



Single Atom Experiments and the Test of Quantum Physics

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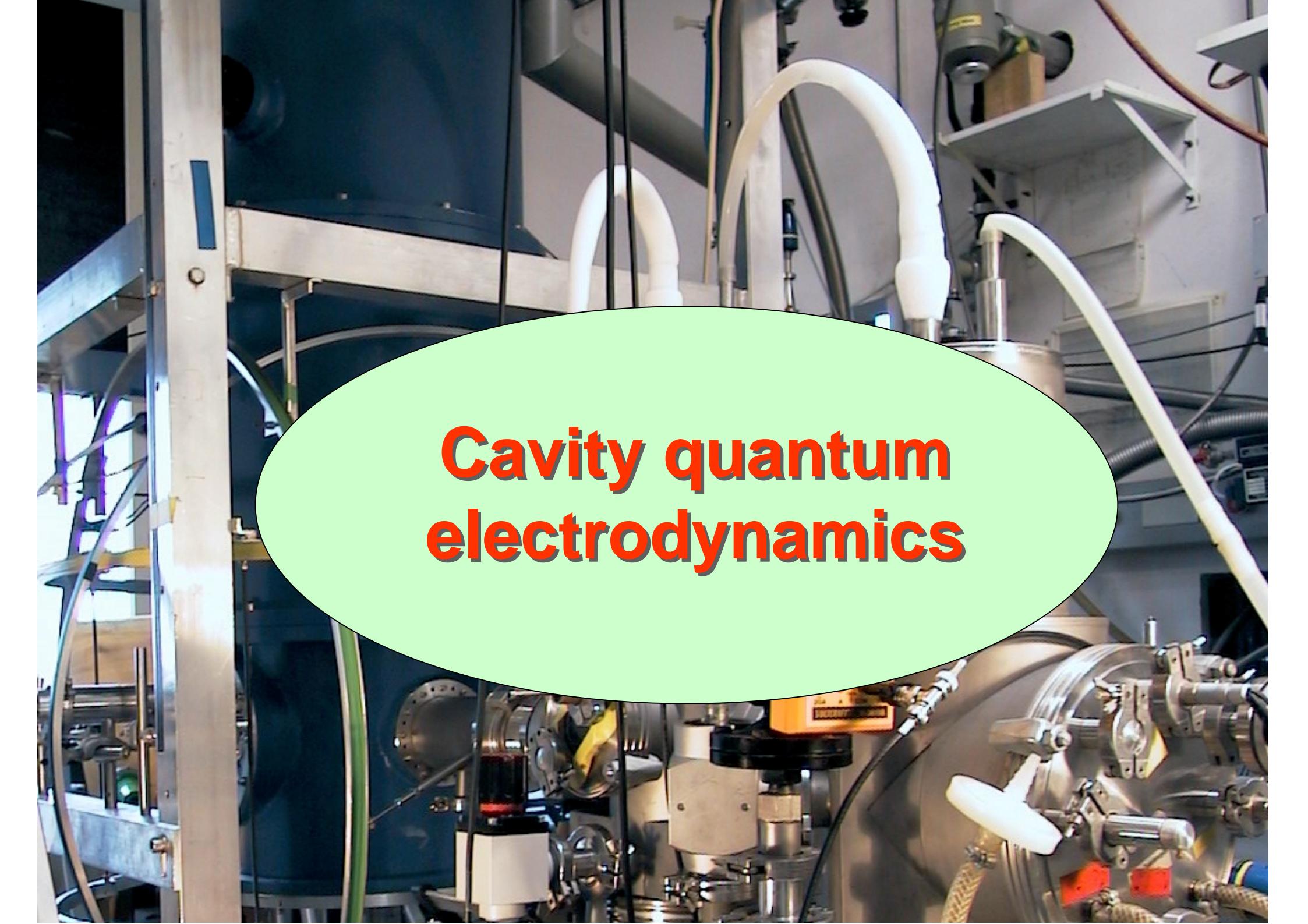
EPS-12: Trends in Physics

„.... You may object that in some cases, after all, we *are* experimenting with single particles. You may point to the linear traces or trajectories in the photographic plate and in the Wilson chamber, the collisions, the stars, the decaying particles etc. This range of phenomena, very fashionable now, seem to form an unconquerable stronghold for the particle view. I shall summarize my attitude towards these things in a few brief remarks.

- We do not in these cases *experiment* with single particles; we are scrutinizing records. We cannot repeat any of them under planfully varied conditions – the typical procedure of the experimenter.

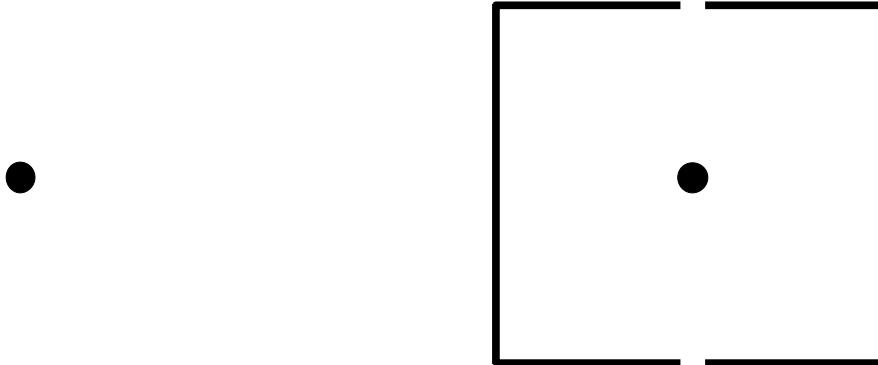
Quantum Interaction of Single Atoms

- Cavity quantum electrodynamics
 - Entanglement of atoms through radiation field
 - Generation of number states of the radiation field
 - Wigner function of a single photon field in a cavity
 - Cavity quantum electrodynamics with trapped ions
- New frequency standard on the basis of a single trapped ion



Cavity quantum electrodynamics

Free Atom versus Atom in Cavity



New field: Cavity quantum electrodynamics

QED Effects

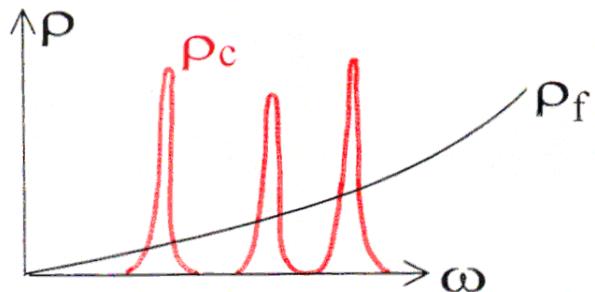
- **Modification of spontaneous emission rate**
- **Level shifts**

Interaction with the Radiation Field

- **Oscillatory energy exchange
(determined by photon statistics)**

Free space versus cavity

γ : emission rate of an atom $e \rightarrow g$



$$\gamma = 2\pi \int d\Omega_k (d_{eg}^k \rho(\omega_k))_{\omega_k=\omega_0}$$

d_{eg}^k : dipole matrix element between states $|e,0\rangle$ and $|g,1_k\rangle$

Free Space:

$$\rho_f(\omega_k) d\omega_k = (\frac{1}{2})(\frac{1}{4\pi}) \frac{\omega_k^2 d\omega_k}{\pi^2 c^3}$$

Cavity:

Atom does not see full vacuum spectrum

$$\gamma_c = \gamma_f \frac{\rho_c(\omega_k)}{\rho_f(\omega_k)} \longrightarrow \gamma_f \frac{(\lambda_k^3)}{V} Q$$

Q: quality factor $Q = \frac{\omega_k}{\Delta\omega_k}$

E. Purcell 1946

K. H. Drexhage 1974

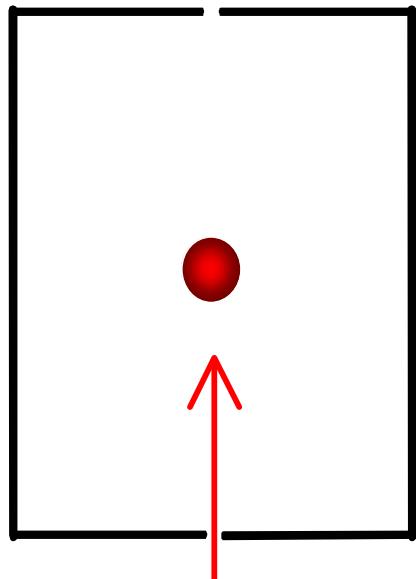
D. Kleppner 1981

S. Haroche 1983

M. Feld 1987

One-Atom Maser

Single atom - Single mode of a cavity



Atom in
excited state

Resonant superconducting cavity

Atom
↑
Cavity field
↓
Cavity walls

coupling constant g (Rabi-frequency)

coupling constant $\frac{\omega}{Q}$

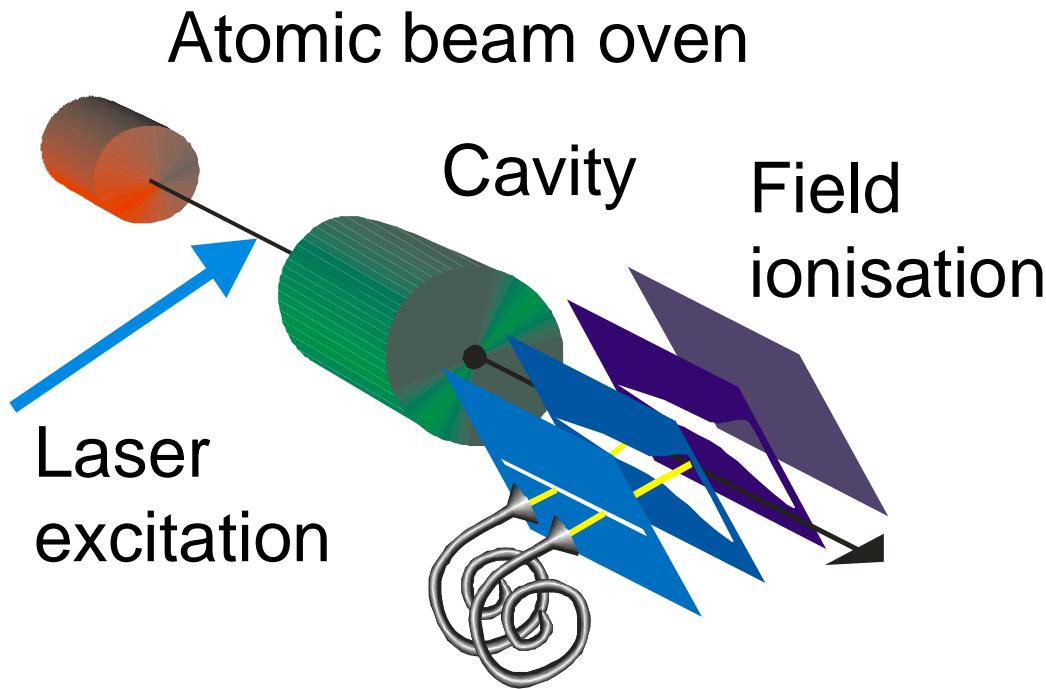
Strong coupling $g > \frac{\omega}{Q}$

Steady state field is generated:

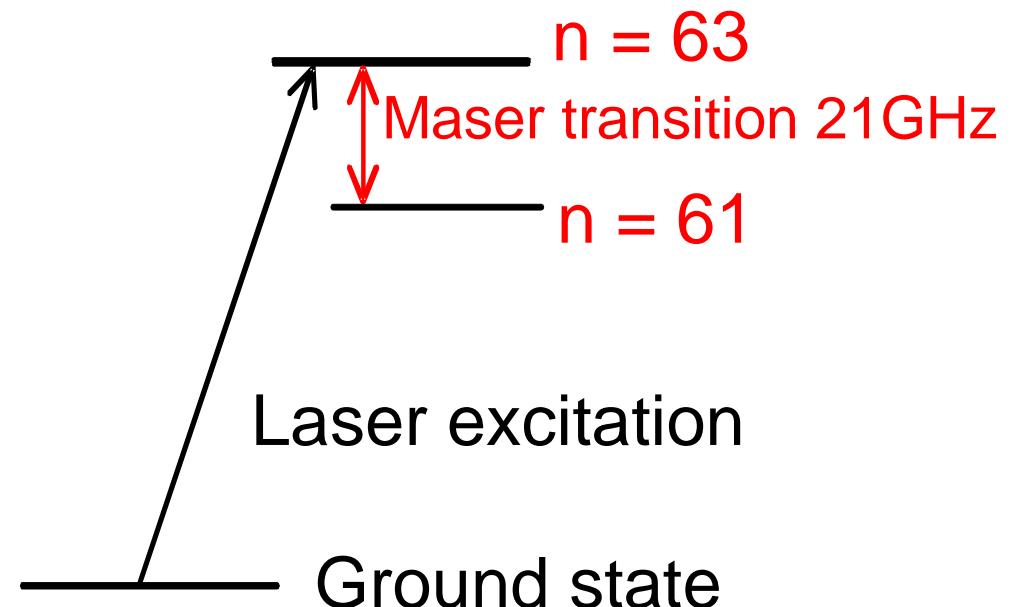
In most of the parameter regions sub-Poissonian statistics is obtained i.e. nonclassical fields

One-Atom Maser

Scheme



Levels



Velocity selected atoms $\frac{\Delta v}{v} = 1 - 4 \%$

Single photon Rabi frequency: $g \approx 40\,000 \text{ s}^{-1}$

Quality factor of cavity $Q = 4 \cdot 10^{10} \rightarrow \tau_{\text{cav}} = 0.3 \text{ s}$

$$g \gg \frac{\omega}{Q}$$

Temperature of the cavity $\approx 140 \text{ mK}$

Maser Resonance

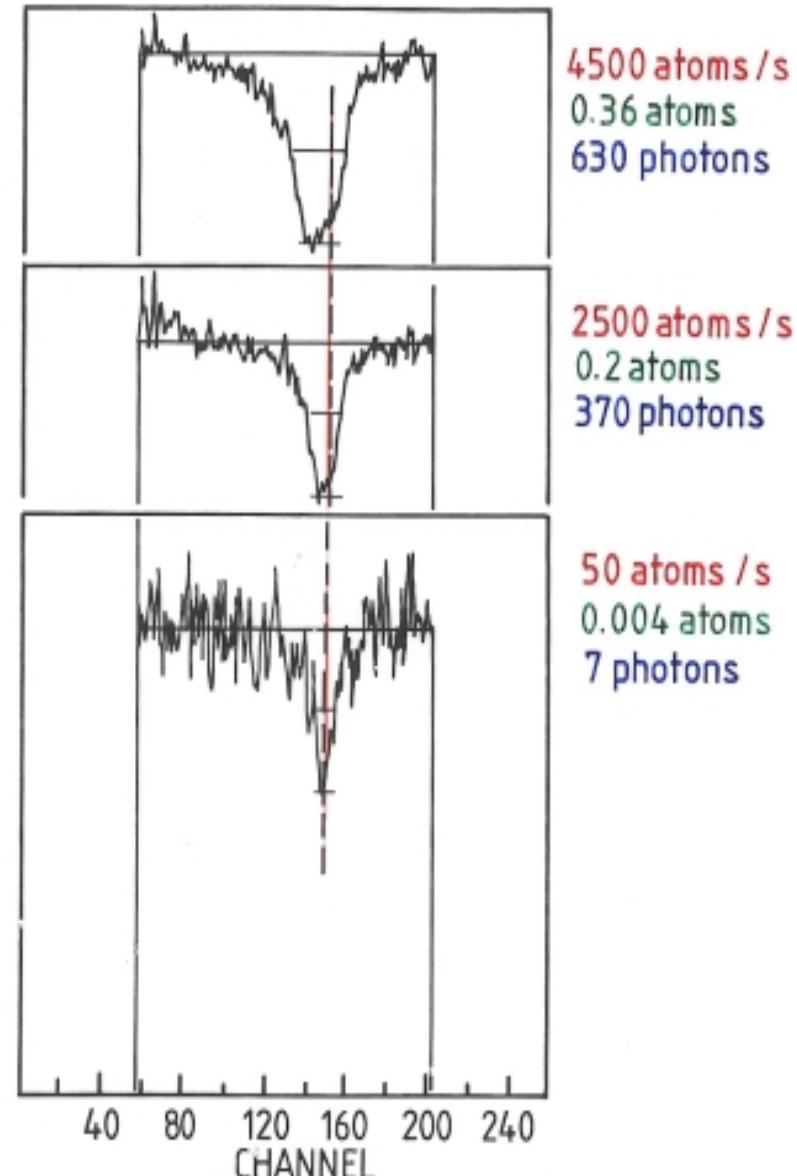
$$Q = 3 \cdot 10^{10}$$

$$T = 0.5 \text{ K}$$

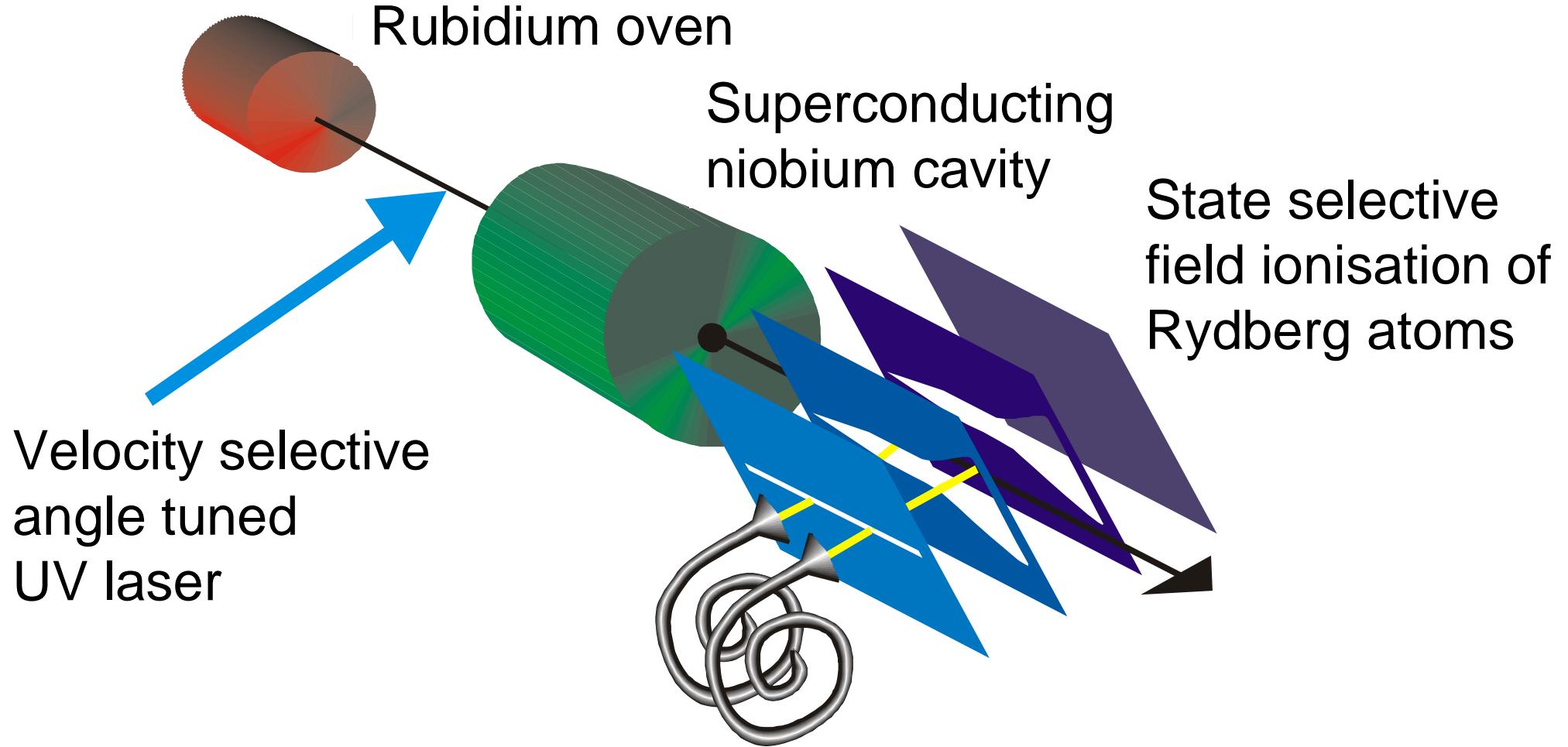
$$\gamma^1 = 0.2 \text{ s}$$

H. Walther, Phys. Rep.
219, 263 (1992)

Rb⁸⁵ 63p_{3/2} – 61 d_{5/2}
Resonance frequency:
21.456 GHz



One-Atom Maser



Atoms leaving the cavity are entangled with the generated field

$$|\bullet, n\rangle \rightarrow \cos(\phi\sqrt{n+1}) |\bullet, n\rangle - i \sin(\phi\sqrt{n+1}) |\bullet, n+1\rangle$$

Experimental Results with the Micromaser

Dynamics of the photon exchange (MPQ)

Test of micromaser theory – photon statistics (MPQ)

Two-photon maser (ENS)

Entanglement atom – field (MPQ-ENS)

atom – subsequent atom(s) (MPQ-ENS)

Bistability and quantum jumps (MPQ)

Excitation exchange between subsequent atoms (ENS)

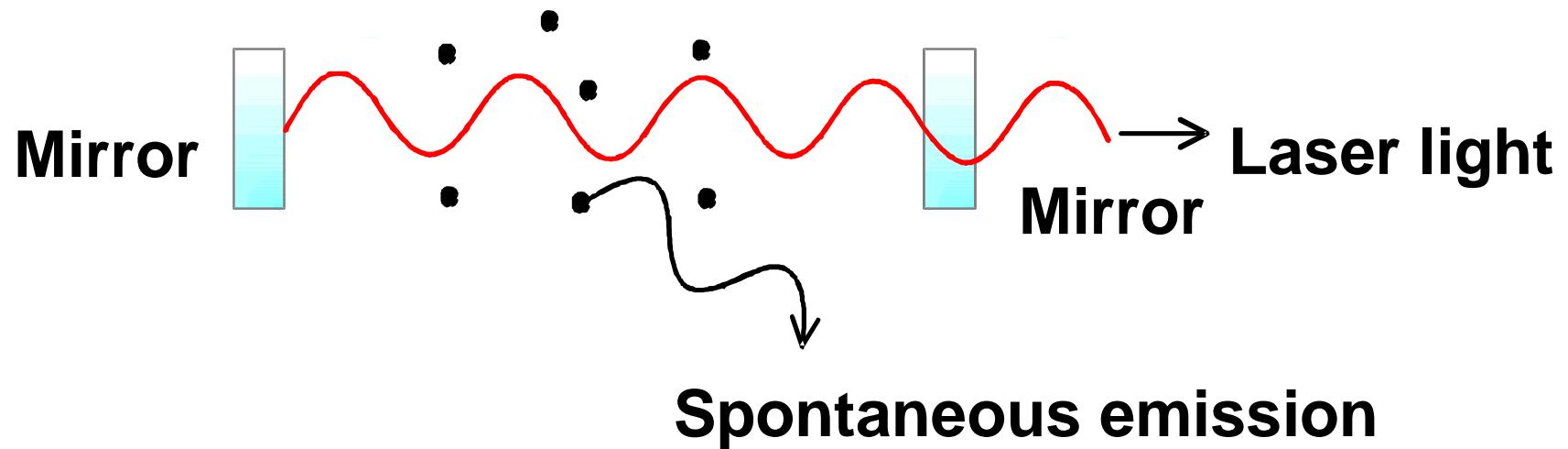
Discrete quantum states of the maser field (MPQ)

Determination of the Wigner function of a single photon (MPQ-ENS)

Generation of photon number states on demand (MPQ)

