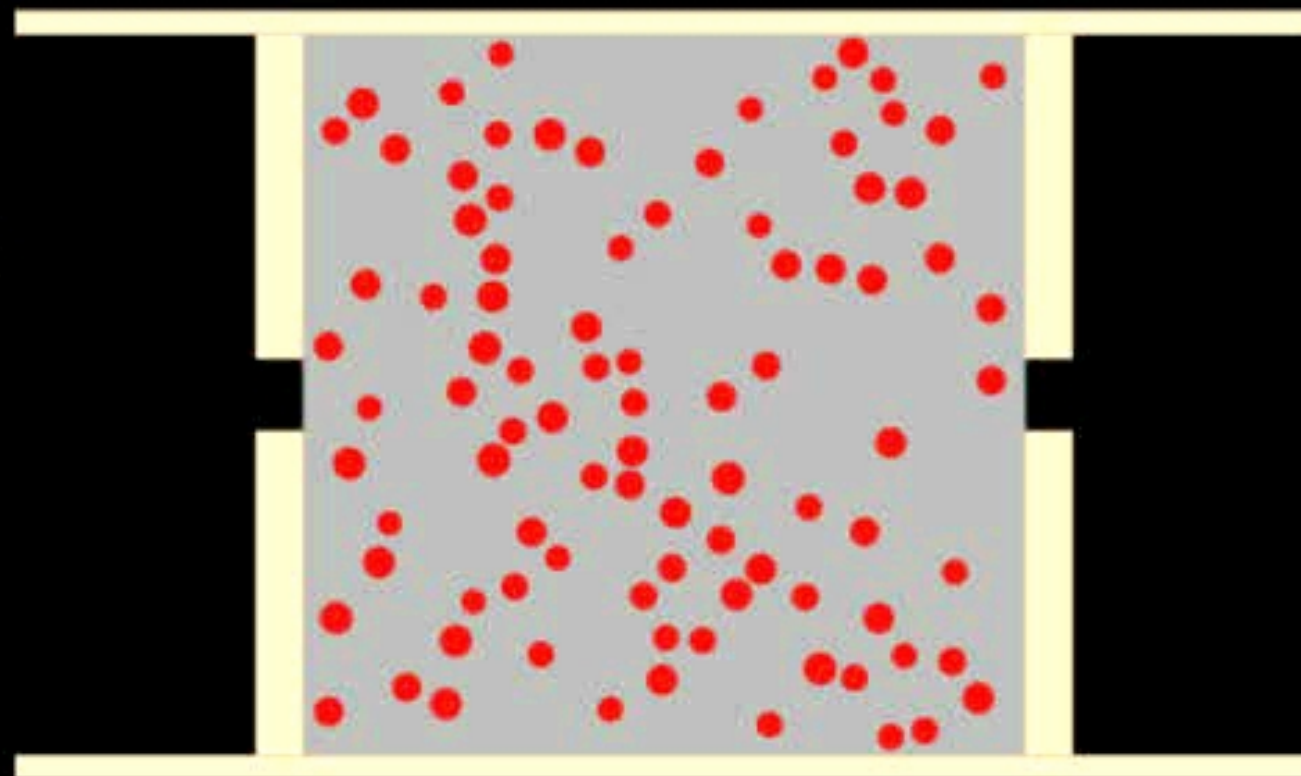




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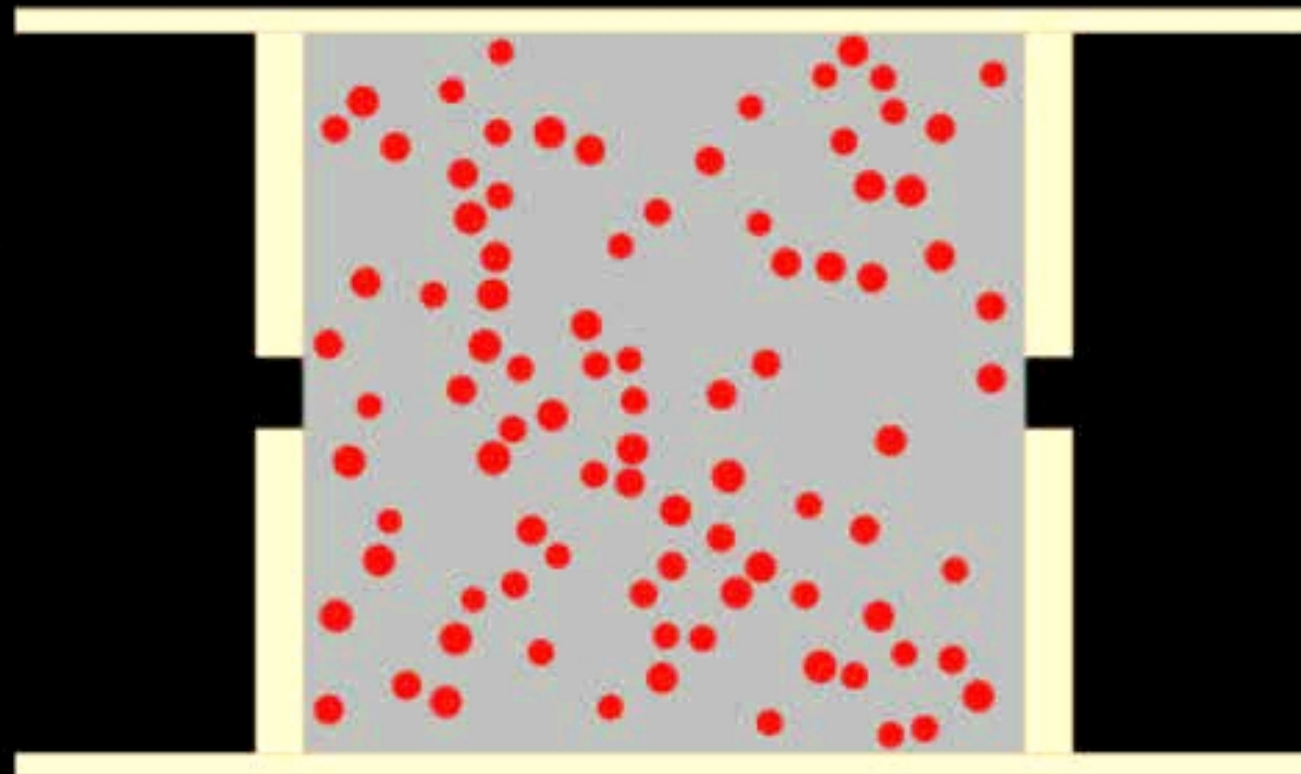
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