Scheme of a Laser

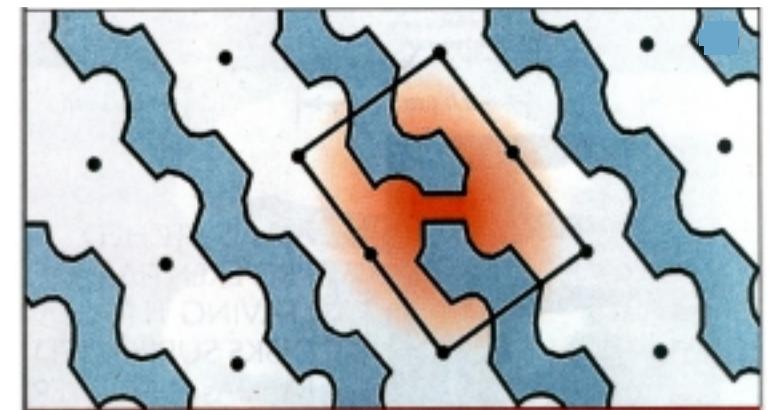
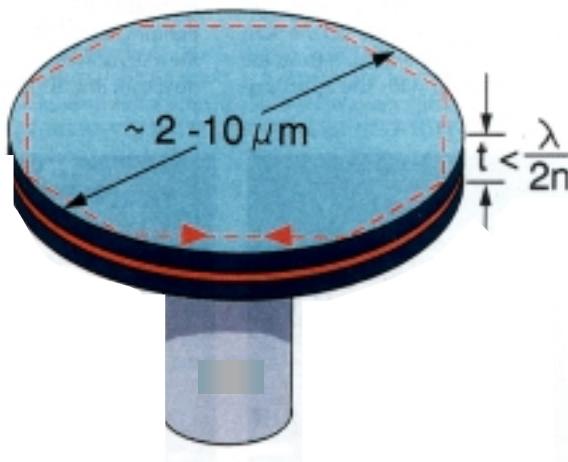
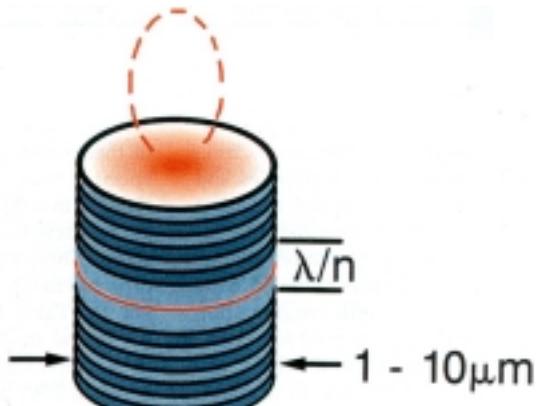
Laser cavity



Stimulated emission causes coherent emission into cavity modes

Spontaneous emission perturbs coherence and causes losses (spontaneous emission occurs in all directions)

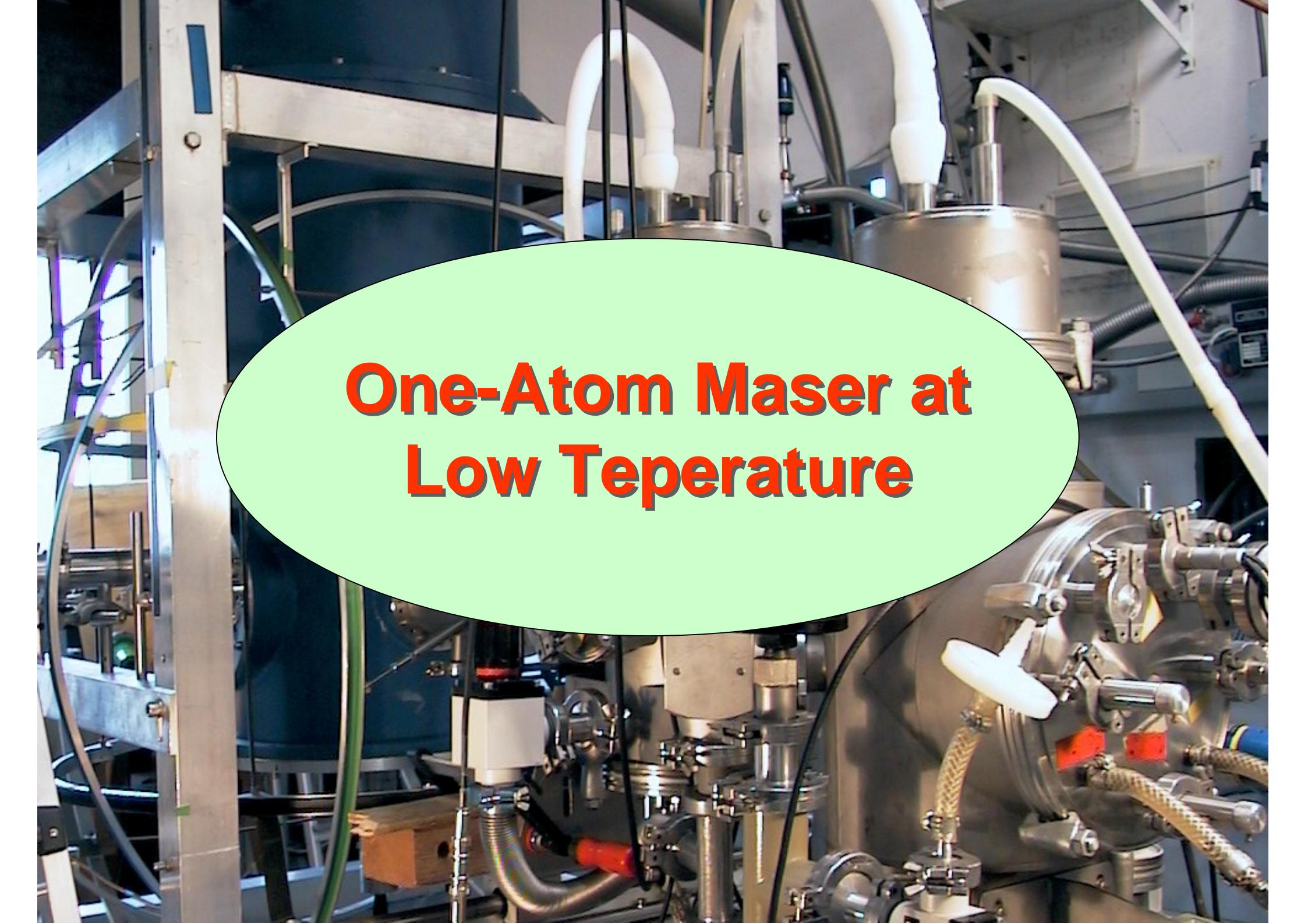
Cavity QED enhances efficiency of laser systems through control of spontaneous emission



Microcavities

Whispering gallery modes

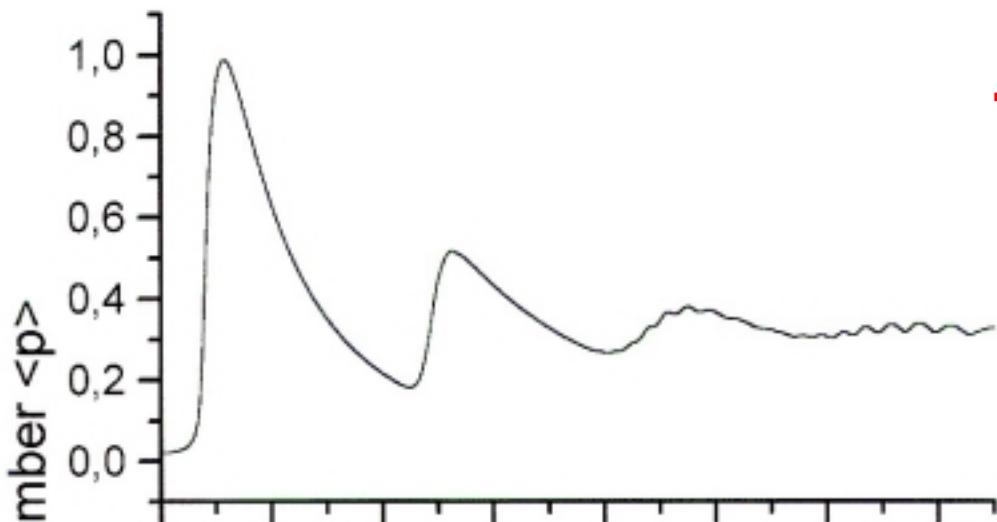
Photonic bandgap structure



A photograph of a sophisticated scientific experiment setup, likely a atomic maser or trap. The scene is filled with intricate metal structures, including cylindrical tanks, a network of white and grey tubes, and a variety of optical and mechanical components. A prominent feature is a large, curved, transparent bell jar or vacuum chamber situated in the upper right. In the lower right foreground, there's a circular component with a central tube and several red rectangular elements. The overall environment suggests a high-tech laboratory or research facility.

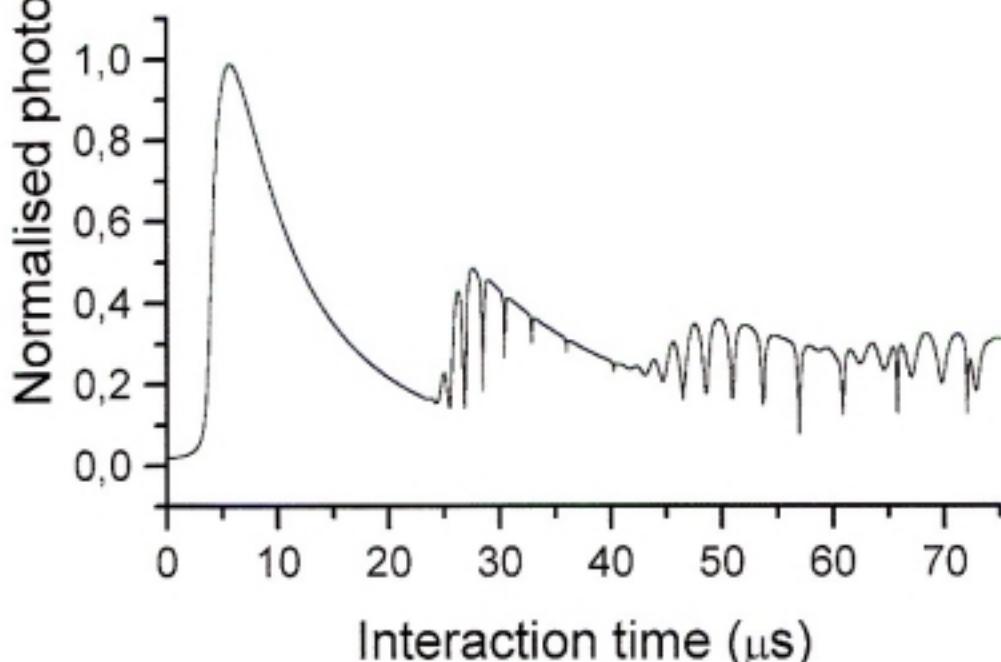
One-Atom Maser at Low Temperature

Low Temperature Behaviour of the One-Atom-Maser



Thermal photon number = 0.1

$N_{\text{ex}} = 50$
 $g = 39 \text{ kHz}$



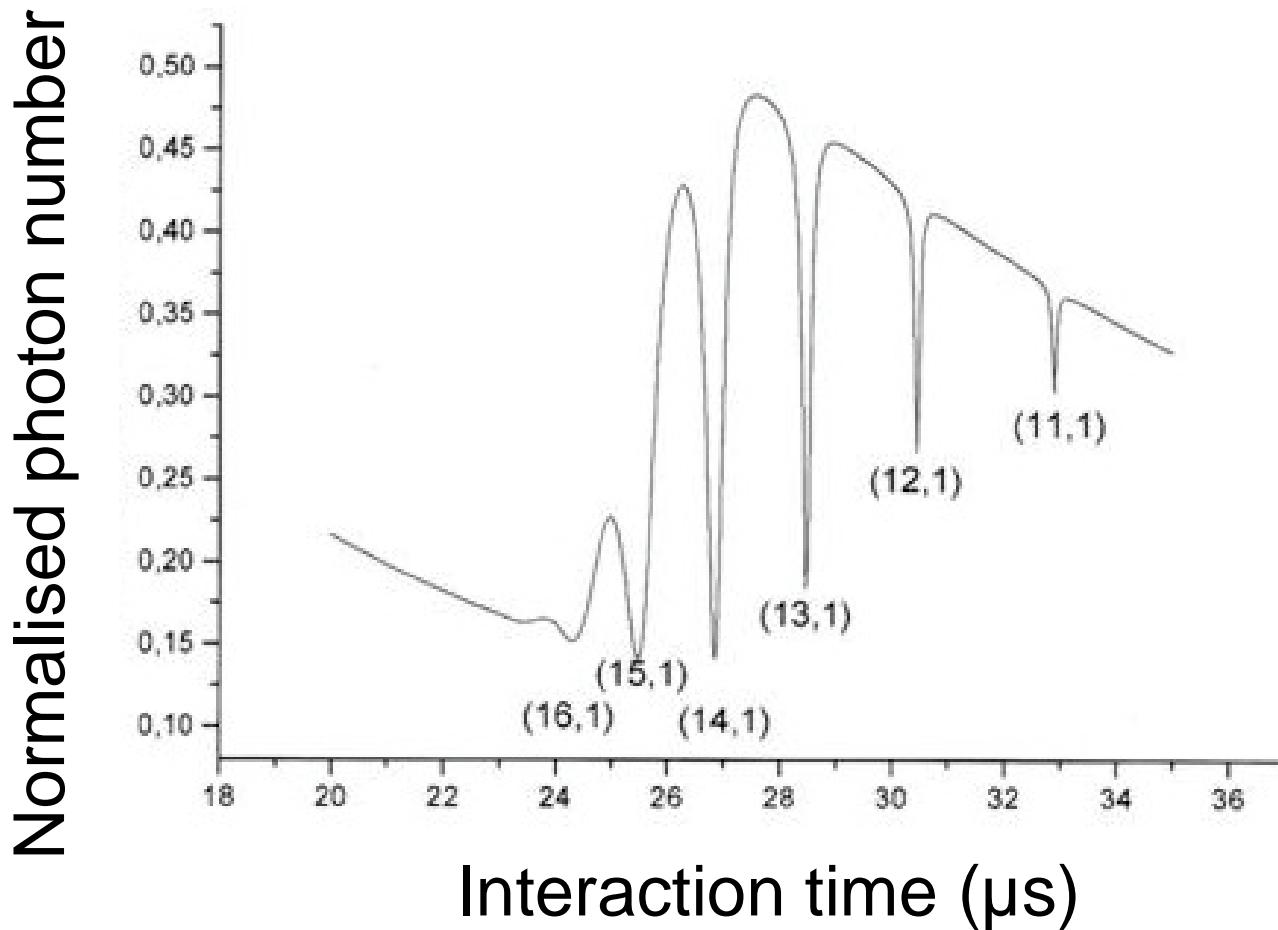
Thermal photon number = 10^{-4}

$N_{\text{ex}} = 50$
 $g = 39 \text{ kHz}$



P.Meystre, G.Rempe,
H.Walther, Opt. Lett. 13,
1078 (1988)

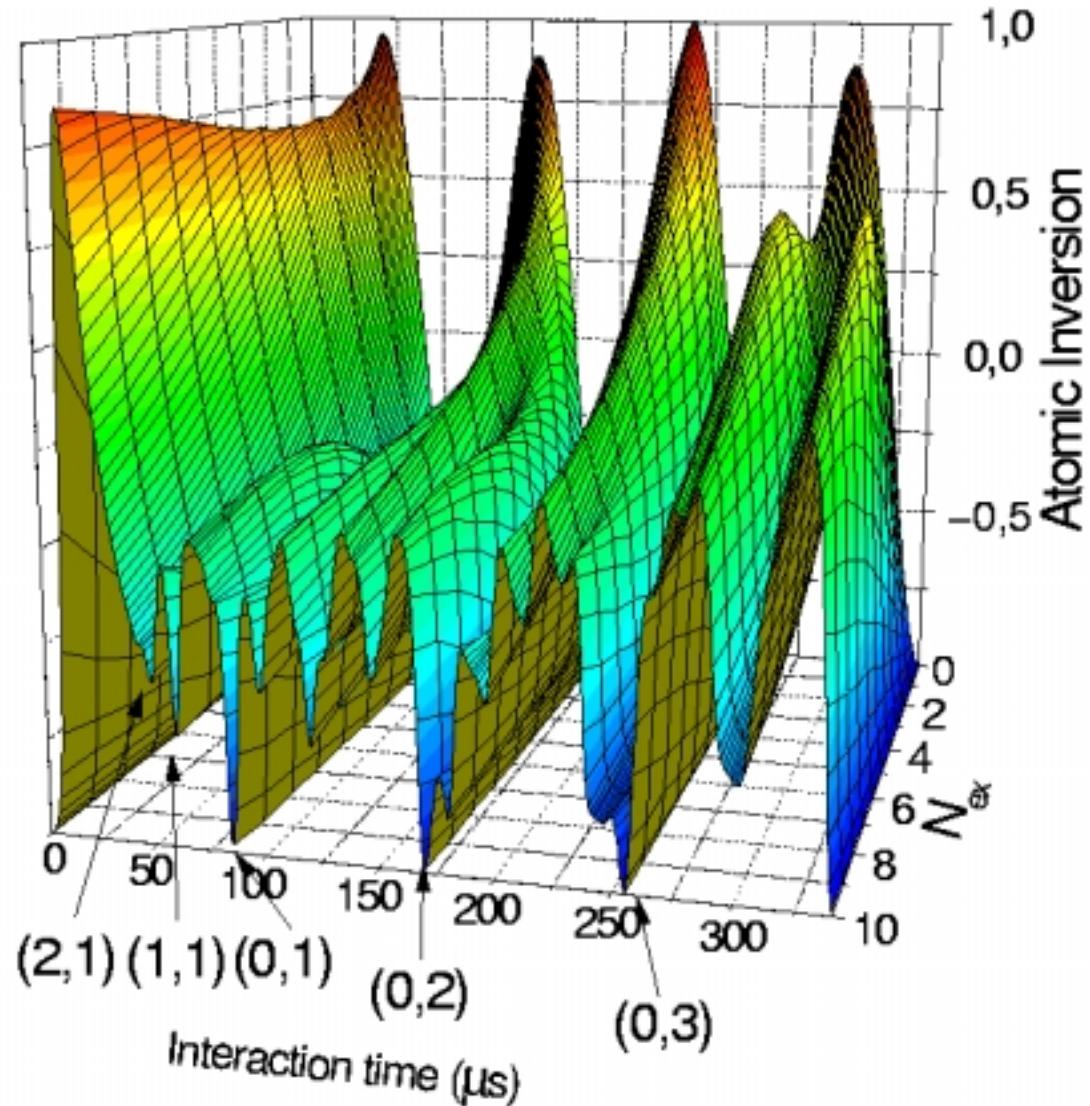
Low Temperature Behaviour of the One-Atom-Maser



Trapping states are characterised by the pair of numbers (n_q, k) that satisfies the relation:

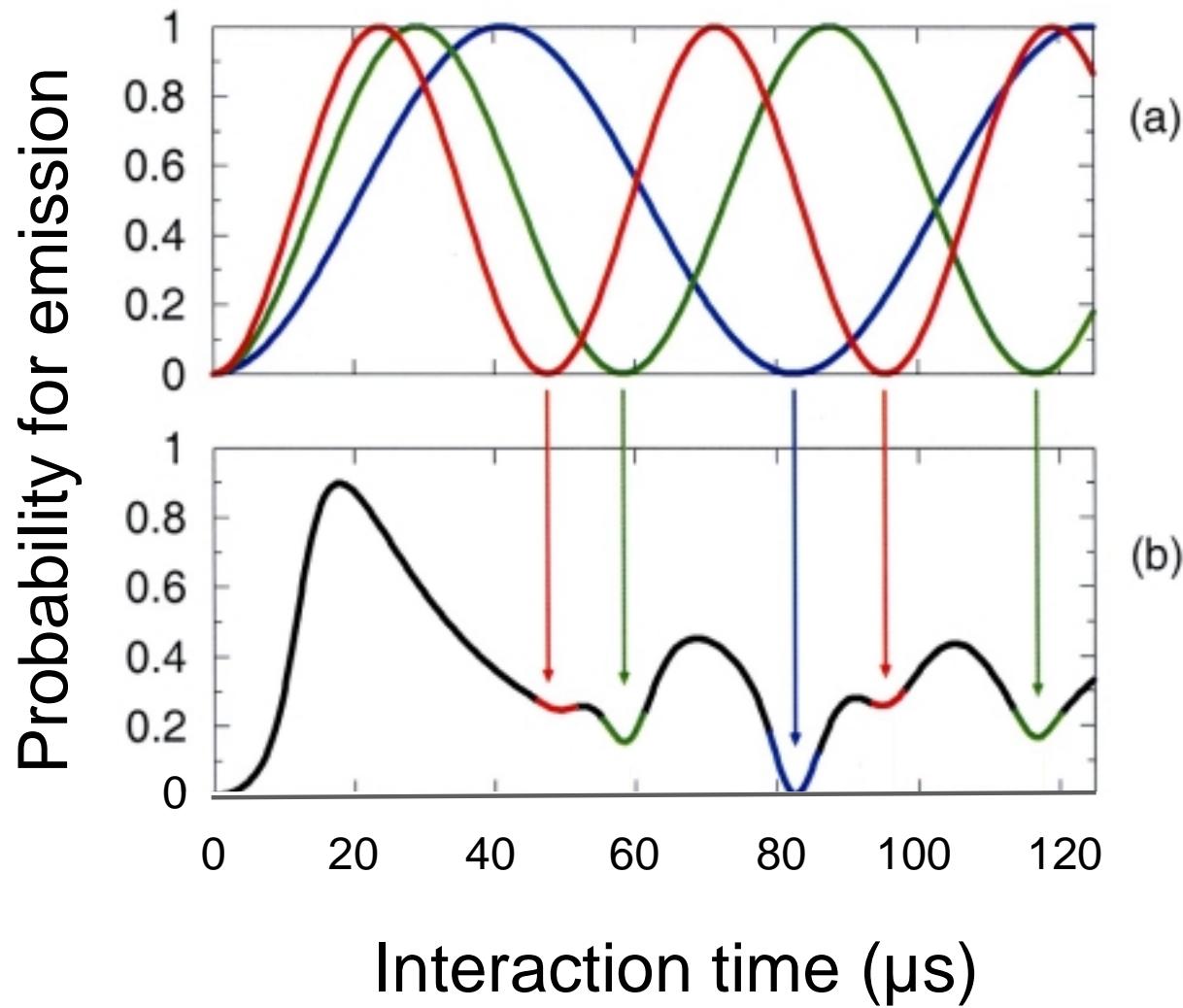
$$\sqrt{n_q+1} \text{ g}t_{\text{int}} = k\pi$$

The Micromaser Pump Curve at Low Temperatures



Trapping states appear as valleys in
the N_{ex} direction

Formation of Trapping States



Trapping state:
 $N_{\text{ex}} = 5, n_{\text{th}} = 0$

(2.1) (0.1) (1.2)
(1.1) (2.2)

Generation of Fock States

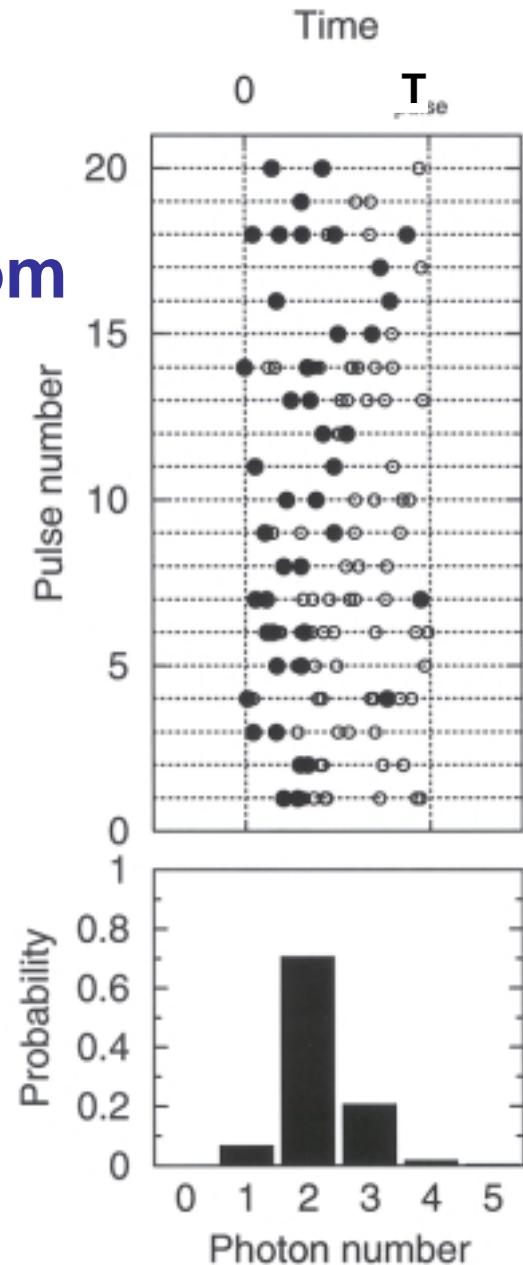
- 1) **State reduction** (dynamical process)
Nature **403**, 743-746 (2000)
- 2) **Trapping states** (steady state generation of Fock states)
Phys. Rev. Lett. **82**, 3795-3798 (1999)
- 3) **Fock state generation on demand** using trapping states
Phys. Rev. Lett. **86**, 3534-3537 (2001)

Pulsed excitation of Rydberg atoms
Interaction time tuned to a trapping state

**When tuned to the (1,1) trapping state
one ground state atom and one photon
is produced per excitation pulse**

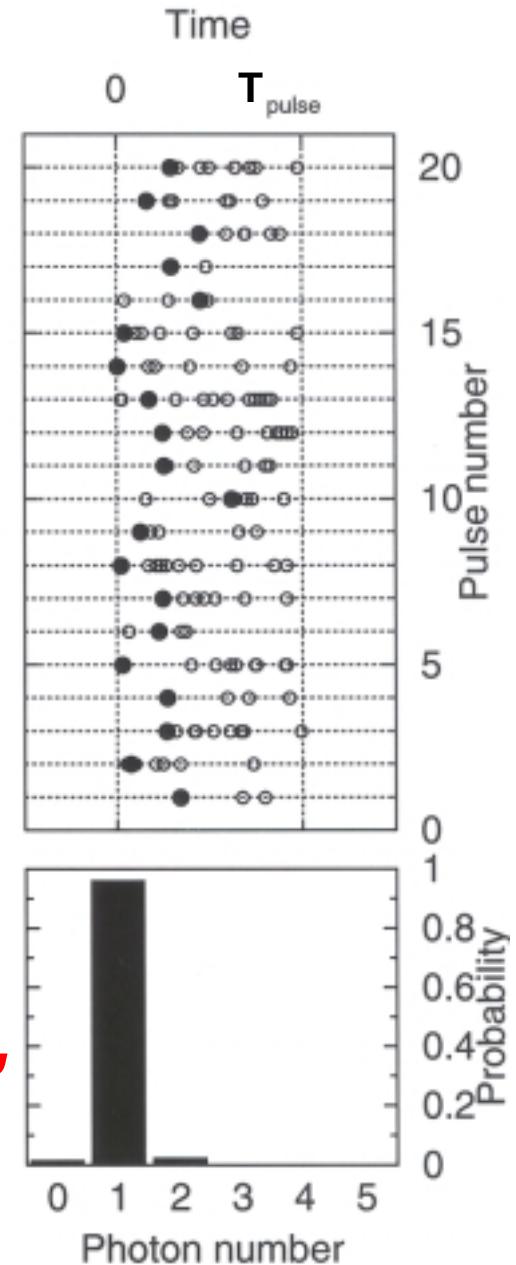
Photon-Fock-States on Demand

$t_{int} = 45 \mu s$
 deviates from
 trapping
 condition



$t_{int} = 58$
 interaction time
 for the (1,1)
 trapping state

*S. Brattke,
 B.T.H. Varcoe,
 H. Walther,
 Phys.Rev.Lett.86,
 3534-3537 (2001)*



New method for the determination of the Wigner function

*P. Lougovski, E. Solano, Z.M. Zhang, H. Walther
H. Mack, W.P. Schleich*

quant-ph/0206083

Determination of the Wigner Function

$P_g(t, \alpha)$ Probability for an atom in the ground state $|g\rangle$
 α determined by displacement field

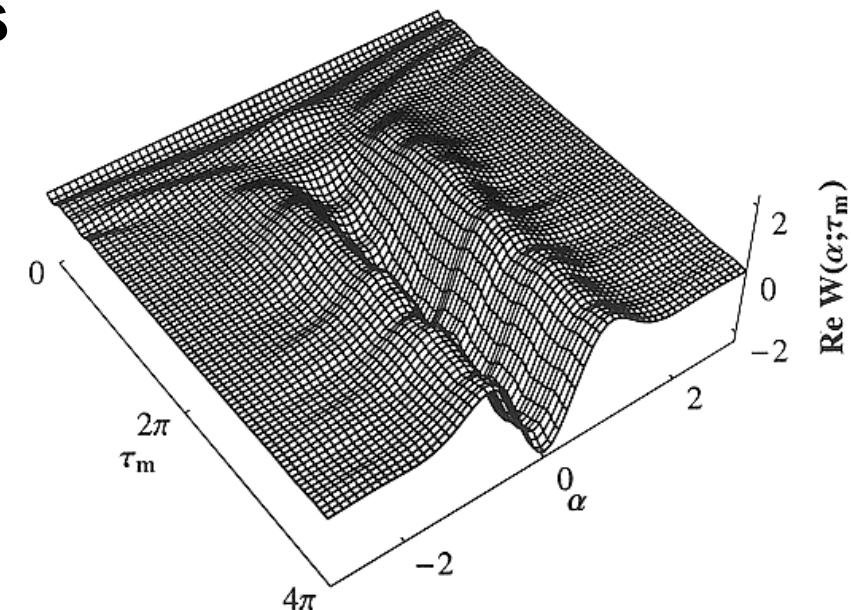
$$P_g(t, \alpha) = \frac{1}{2} - \frac{1}{2} \sum_{n=0}^{\infty} P_n(\alpha) \cos(2\sqrt{n+1}\Omega t)$$

Wigner-function obtained from the Fresnel transform of Rabi oscillations

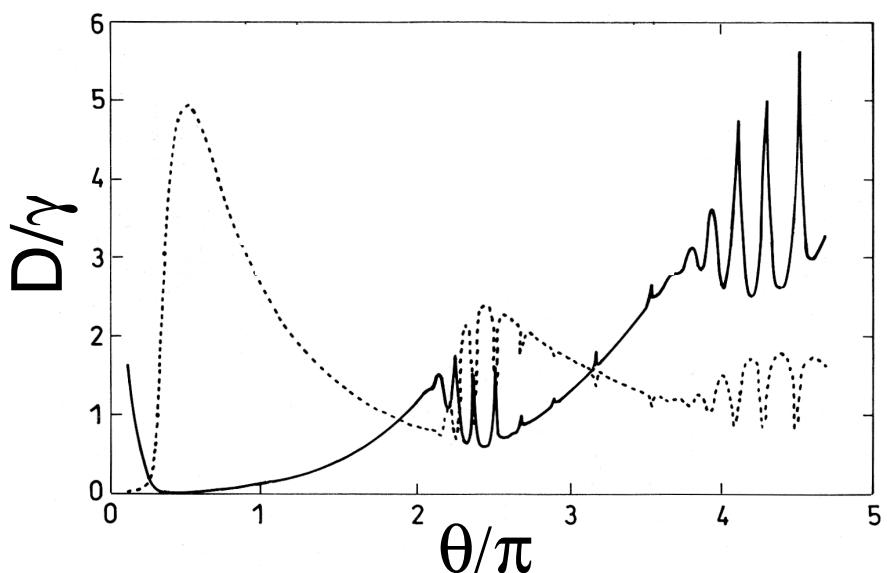
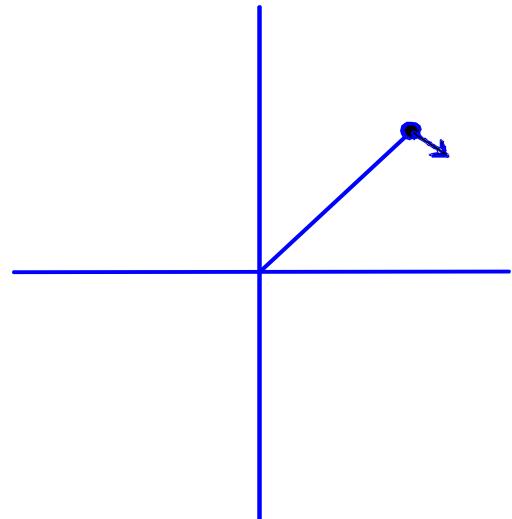
$$W(\alpha) \approx \frac{8}{\pi\sqrt{i}} \int_0^\infty d\tau e^{i\tau^2/\pi} [P_g(\tau, \alpha) - \frac{1}{2}]$$

Rabi oscillations
 of a one-photon
 number state

W(0) = -1.9



Phase space representation:



Spectrum of the maser:

$$s(\omega) = \left| \int_0^{\infty} \langle E(t) \rangle e^{-i\omega t} dt \right|^2$$

$$s(\omega) = \frac{\langle E(0) \rangle}{(\omega - \omega_F)^2 + \frac{1}{4}D}$$

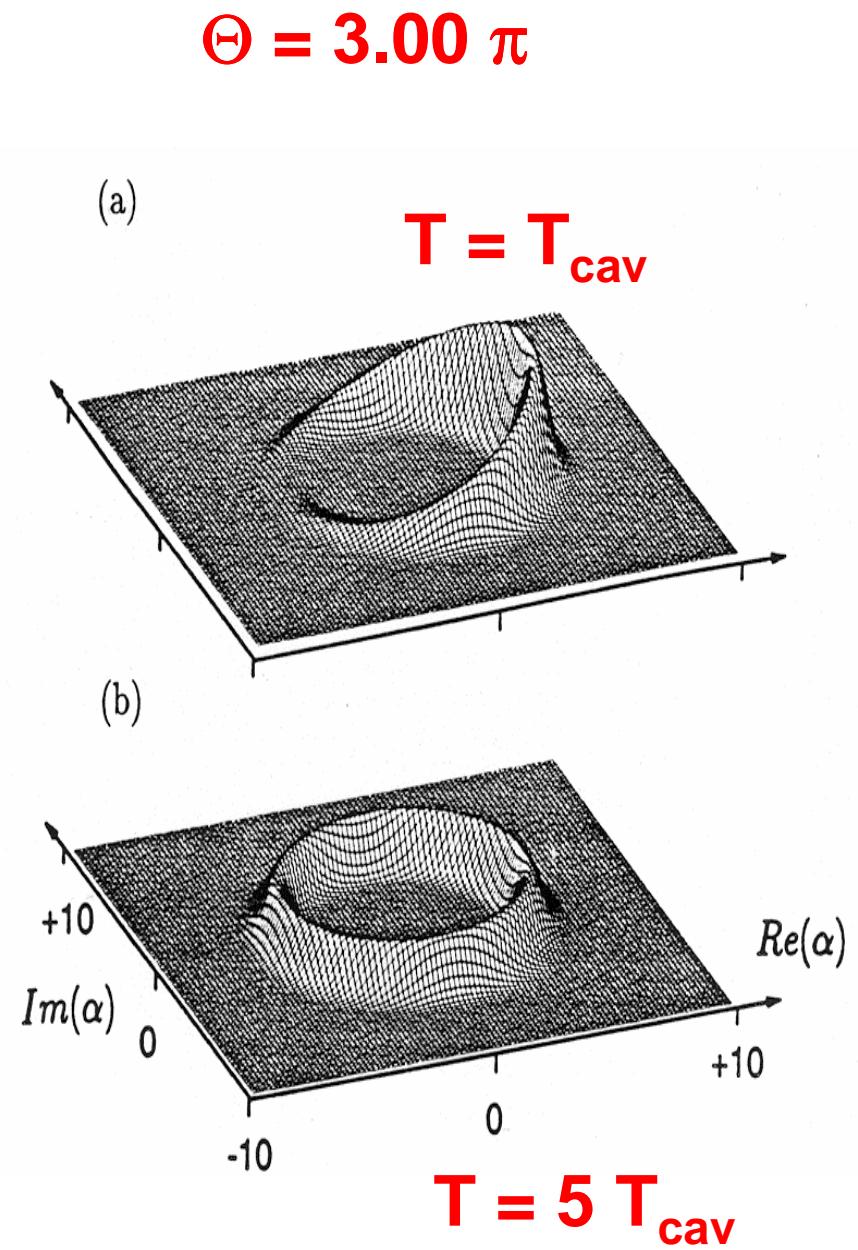
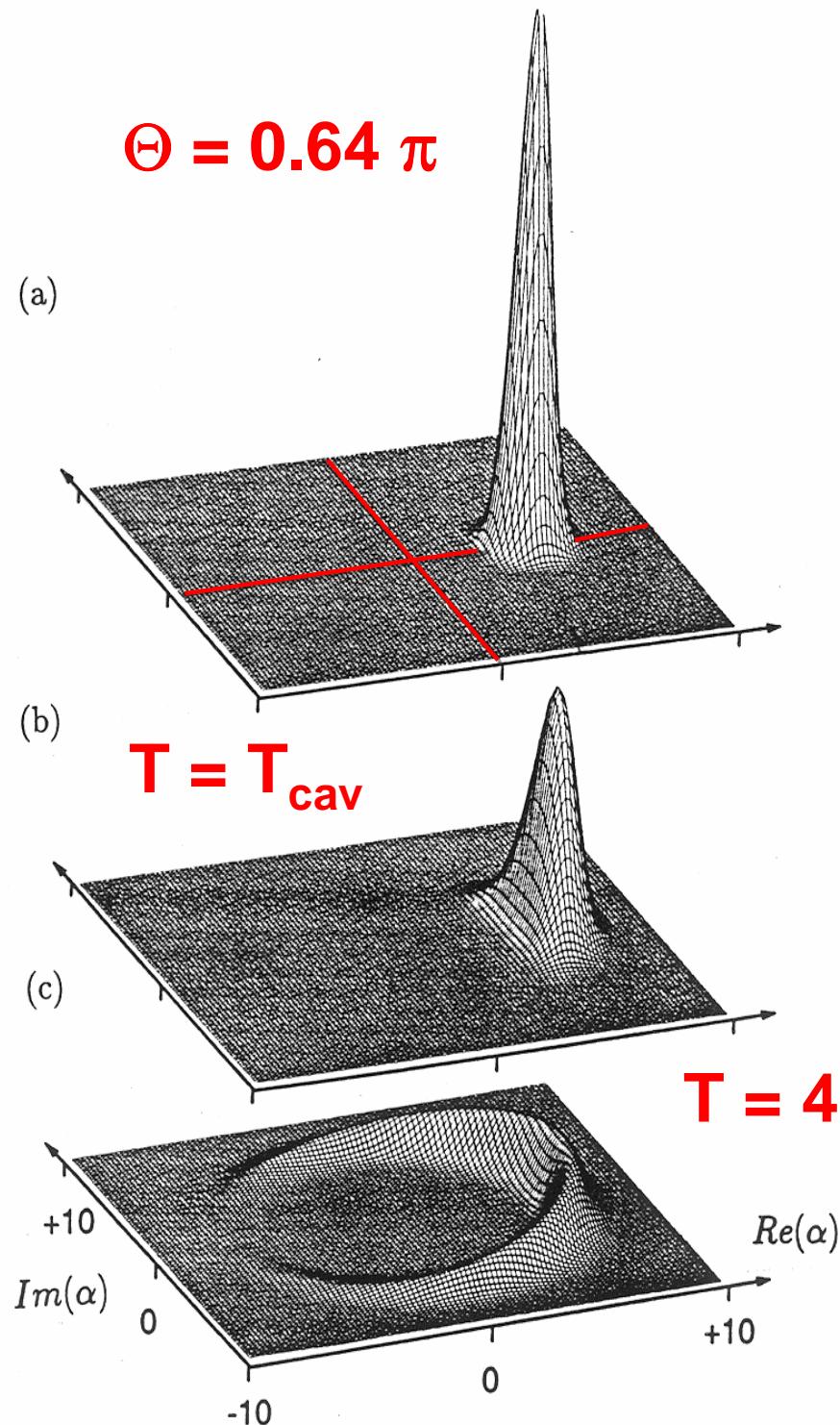
$$D \approx \frac{\alpha + \kappa(2n_{th} + 1)}{4 \langle n \rangle}$$

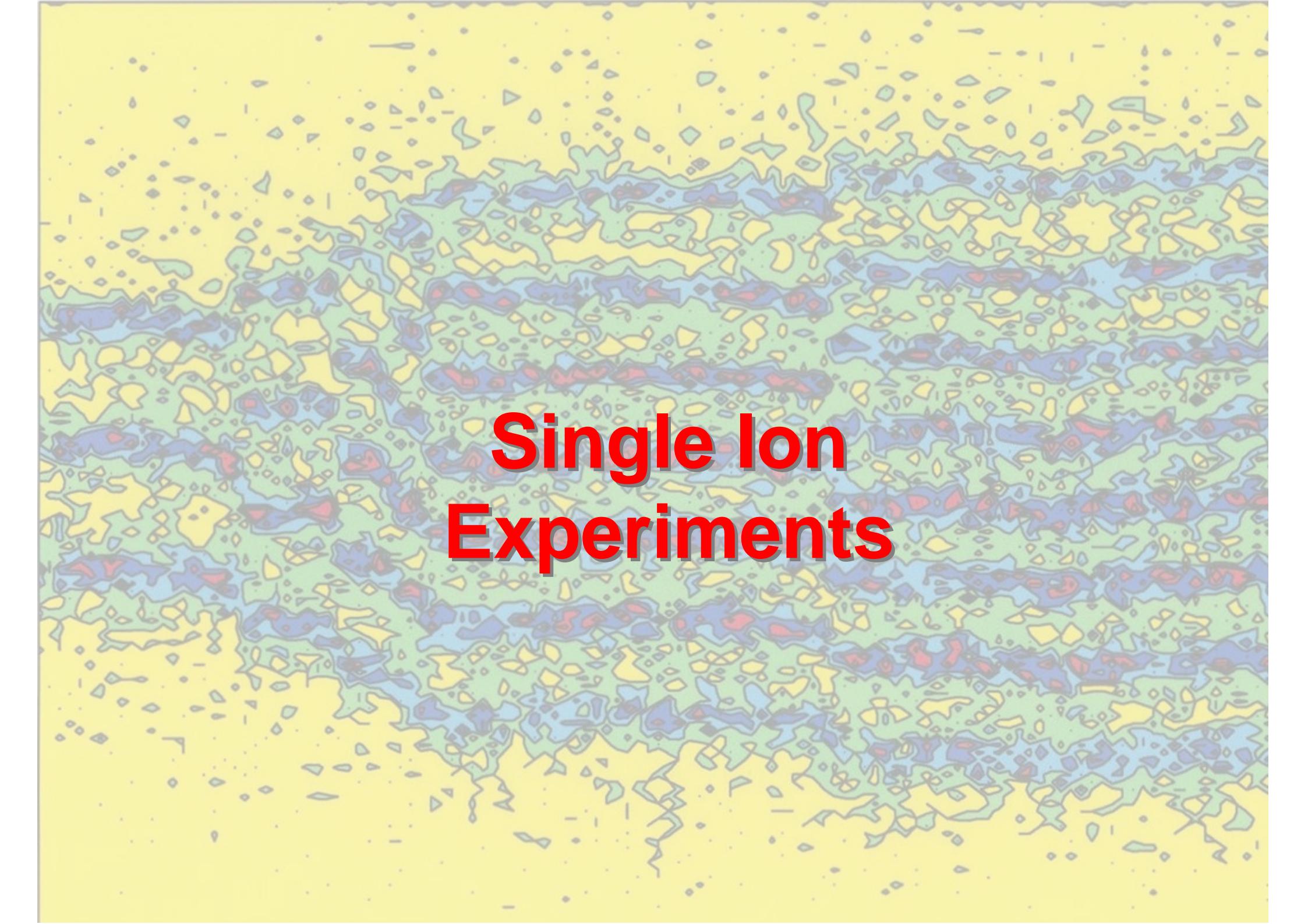
$\alpha = \kappa \theta^2$ $\theta = g t_{int} \sqrt{N}$ pumping parameter

κ : cavity decay rate

n_{th} : thermal field

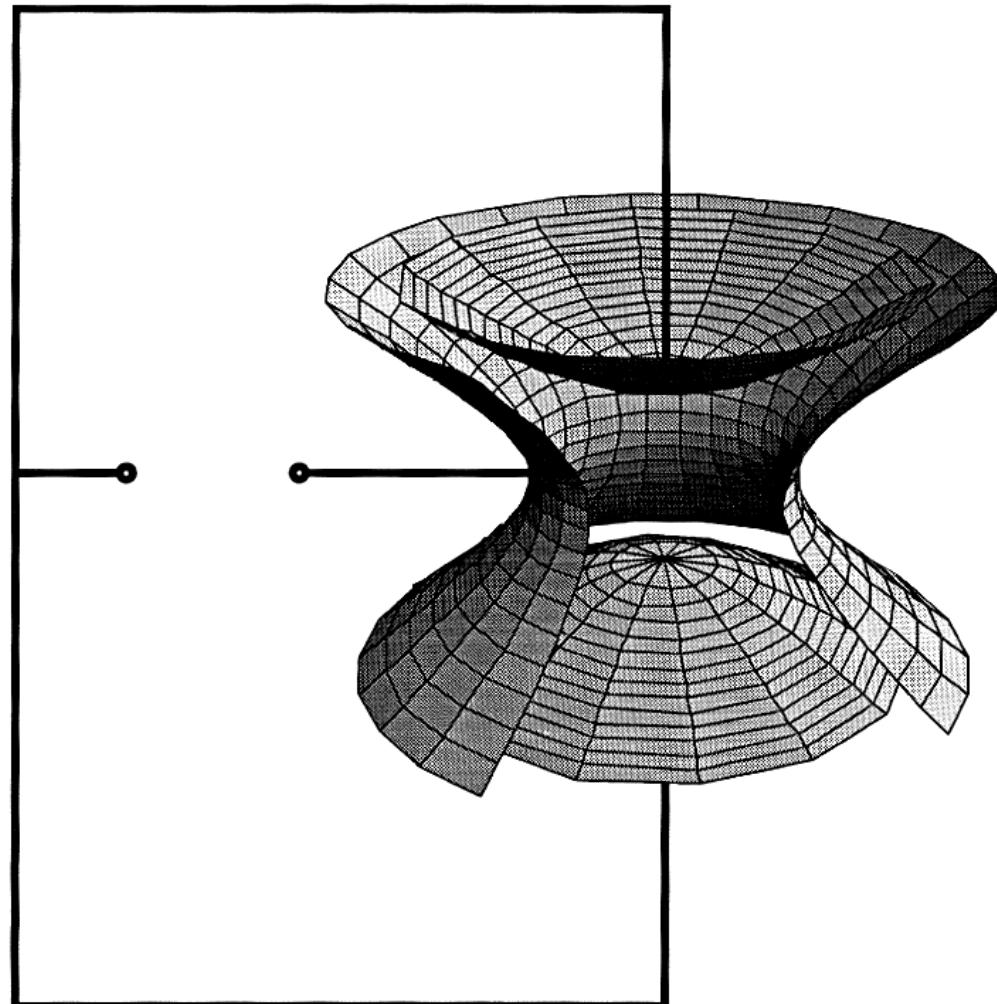
M.O.Scully, H.Walther, G.S.Agarwal,
T. Quang, W. Schleich,
Phys. Rev. A 44, 5992 (1992)