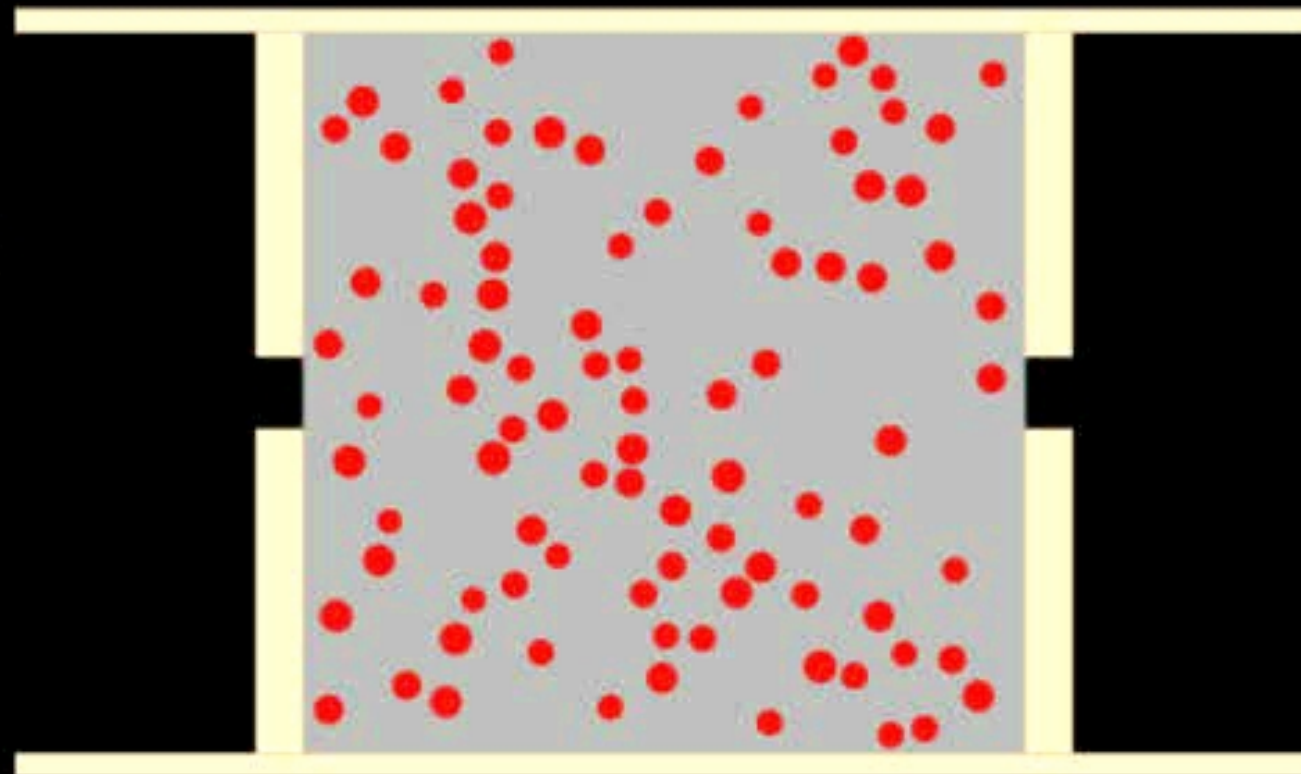
Nincs követés

[vissza](#)

$t = 0$
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 $V0 = 5$



Teljes követés

[vissza](#)