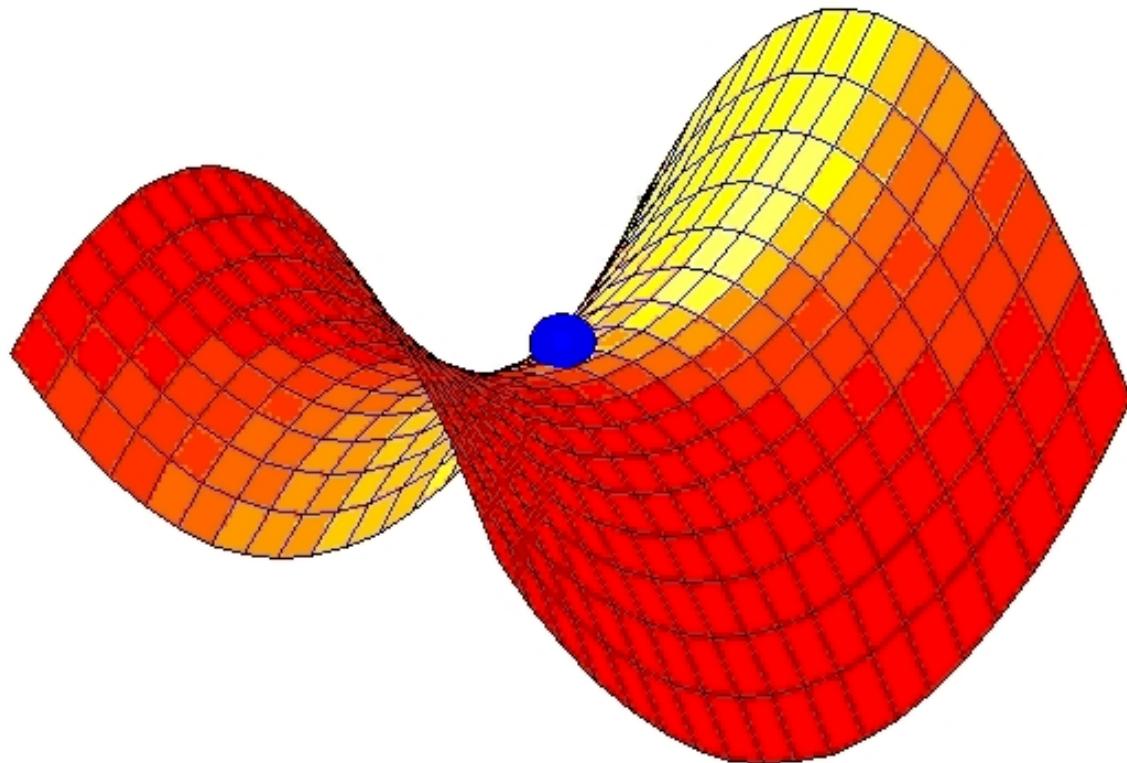


Single Ion Experiments

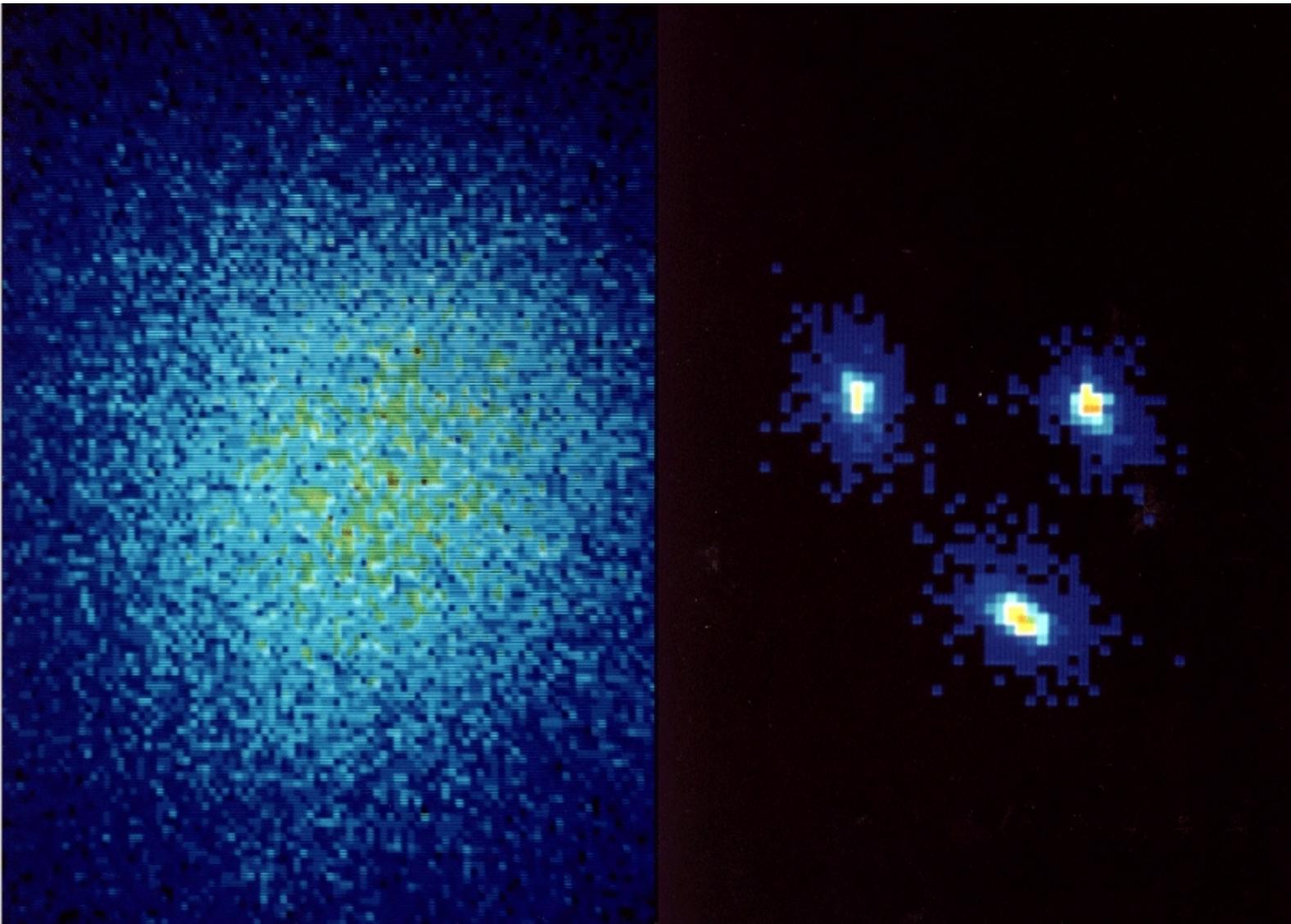
$$U_0 + V_0 \cos \omega_{RF}$$



Principle of the Paul trap



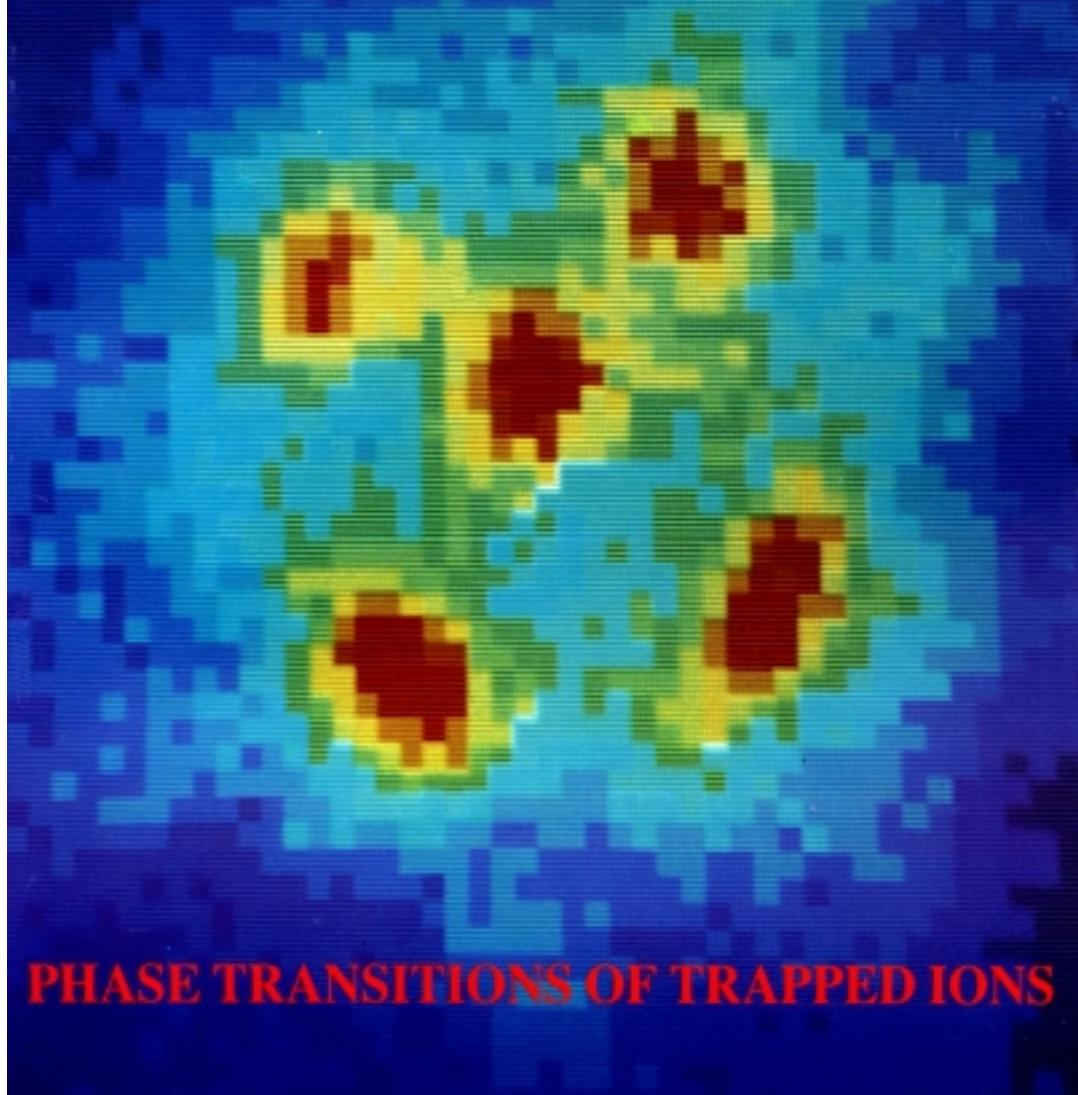
Magnesium Ions in a Paul-Trap



nature

INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

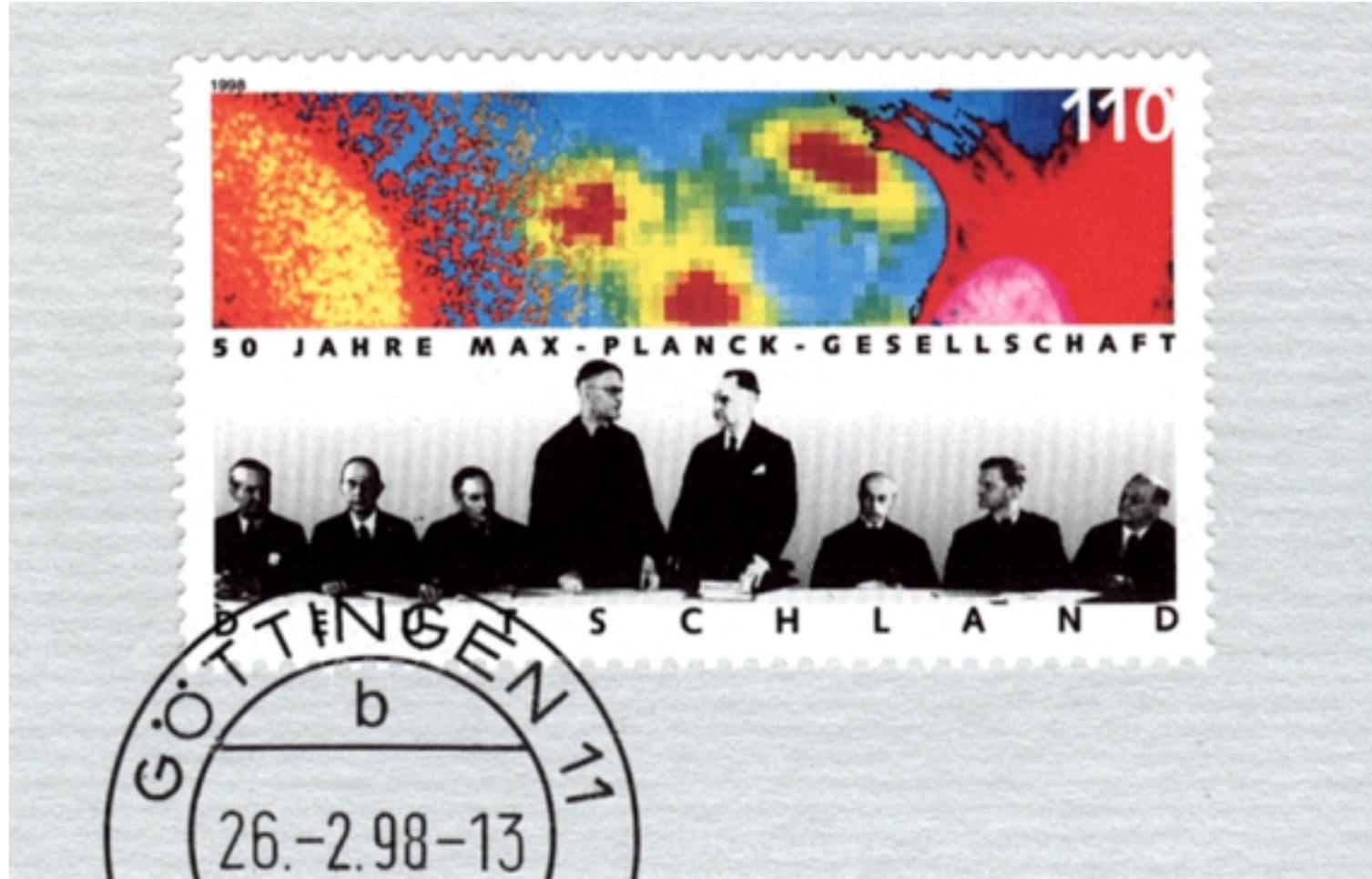
Volume 334 No. 6180 29 July 1991 £1.95



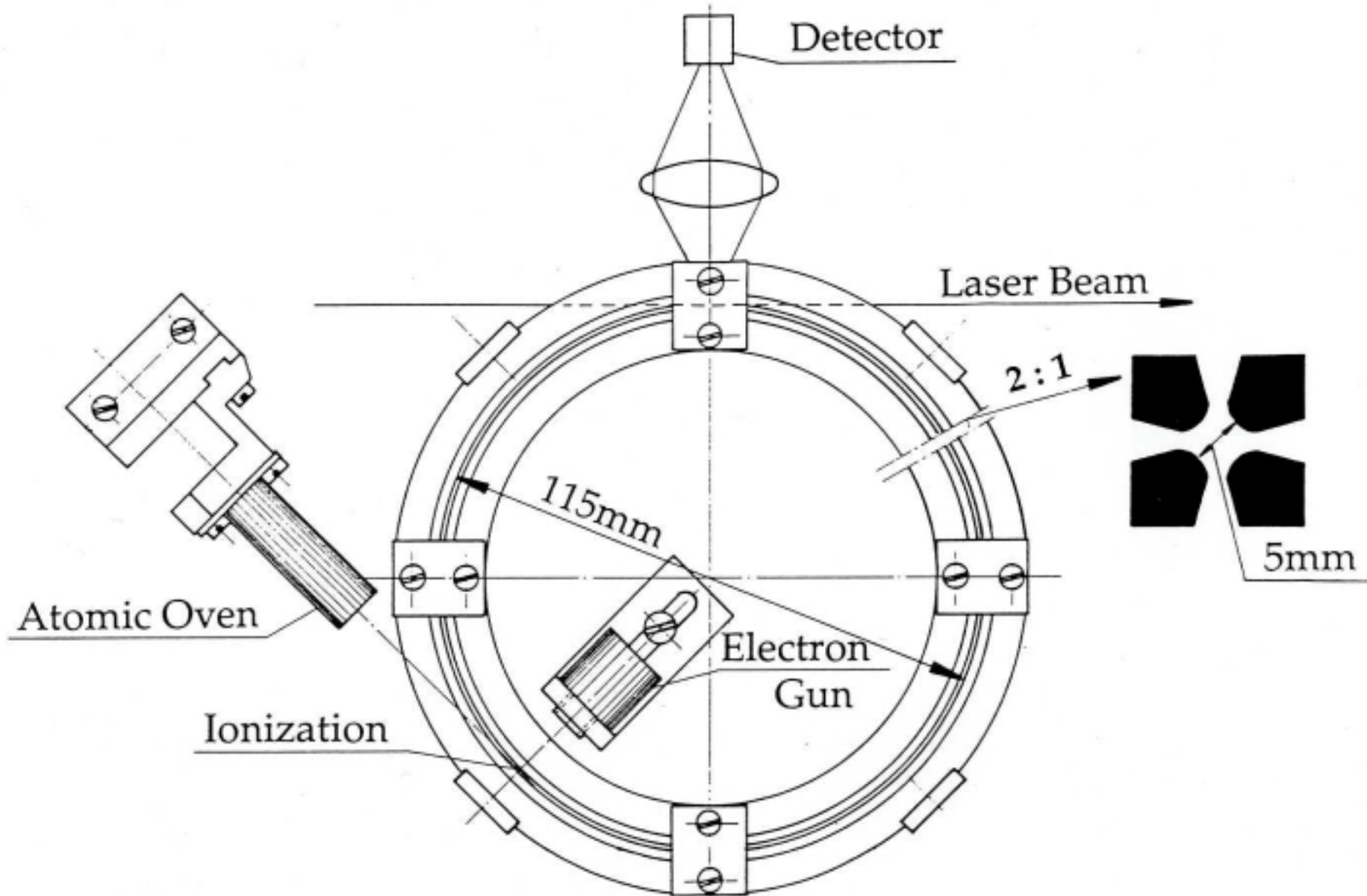
PHASE TRANSITIONS OF TRAPPED IONS



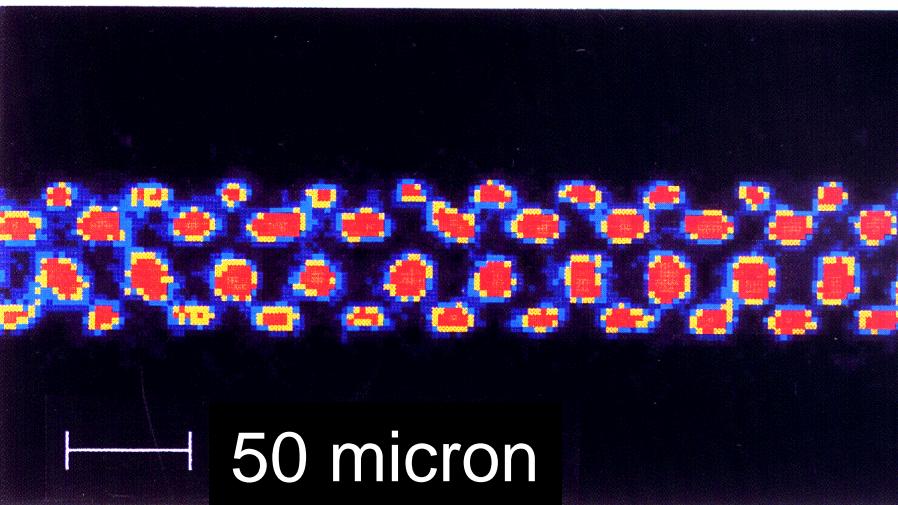
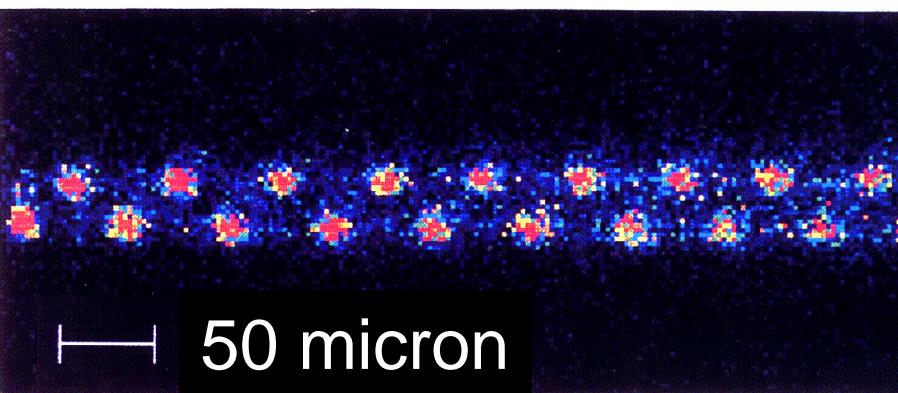
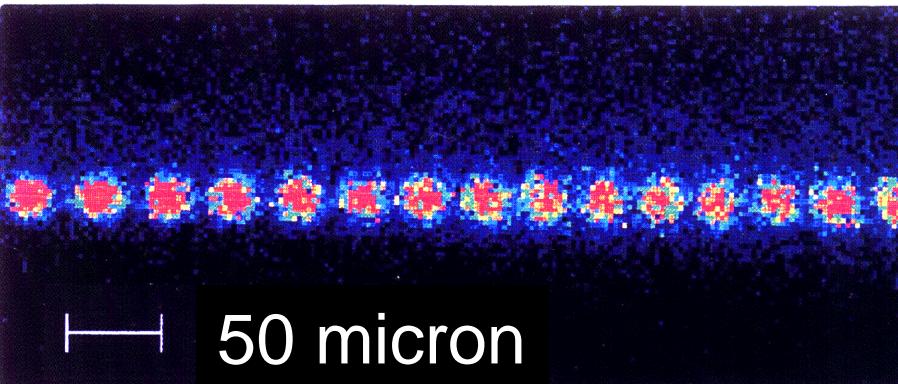
Stamp for the 50th Anniversary of the Max-Planck Society



Scheme of the Ring-Trap



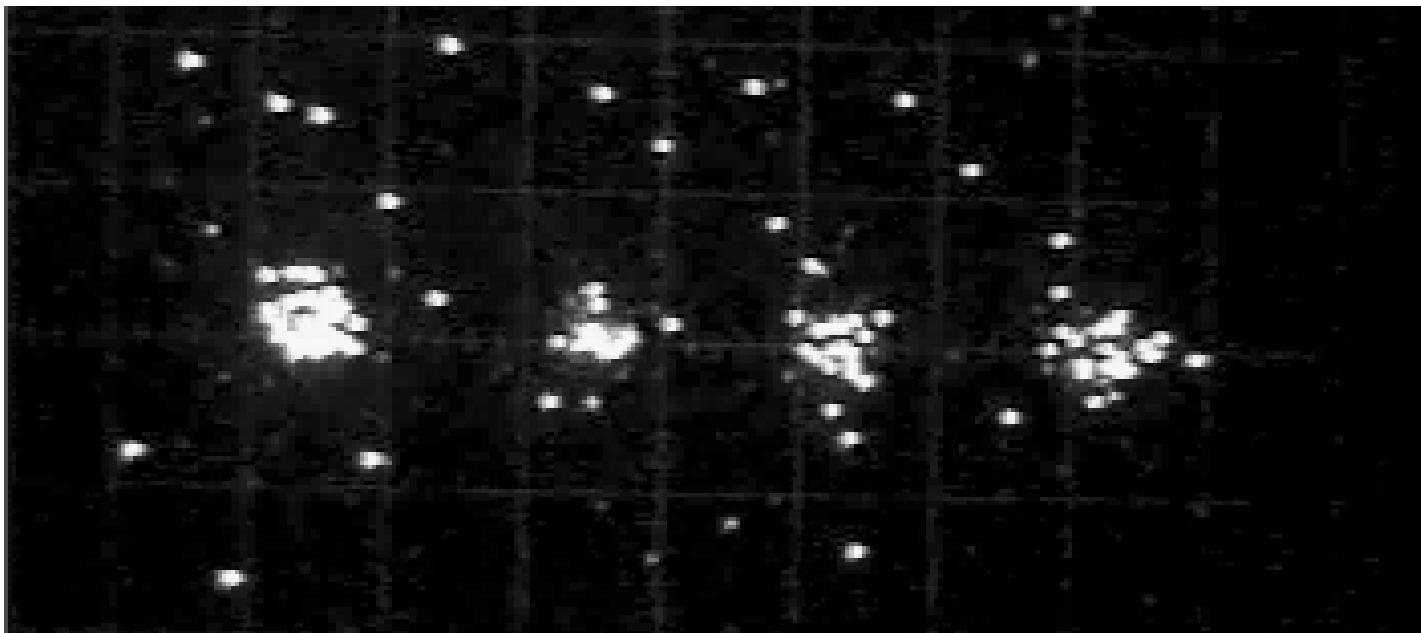
Ordered Ion Structures in the Ring-Trap



**G. Birkl, S. Kassner,
H. Walther,
Nature 357, 310 (1992)**



Ion Chain



History: Feynman: simulation of quantum systems
Deutsch, Josza, Shor, Grover: considerable speedup
Bennett et al.: quantum cryptography

Classical Bit: A classical bit is represented by
0, 1 a voltage (one or zero)



Quantum Bit (Q-Bit): Represented by a quantum object – **two level system** (spin-orientation, polarization, atomic or molecular states)
 $|0\rangle, |1\rangle$



Superpositions are possible $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

→ **N classical bits represent one out of 2^N values**
N quantum bits can represent 2^N values simultaneously



Chain of 50 ions ($1,125,899,906,842,624$ numbers may be simultaneously processed.)

Quantum Logic

In order to produce logic operations strong interaction between parameters representing information is necessary.

Gates described by unitary transformations.

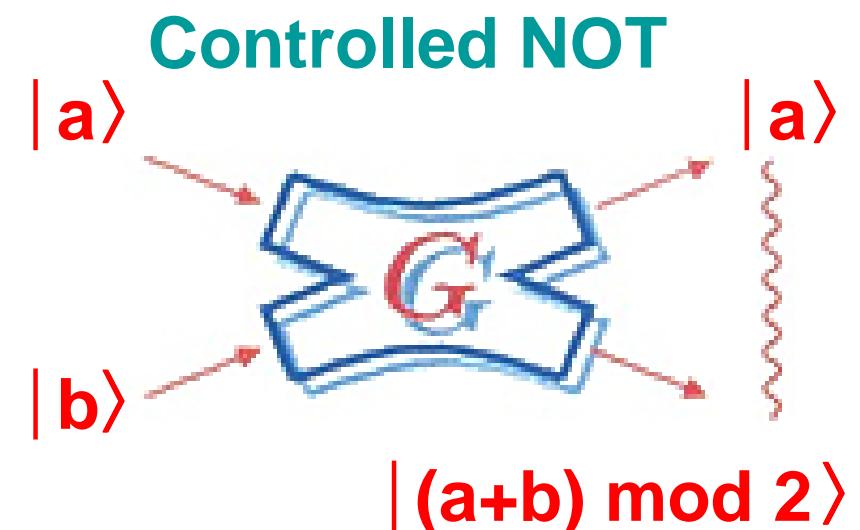
Controlled NOT is basic gate

b = target bit

a = control bit

a = zero: b is not changed

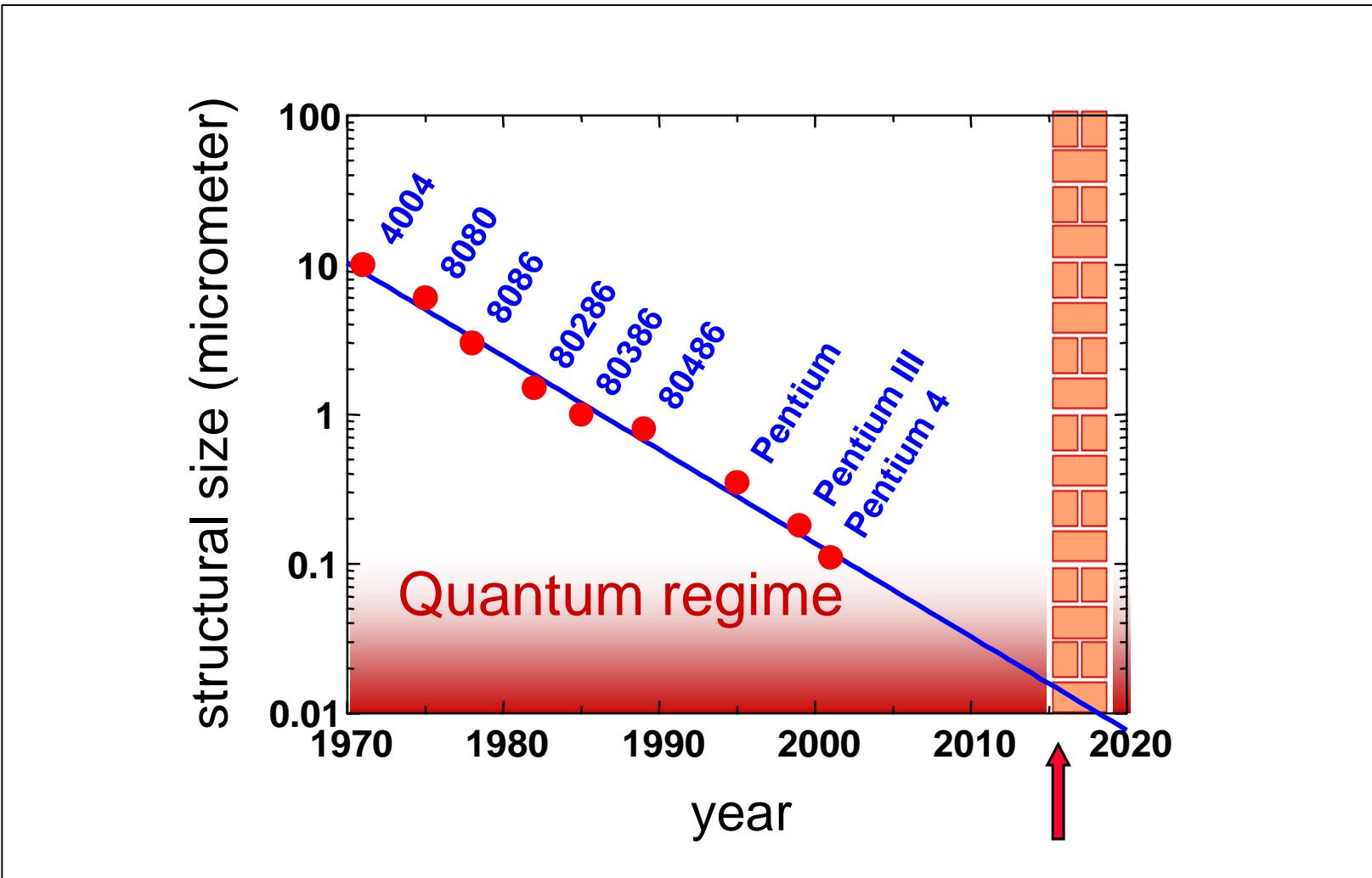
a = one: b is negated

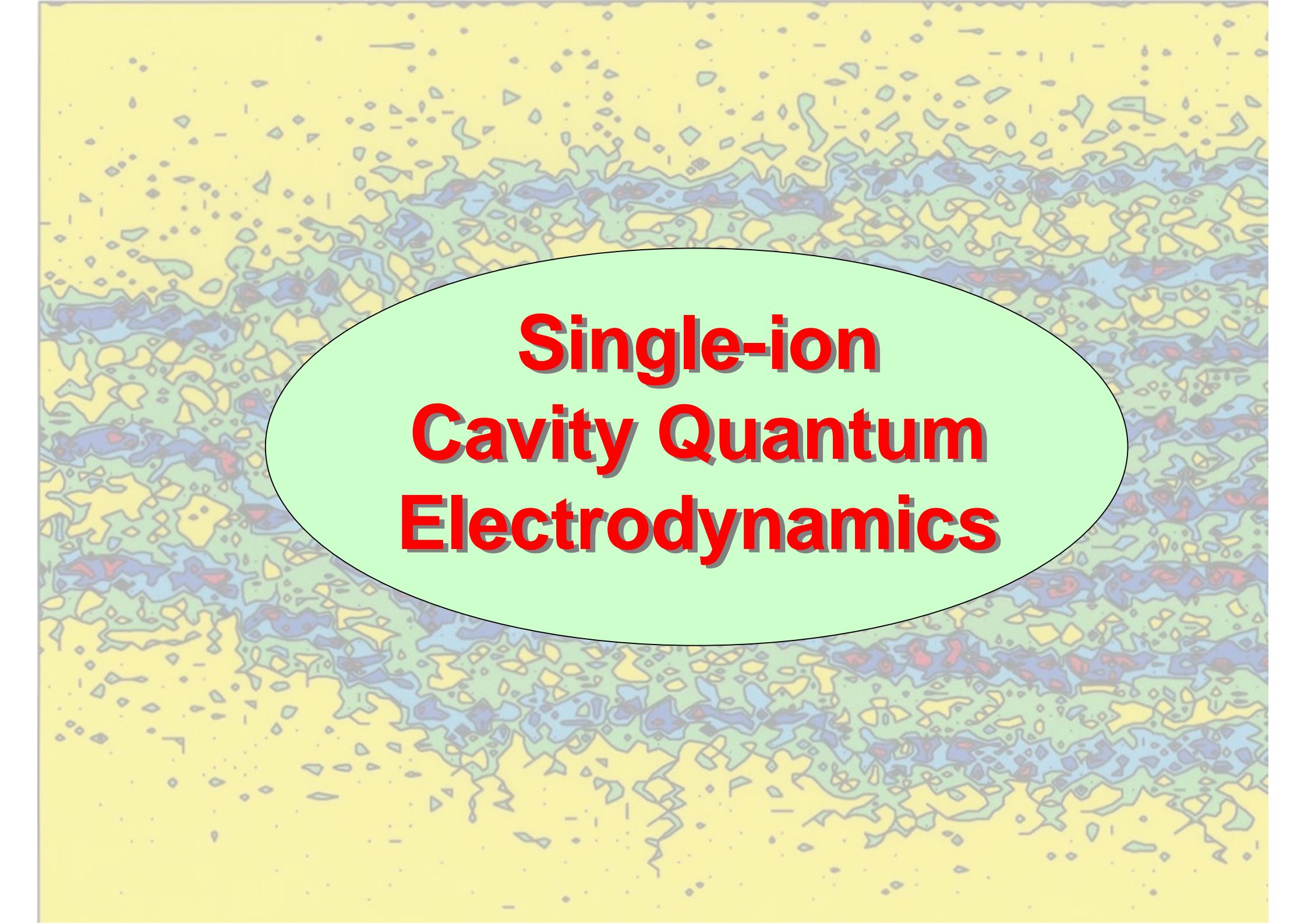


Controlled NOT operation changes a **superposition** into an **entangled state**

$$(|0\rangle + |1\rangle) \otimes |1\rangle \Rightarrow |0\rangle|1\rangle + |1\rangle|0\rangle$$

Development of Microelectronics - Moore's Law

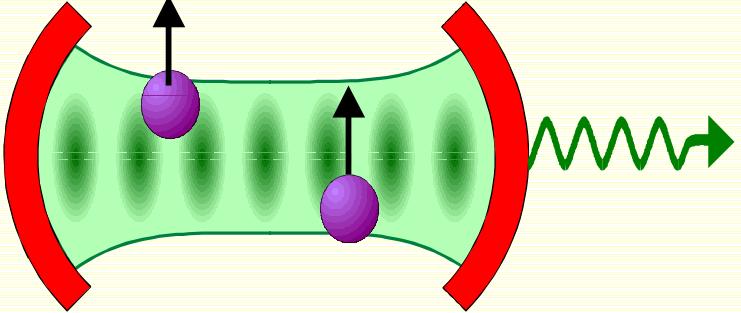




**Single-ion
Cavity Quantum
Electrodynamics**

Single-Ion Cavity QED

Single mode cavity QED



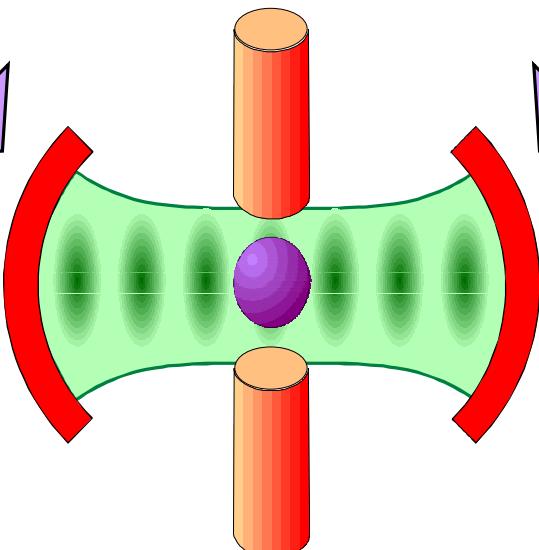
strong atom-field coupling

Single ion trapping

- **sub-wavelength position control**
- **unlimited observation time**



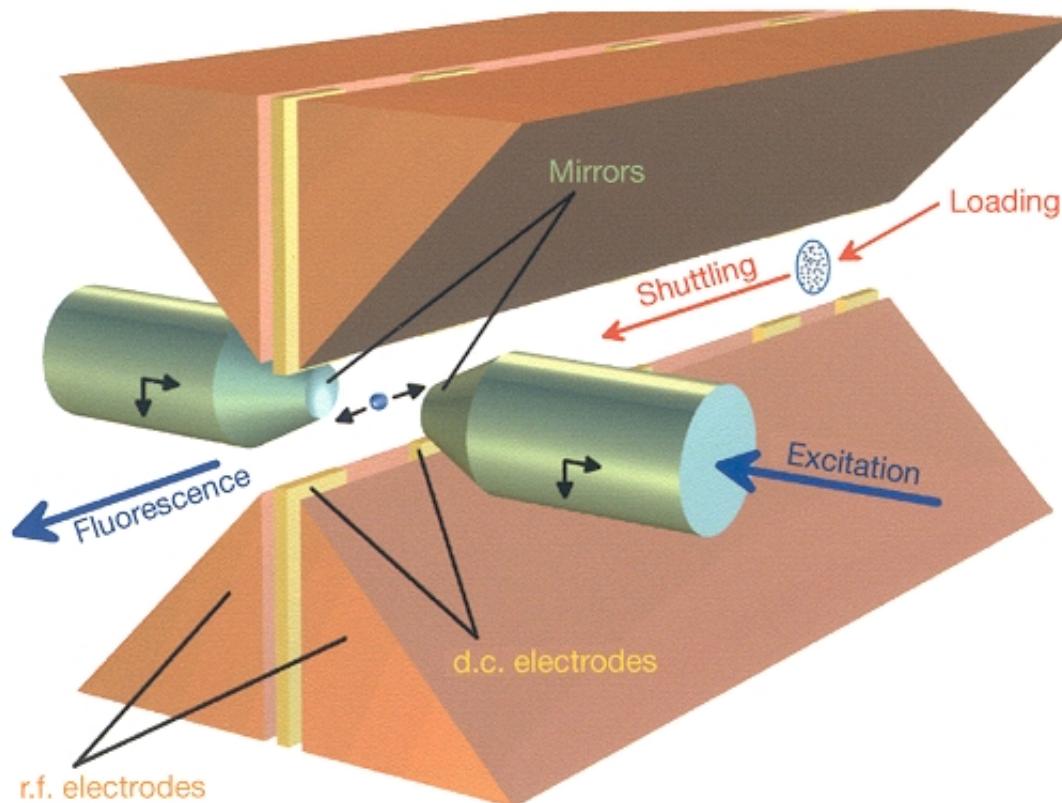
Combine the technologies:



- **deterministic ion-field interaction**
- **single-photon gun**
- **single-ion laser**

Setup: ion trap and optical cavity

How to place the ion between the mirrors?



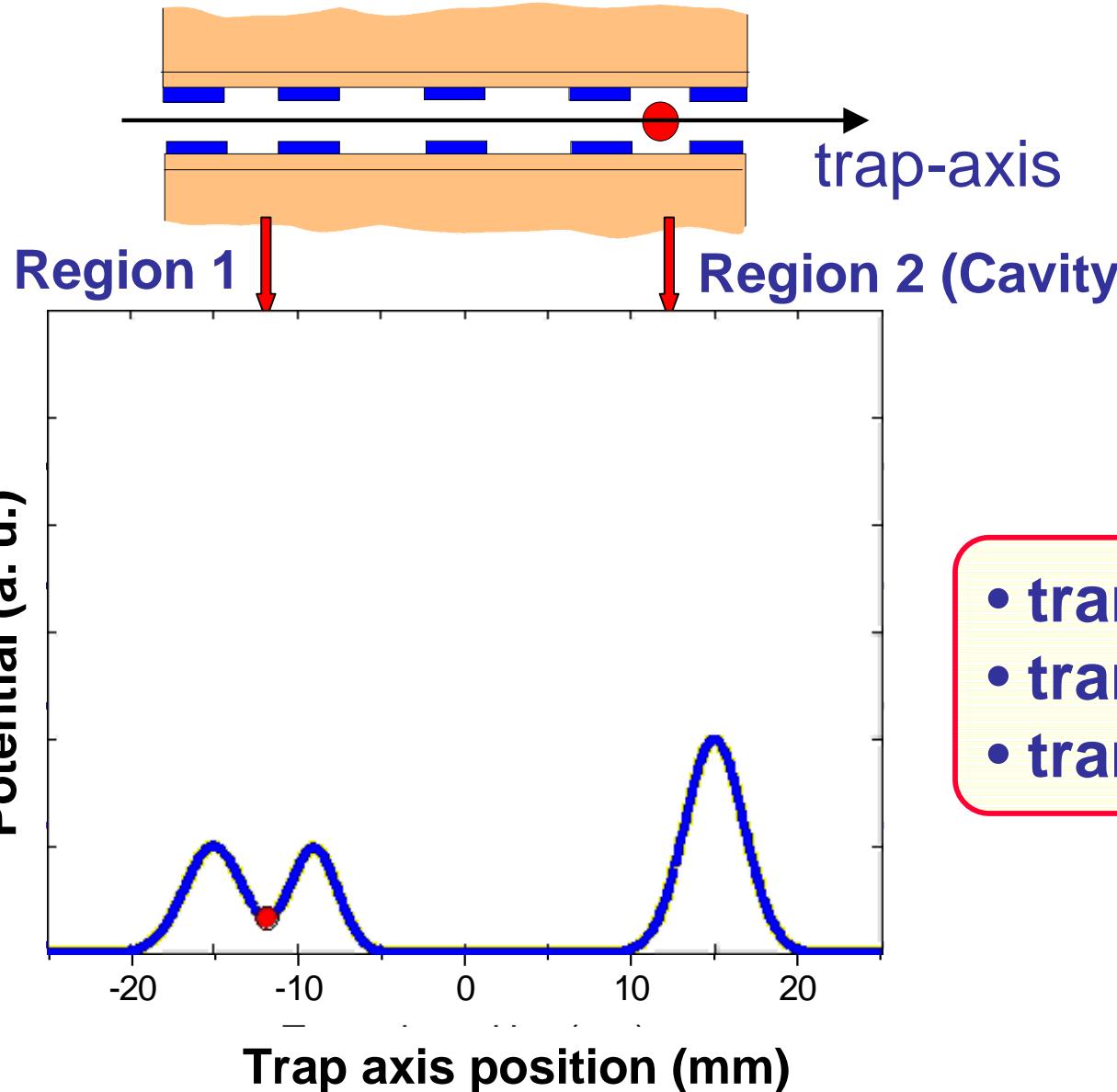
Nature 414, 49 (2001)

Trap design:

- Linear RF trap with open electrode configuration
- separate loading region
- Ion transfer by DC fields

→ no coating or charging of the dielectric mirrors even at small cavity length

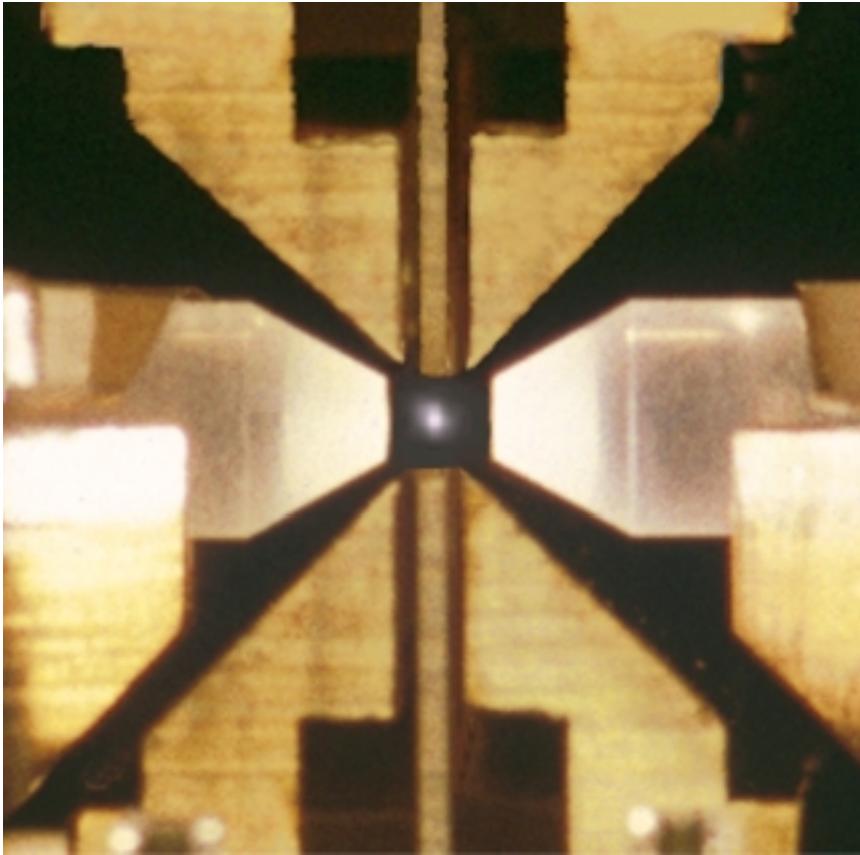
Ion Transfer from Loading Region to Cavity



axial ion position
controlled by 5
DC-electrodes

- transfer distance: 25 mm
- transfer time: 4 ms
- transfer efficiency: 50 %

A Single Ion in a Cavity



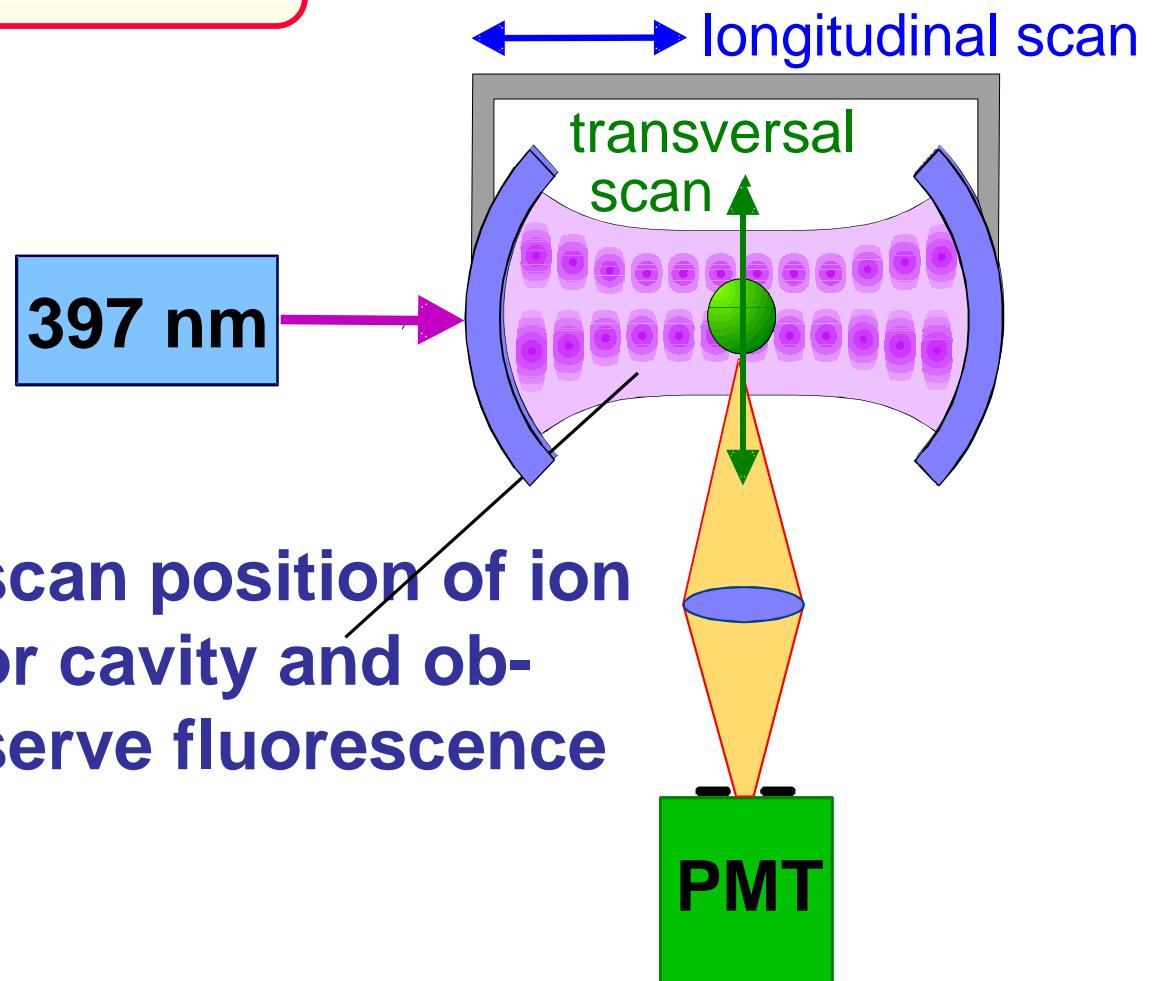
Single-Ion Mode Mapping (SIMM)

single $^{40}\text{Ca}^+$ ion as a nanometric probe of the electromagnetic field:

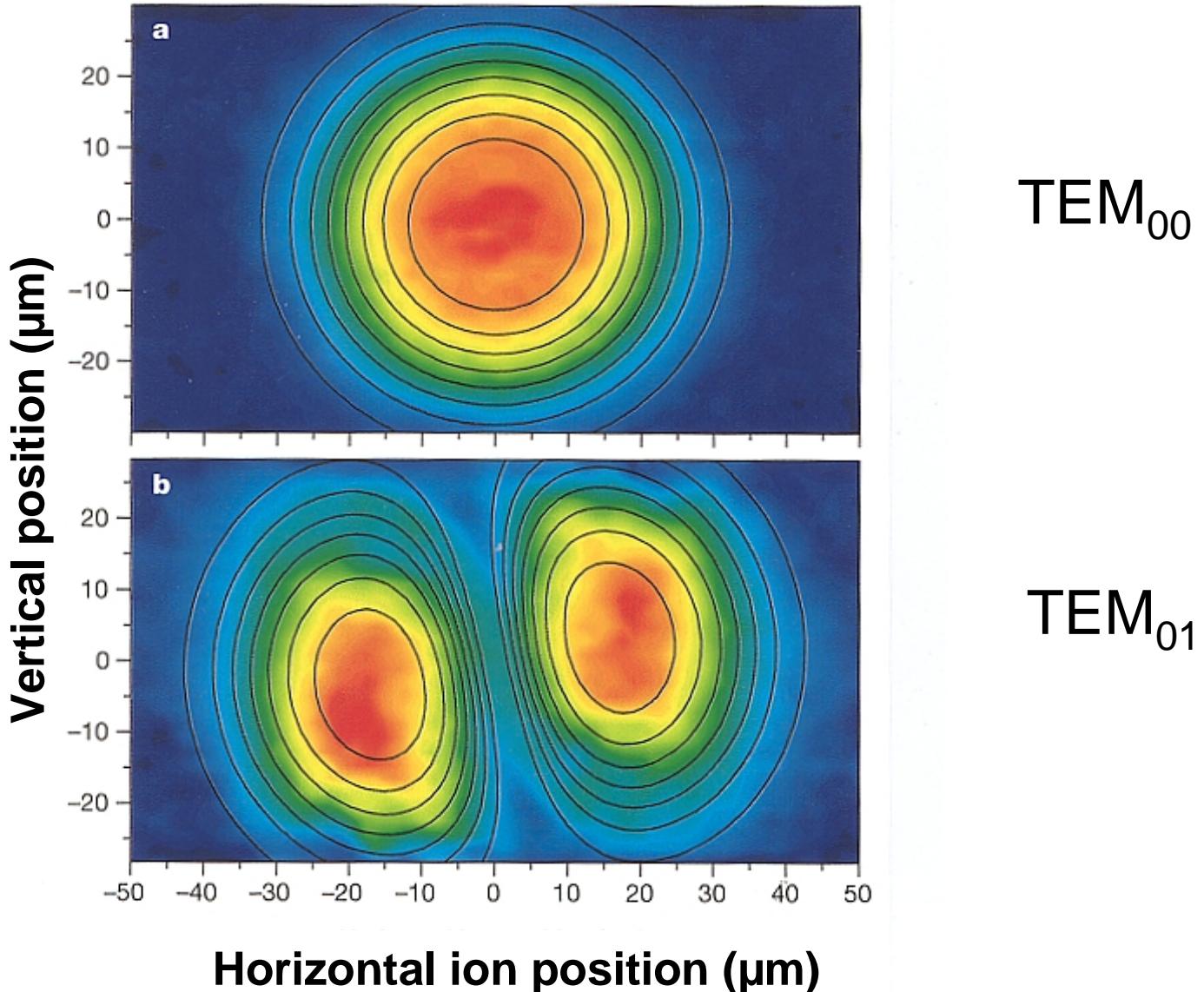


Test of the deterministic interaction of ion and cavity field

- resolution down to 10 nm
- first step towards single-ion cavity QED



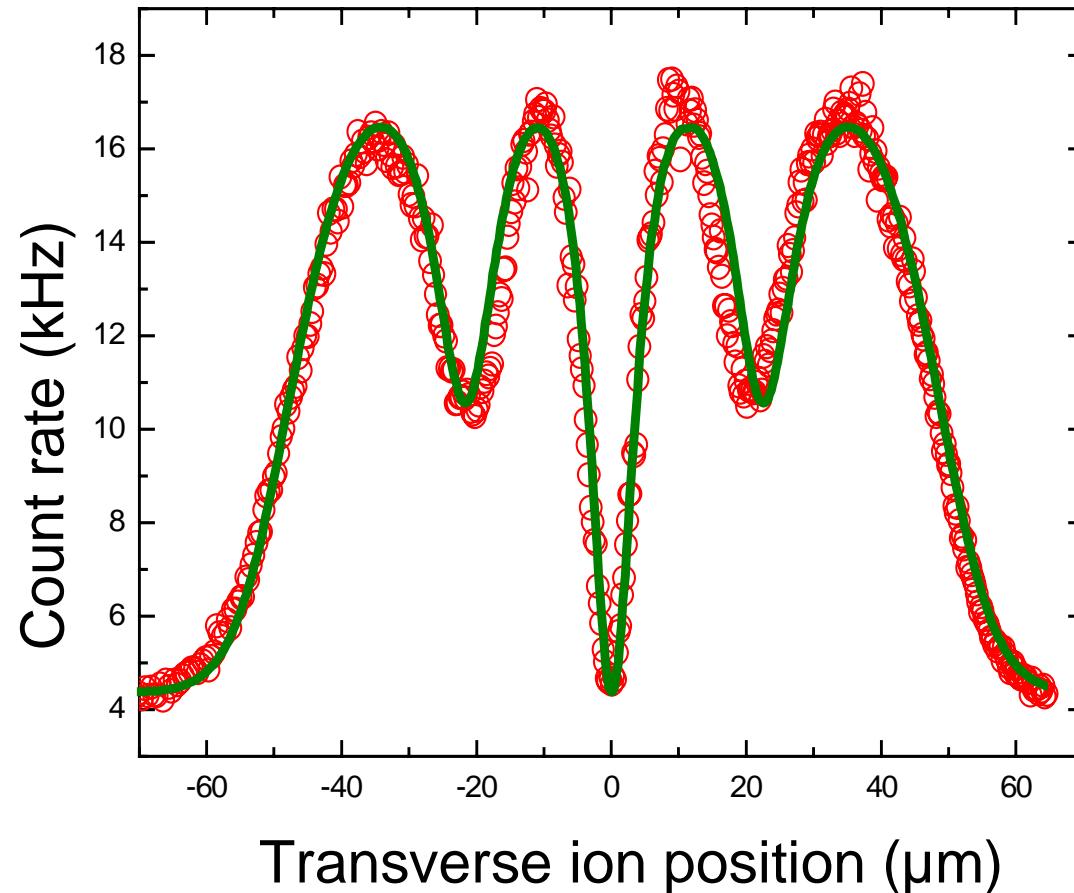
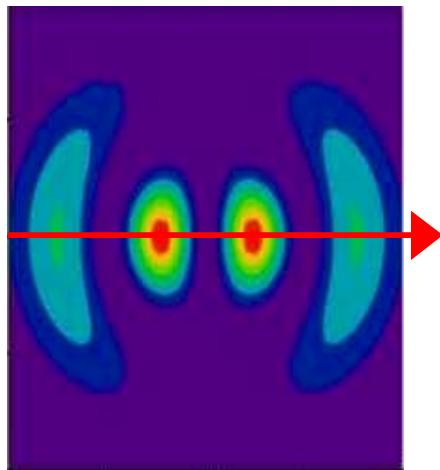
Two-Dimensional Images of the Cavity Field



Higher-Order Gauss-Laguerre Modes

Example:

TEM_{11} Mode

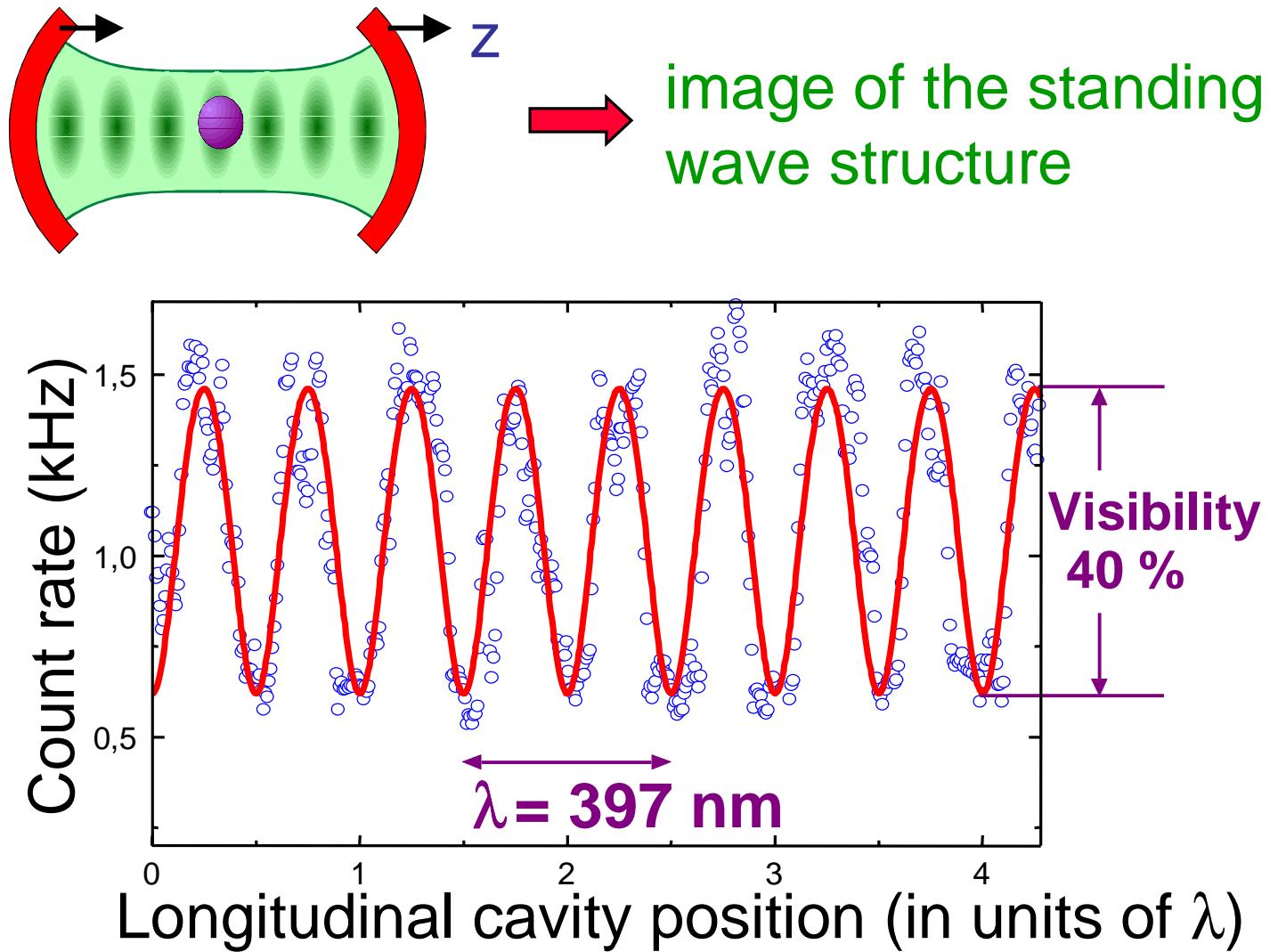
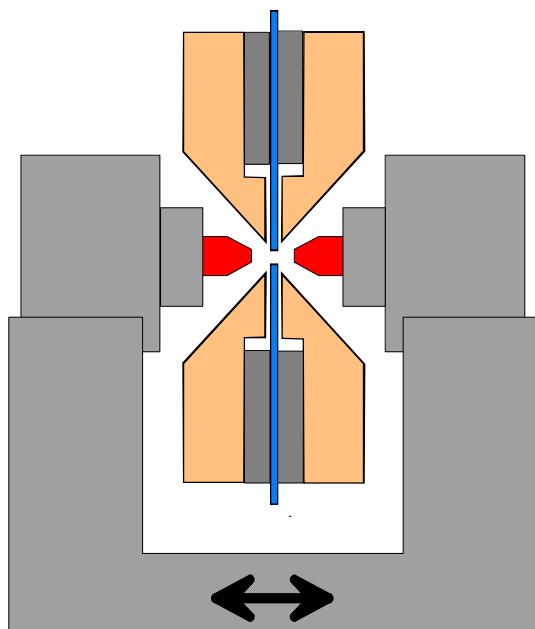


Fit to fluorescence intensity includes:

- Saturation (up to 60 I_s inside cavity)
- Residual ion motion (thermal)

Longitudinal Cavity-Mode Mapping

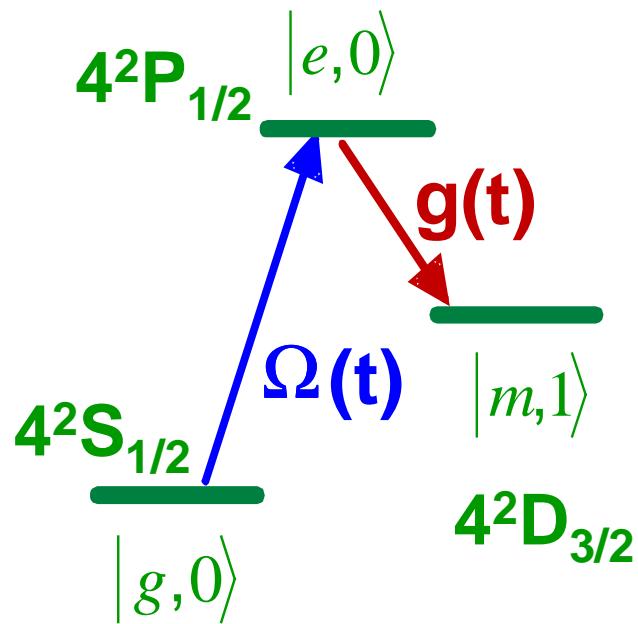
Translation of the cavity along its axis:



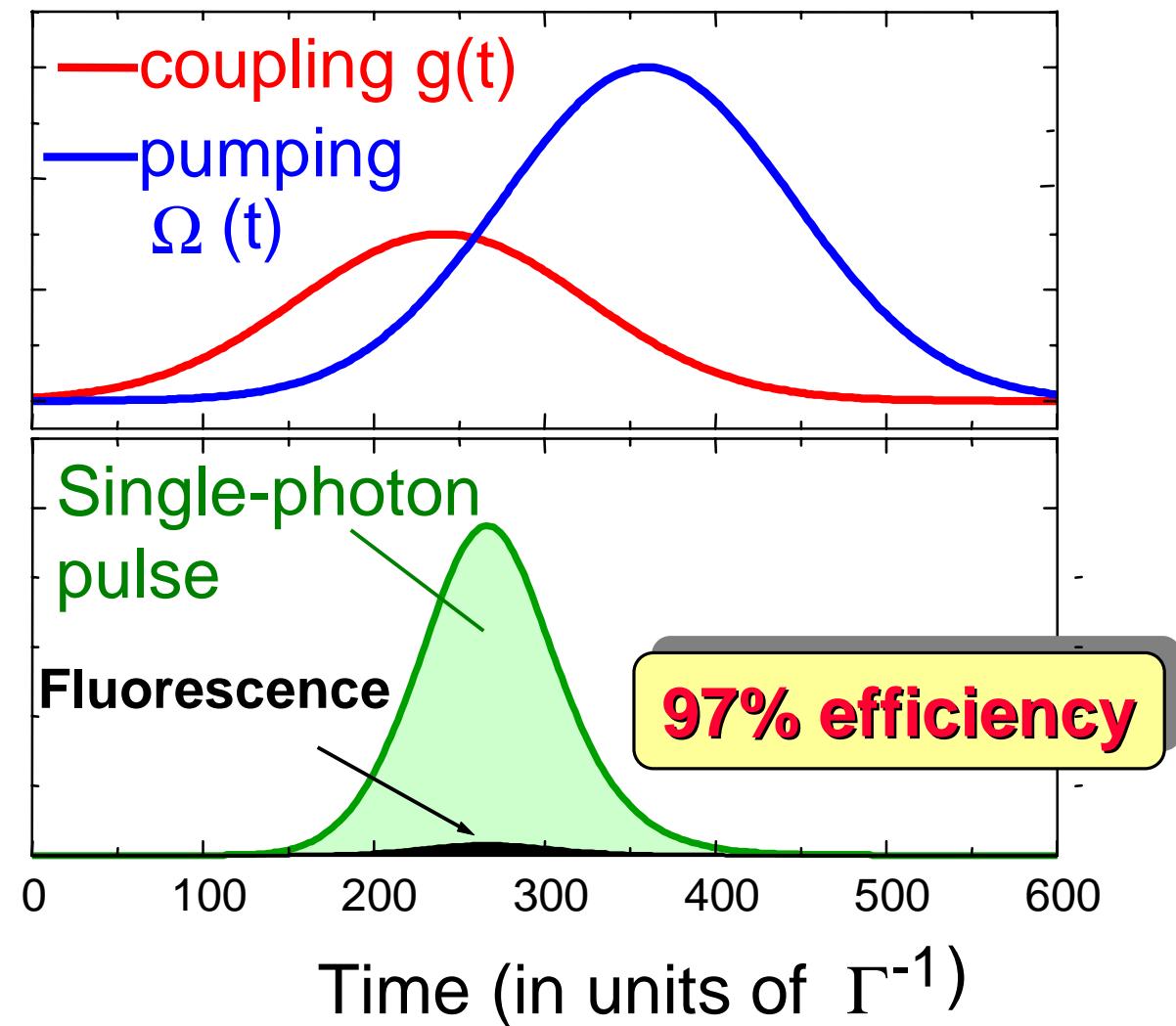
Resolution determined by wavefunction or residual motion of ion

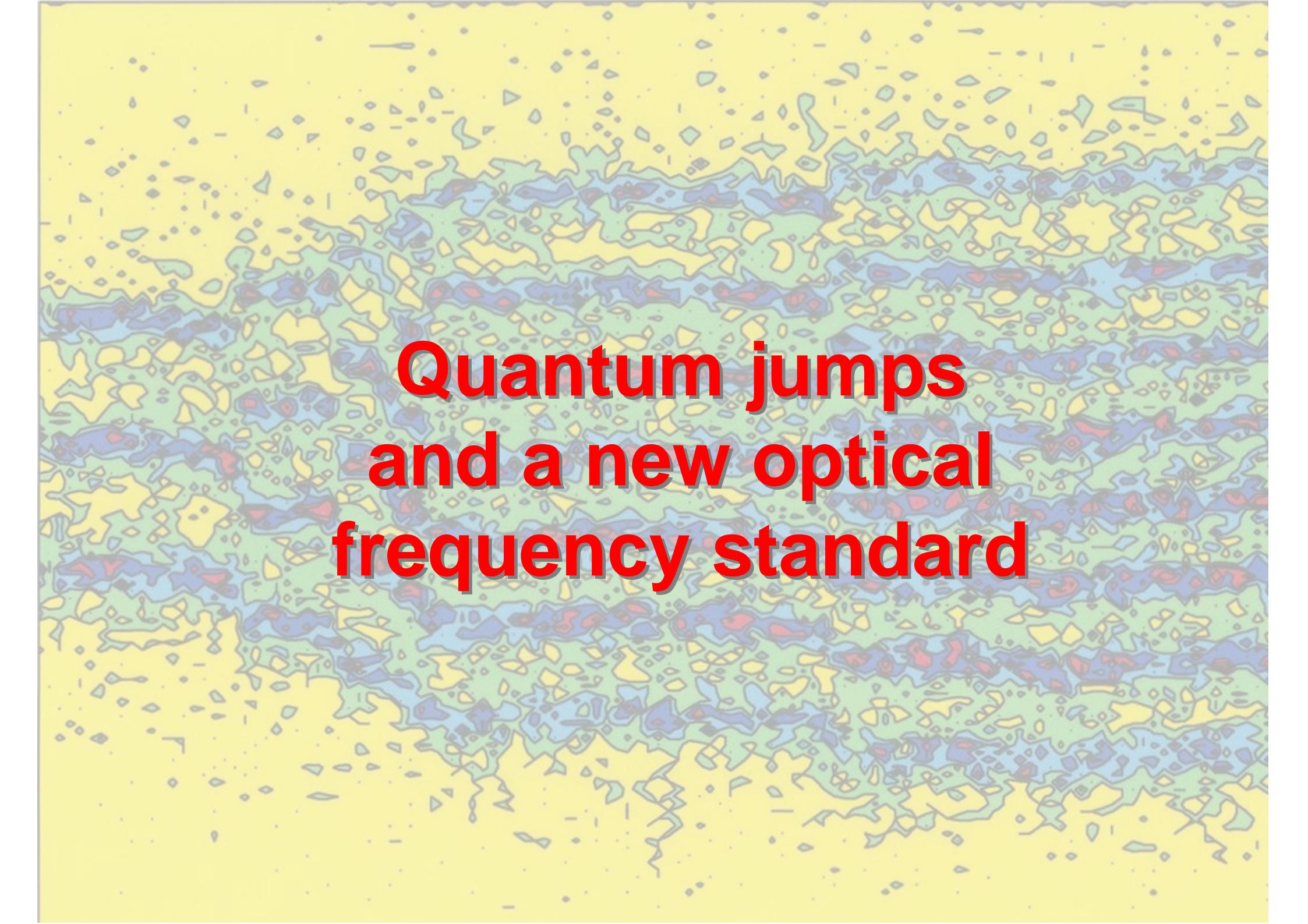
Deterministic Single Photon Gun

Method: adiabatic passage



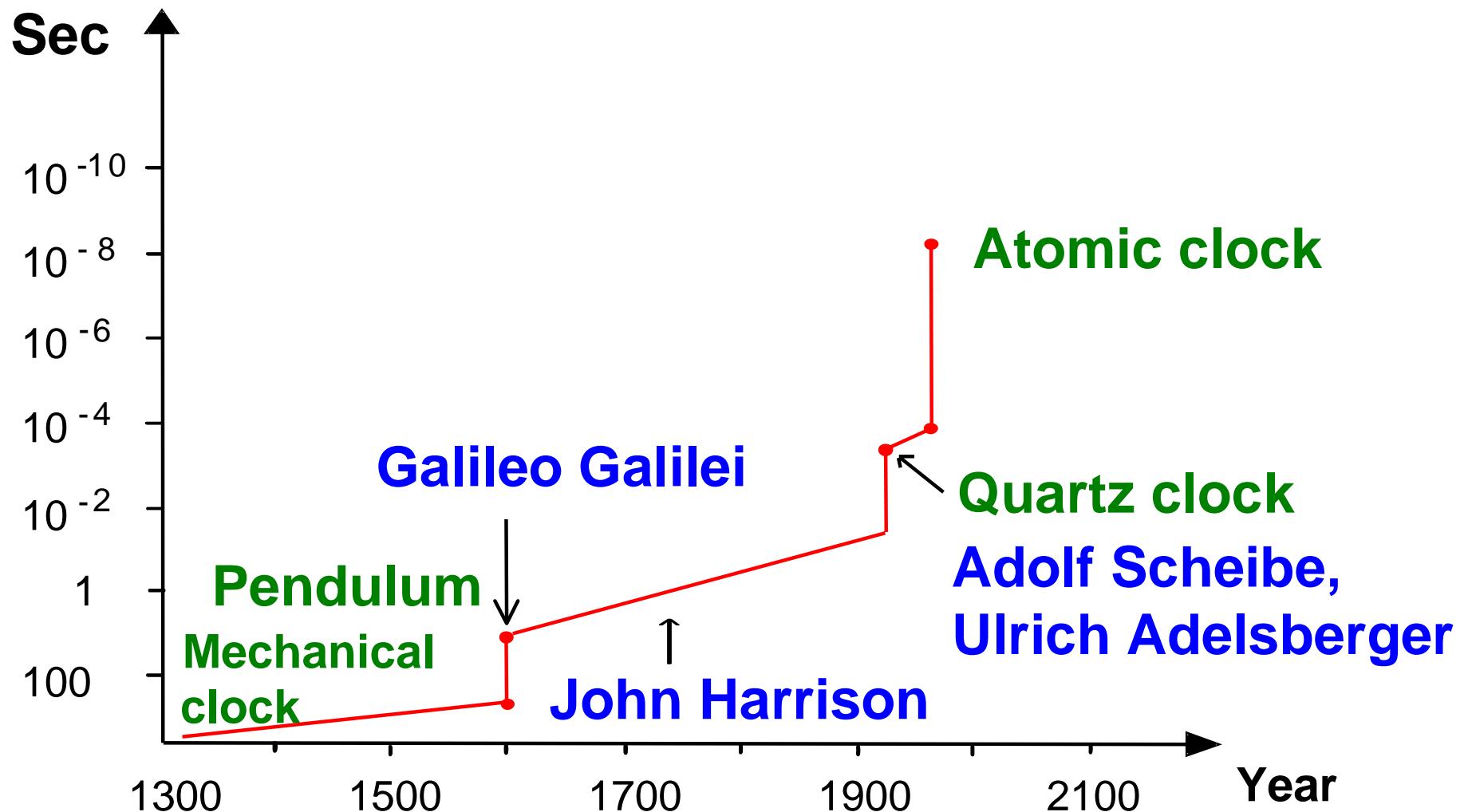
Fourier-limited single photon pulse





**Quantum jumps
and a new optical
frequency standard**

Deviation per day



Accuracy of Present Day Clocks

Accuracy

Mechanical clock

~1s/month

$3.8 \cdot 10^{-7}$

Quartz-clock

1s/30 years

$1 \cdot 10^{-9}$

Atomic-clock

Cs-clock

1s/ 10^6 years

$3 \cdot 10^{-14}$

PTB Braunschweig

Global positioning system

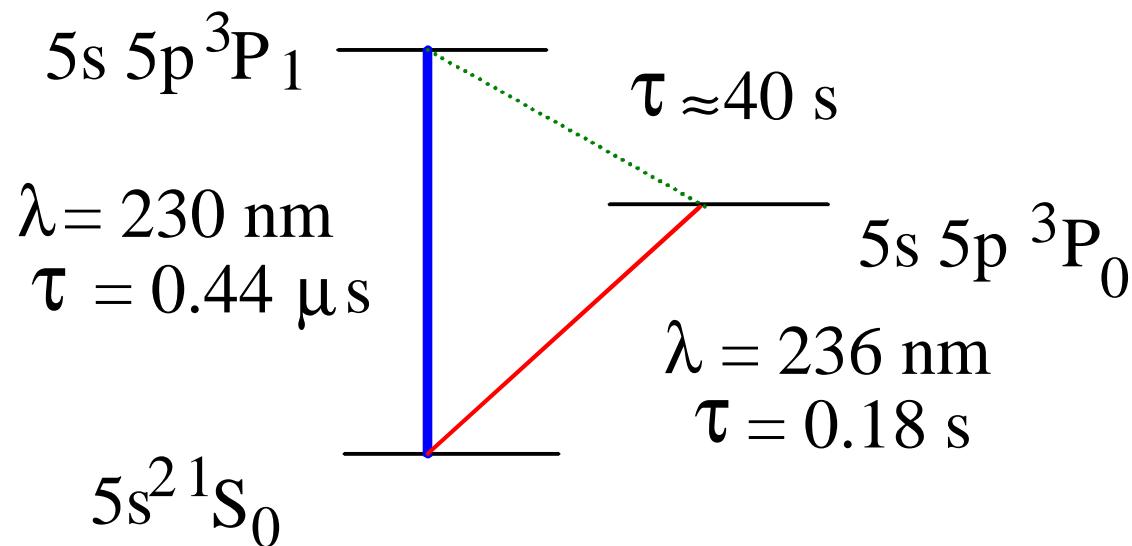
**Integrated services digital network (ISDN)
(coding in multiplex operation)**