Motion driven by fluctuations

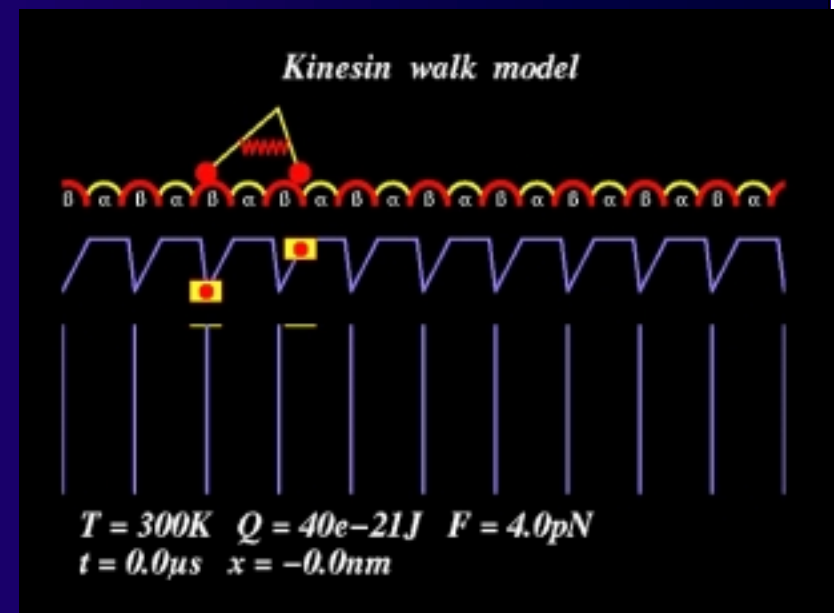
Molecular motors:

Protein molecules moving in a strongly fluctuating environment along chains of complementary proteins

Our model:

Kinesin moving along microtubules (transporting cellular organelles).

„scissors” like motion in a periodic, sawtooth shaped potential

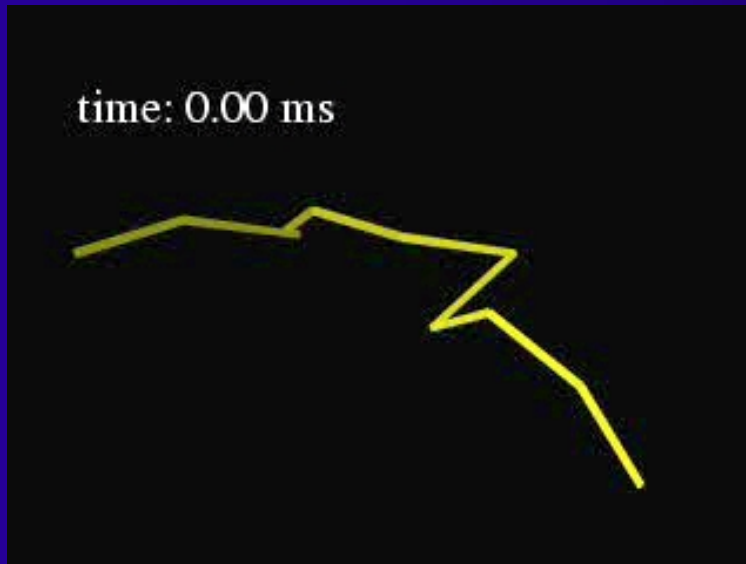


Translocation of DNS through a nuclear pore

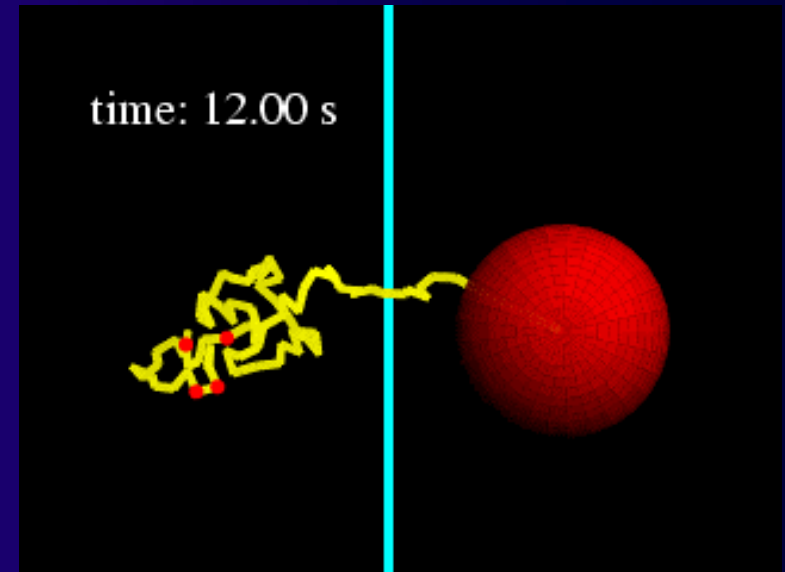
Transport of a polymer through a narrow hole

Motivation: related experiment, gene therapy, viral infection

Model: real time dynamics (forces, time scales, three dimens.)



duration: 1 ms
Length of DNS: 500 nm



duration: 12 s
Length of DNS : 10 μm



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Collective motion



Humans

Ordered motion (Kaba stone, Mecca)