Long-baseline-interferometry in radio-astronomy

**Precise measurement of fundamental constants and
of phenomena of basic importance**

- **Test of a possible time variability of
fundamental constants**

Energy Levels of the In⁺-Ion



Perturbing effects

Zeeman splitting $2 \cdot 10^{-17}$

Second order Doppler effect $\approx 10^{-18}$

Quadratic Stark effect $\approx 10^{-19}$

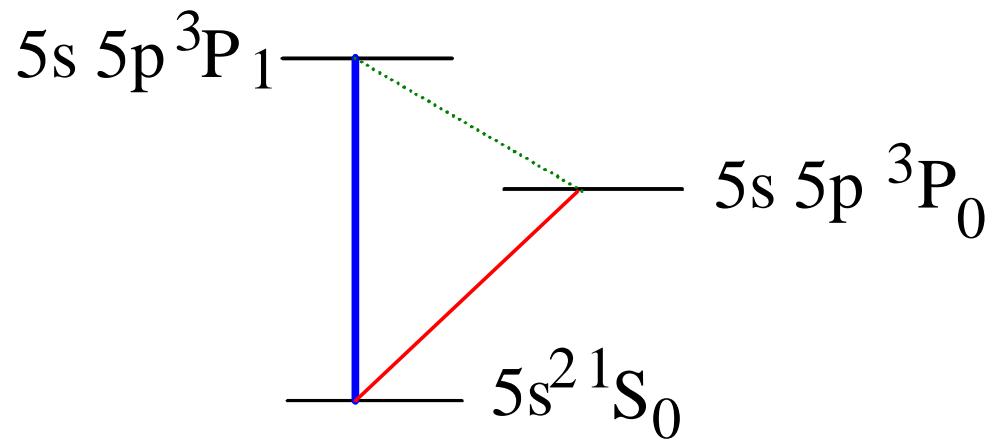
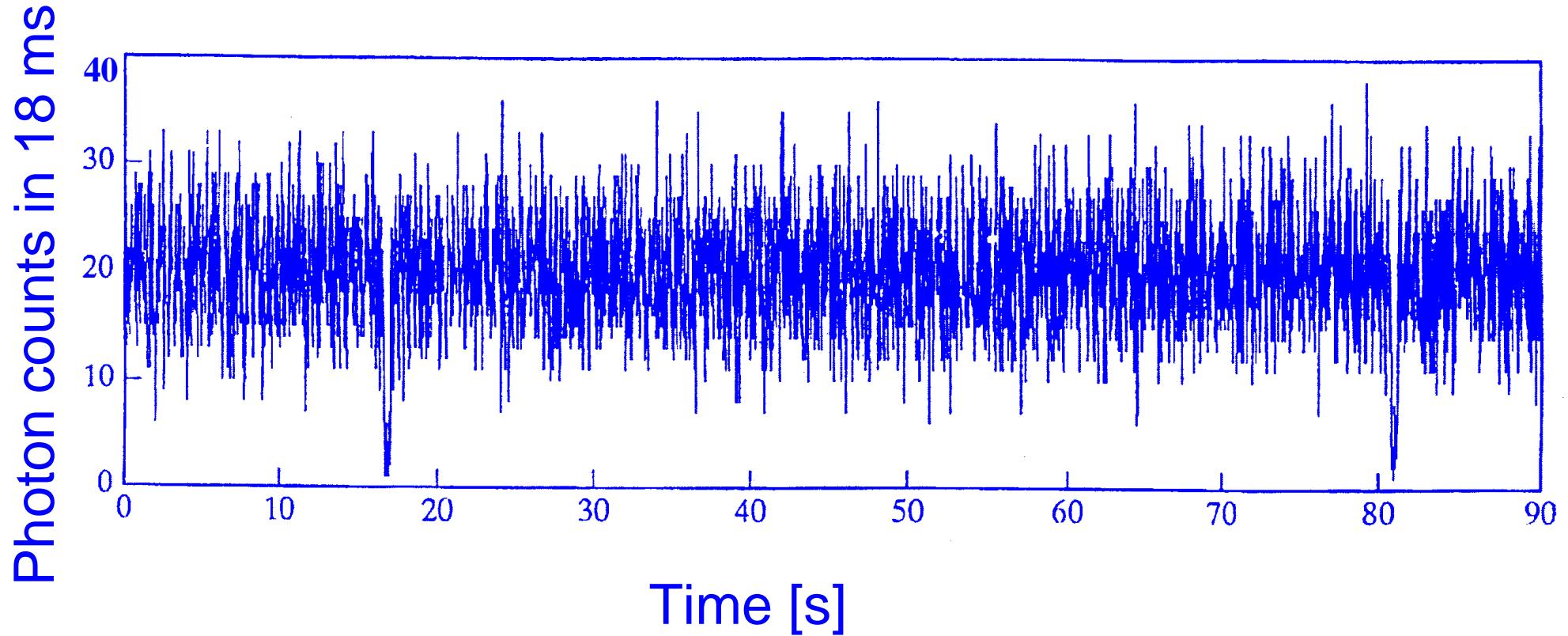
Residual gas collisions $\approx 10^{-17}$

Other ions: Yb⁺

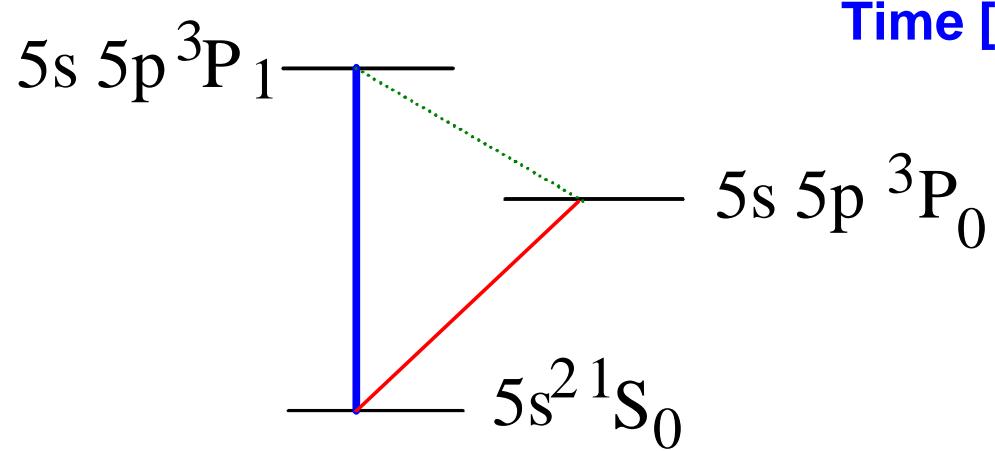
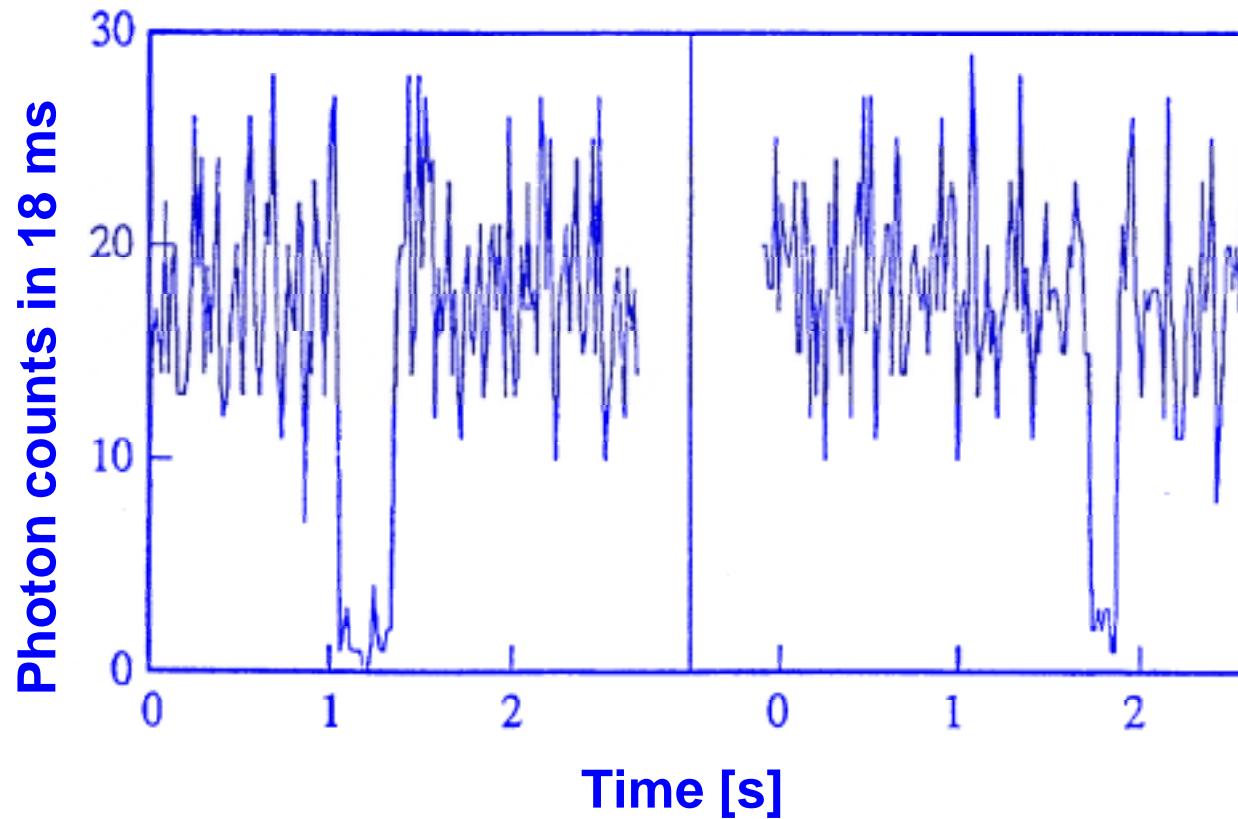
Hg⁺

NIST, NPL, PTB

Quantum Jumps of the In⁺ Ion

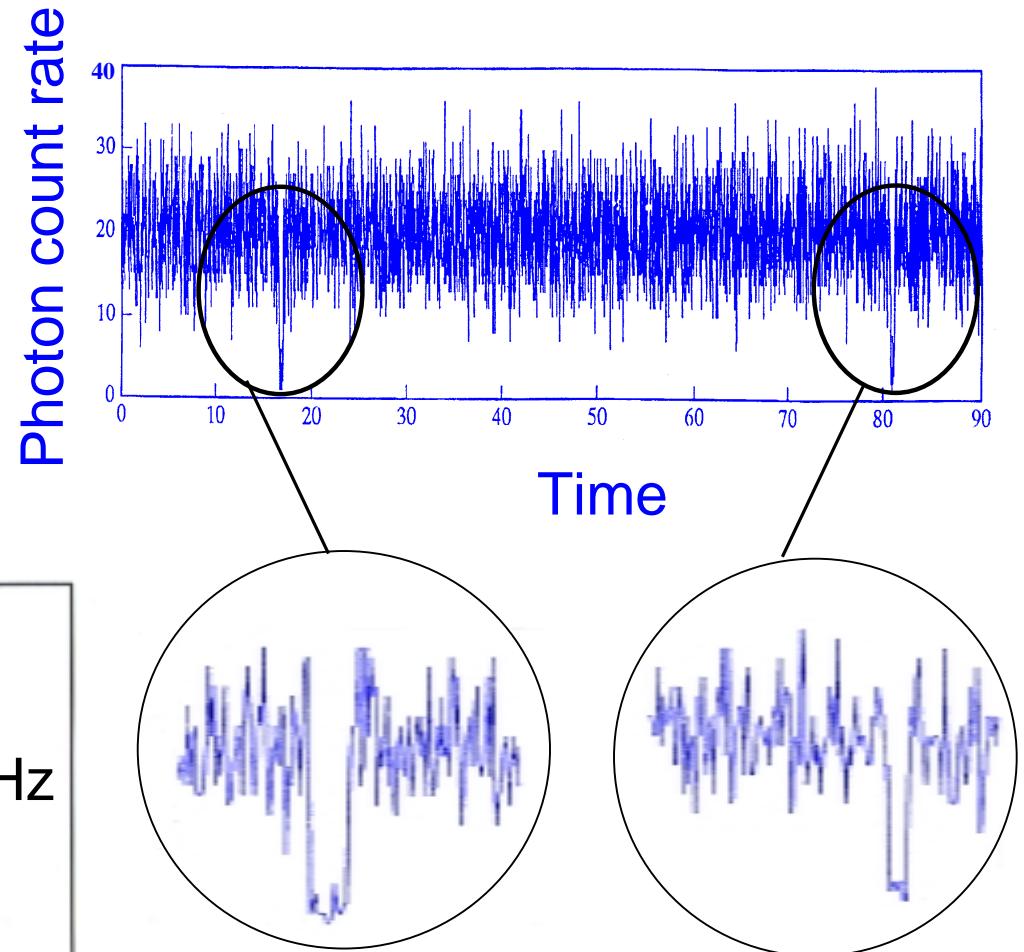
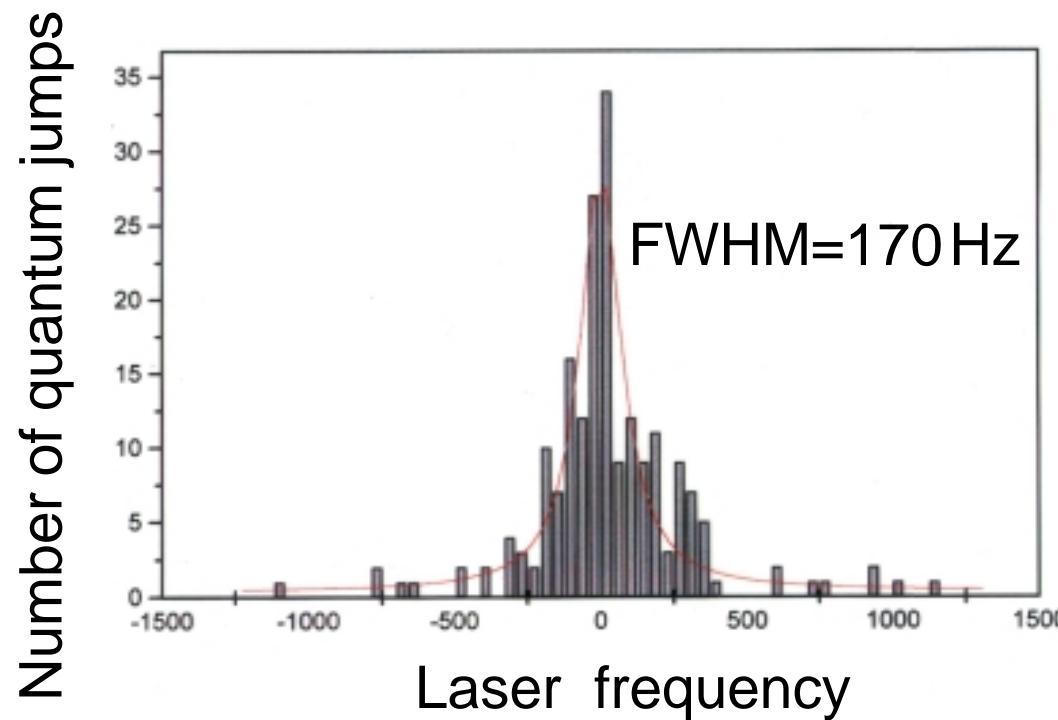
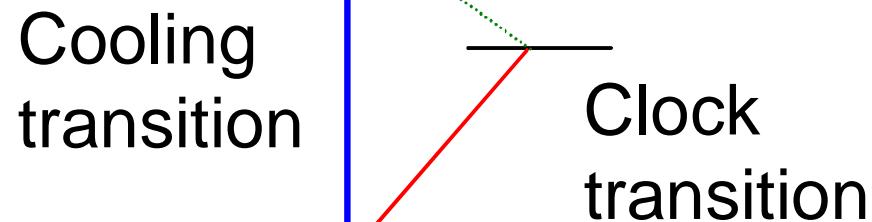


Quantum Jumps of the In^+ Ion



Quantum Jumps and Atomic Clock

Energy levels of In^+



Summary

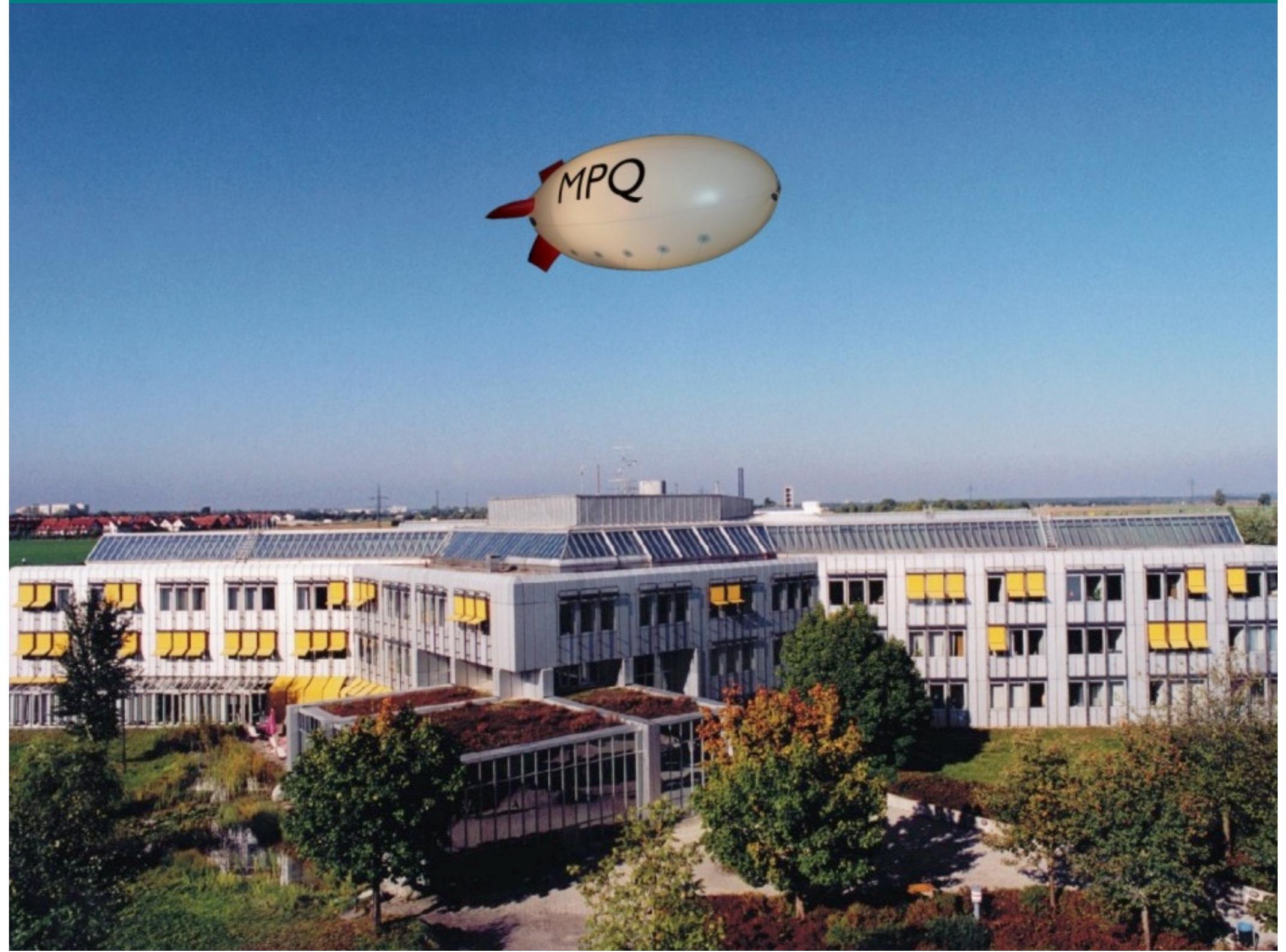
Cavity quantum electrodynamics with single atoms

One-atom maser or micromaser:

- **source for number states
of the radiation field**
- **photons on demand**
- **state reconstruction and phase
diffusion of the maser field**

Trapped ions:

- **single ion as a nanoprobe of
an optical field**
- **single photon sources and
single ion laser**
- **realization of a new frequency
standard**





Strong Coupling Experiments

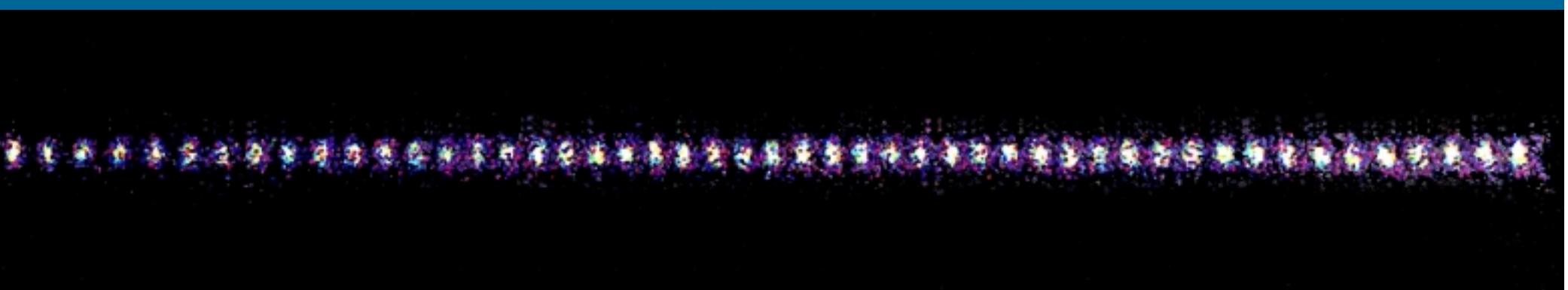
$g \gg (\kappa, \gamma)$

	$g/2\pi$	$\kappa/2\pi$	$\gamma/2\pi$	R_{th} (atoms/s)
Walther et al. 1985,1990	7 kHz	0.4 Hz	500 Hz	1.5
Haroche et al. 1994	48 kHz	400 Hz	5 Hz	$3 \cdot 10^4$
Kimble et al. 1994	7.2 MHz	0.6 MHz	5 MHz	$5 \cdot 10^6$
Rempe et al. 2000				
Feld et al. 1994	340 kHz	190 kHz	50 kHz	$8 \cdot 10^6$

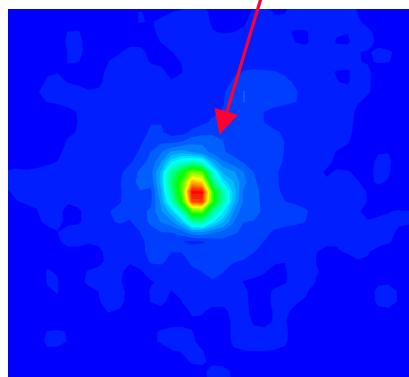
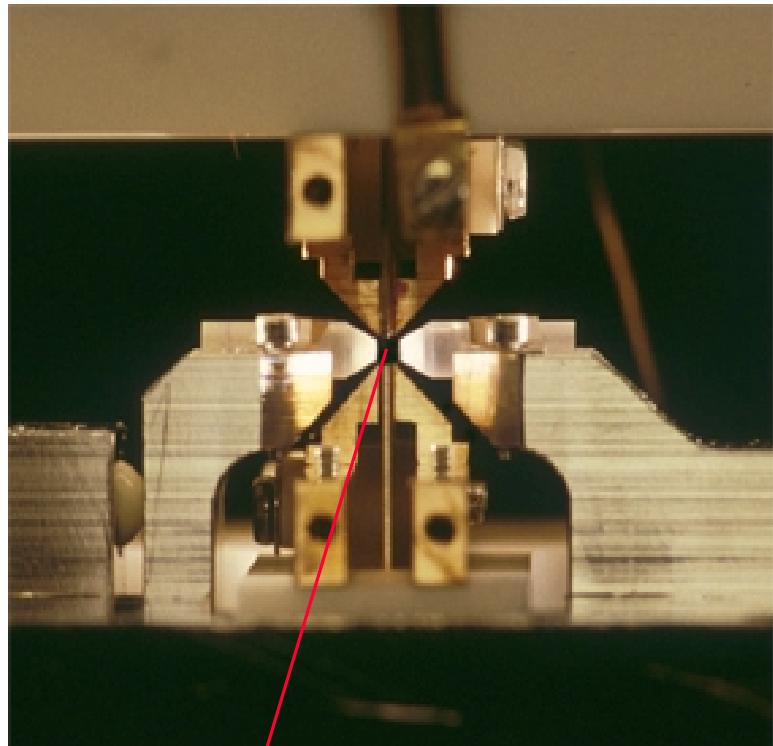
κ : decay rate of cavity field

γ : spontaneous decay of atomic polarization

R_{th} : pumping rate at threshold



Single $^{40}\text{Ca}^+$ Ion in an Optical Cavity



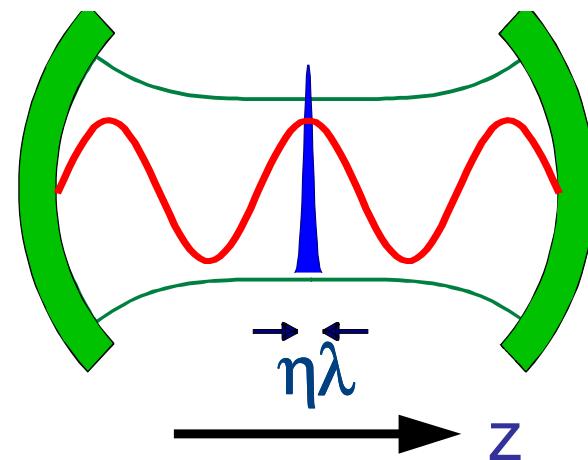
resonator:

length: $L = 1 \dots 6 \text{ mm}$

waist: $w_0 = 16 \dots 24 \mu\text{m}$

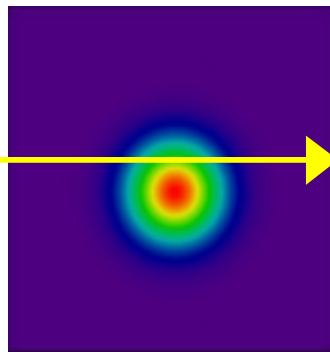
damping: $\kappa/2\pi = 250 \dots 4000 \text{ kHz}$

coupling: $g/2\pi = 5.8 \dots 2.7 \text{ MHz}$

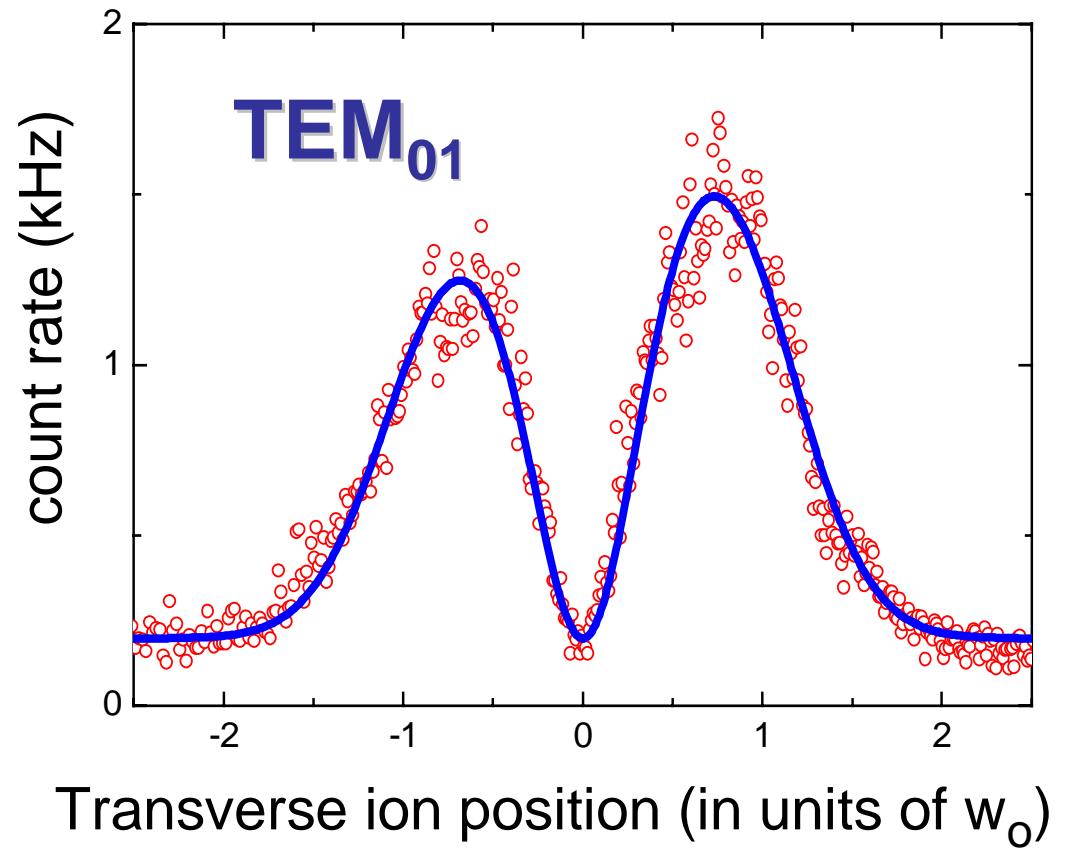
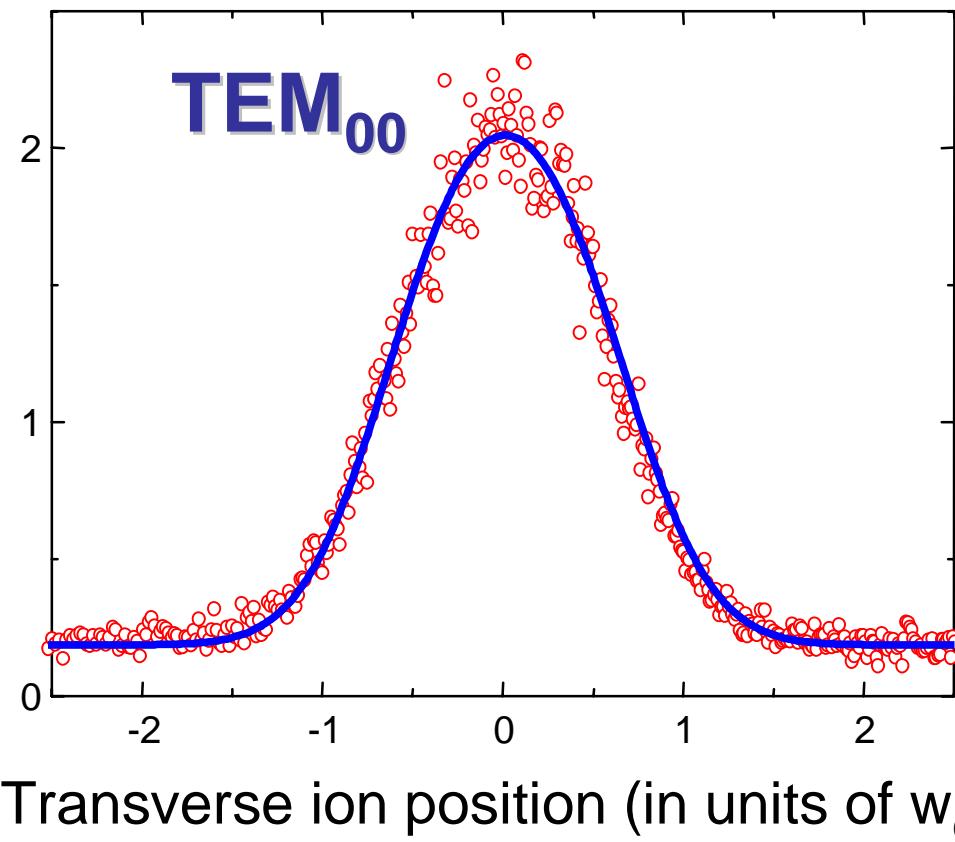
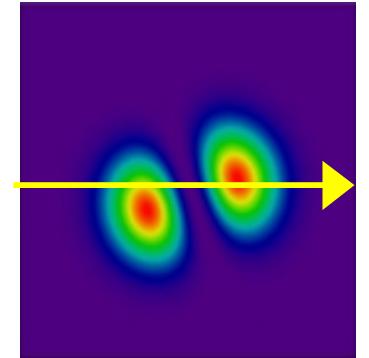


After compensation localisation in z-direction
of RF-micromotion: $\sim \lambda/10$ (Lamb-Dicke regime)

Transverse Scans of Hermite-Gauss-Modes

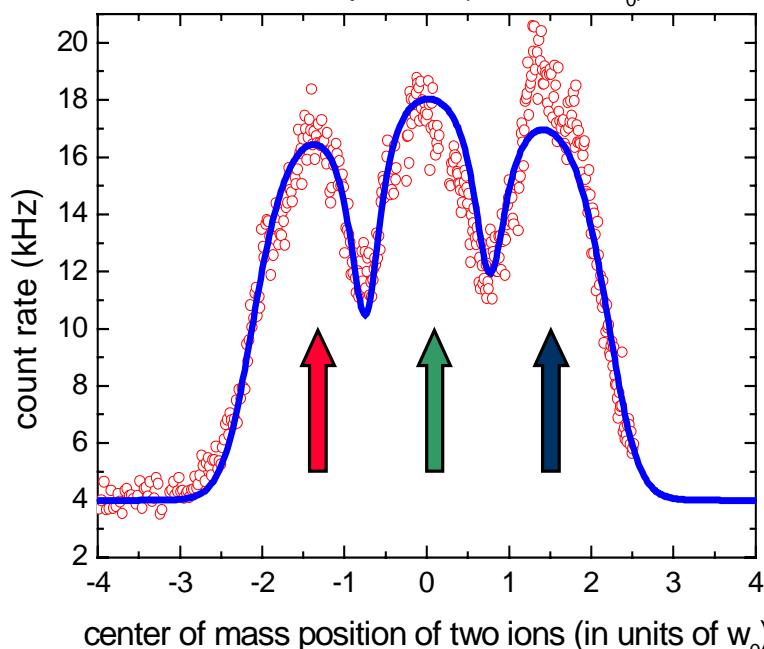
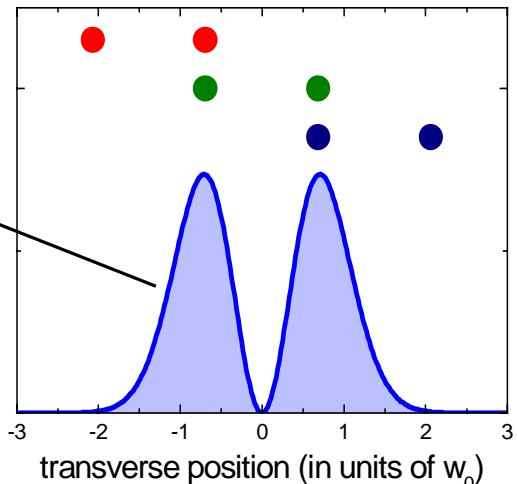


Axial trap frequency: $\omega_z/2\pi=300$ kHz
Transverse resolution: 170 nm
Cavity waist: $W_0=24\mu\text{m}$

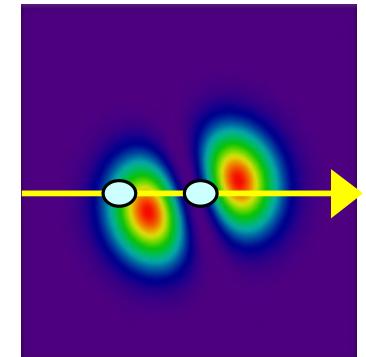


Two Ions in the Cavity Mode

TEM₀₁



a crystal of two Ca⁺ ions
is scanned transverse
to the cavity axis:



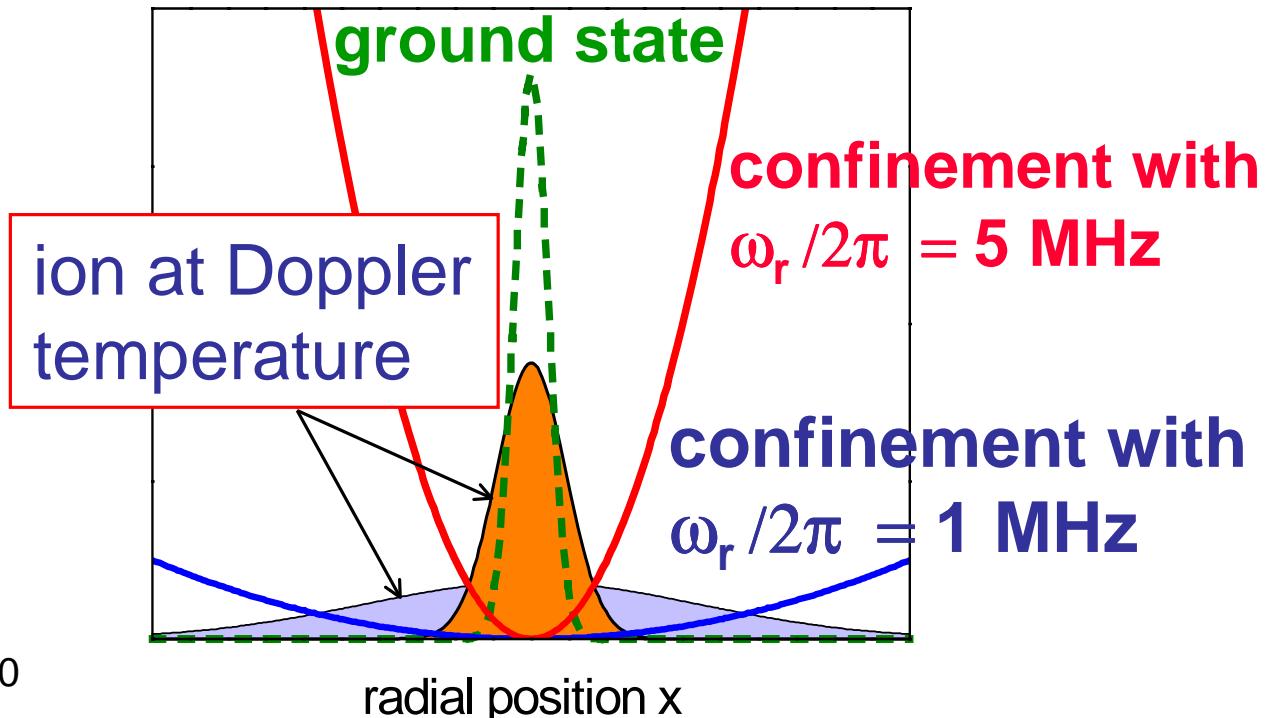
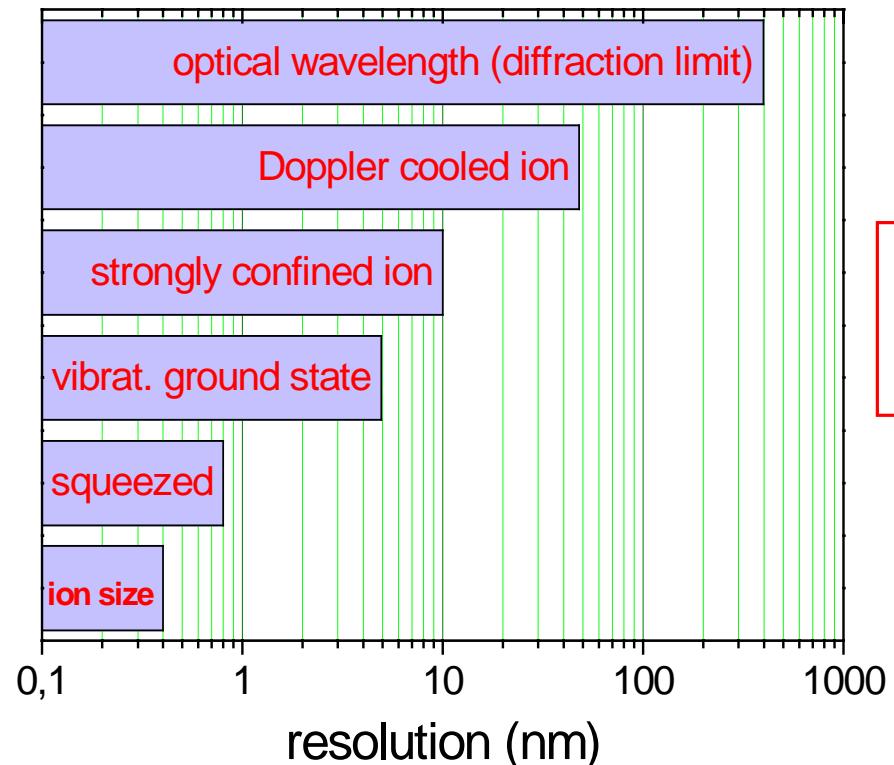
ion separation $\Delta x = 33 \mu\text{m}$
 \approx peak separation of TEM₀₁ mode

Applications:

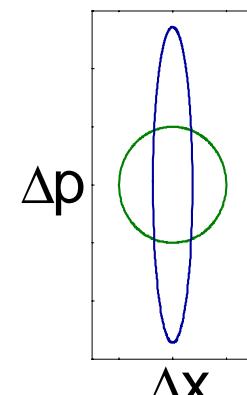
- entanglement of two ions
- quantum state transfer
- two-bit quantum gate

Cavity-mediated detuned Raman coupling leads to a fidelity in two-ion coupling of 93%

Resolution of Single Ion Mode Mapping



- resolution well below diffraction limit achieved
- sub-nm resolution feasible using:
 - strong confinement
 - ground state cooling
 - motional squeezing

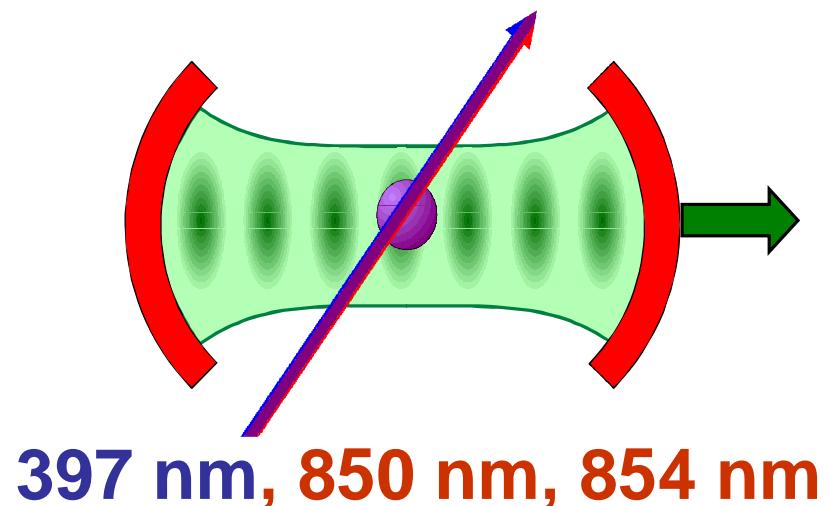


increased resolution by squeezing the ground state

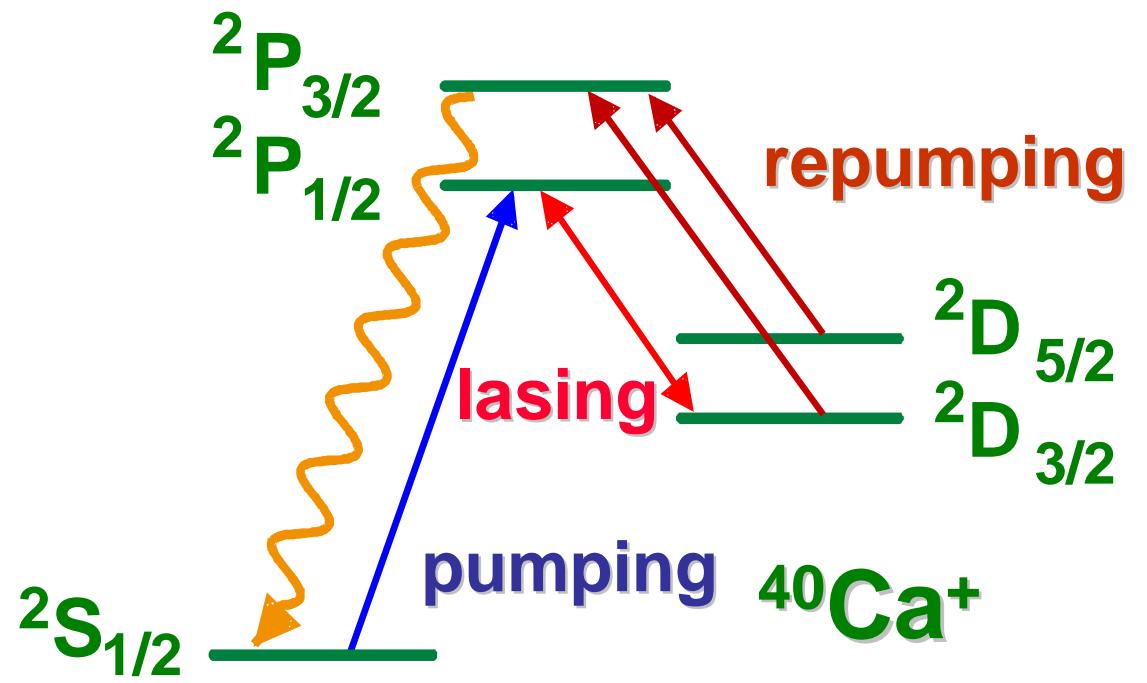
Single-Ion Laser

- single ion as gain medium
- ultralow-loss cavity (finesse $\sim 300\,000$)
- continuous pumping / repumping
- strong atom-cavity coupling ($L \sim 1$ mm)

$$g > K, \gamma$$

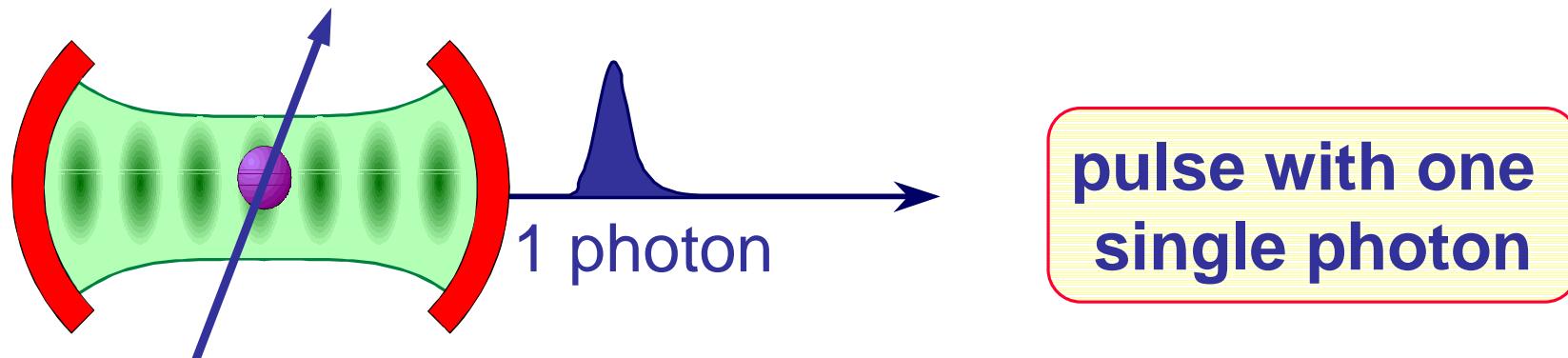


cw source of
non-classical light



Deterministic Single Photon Gun

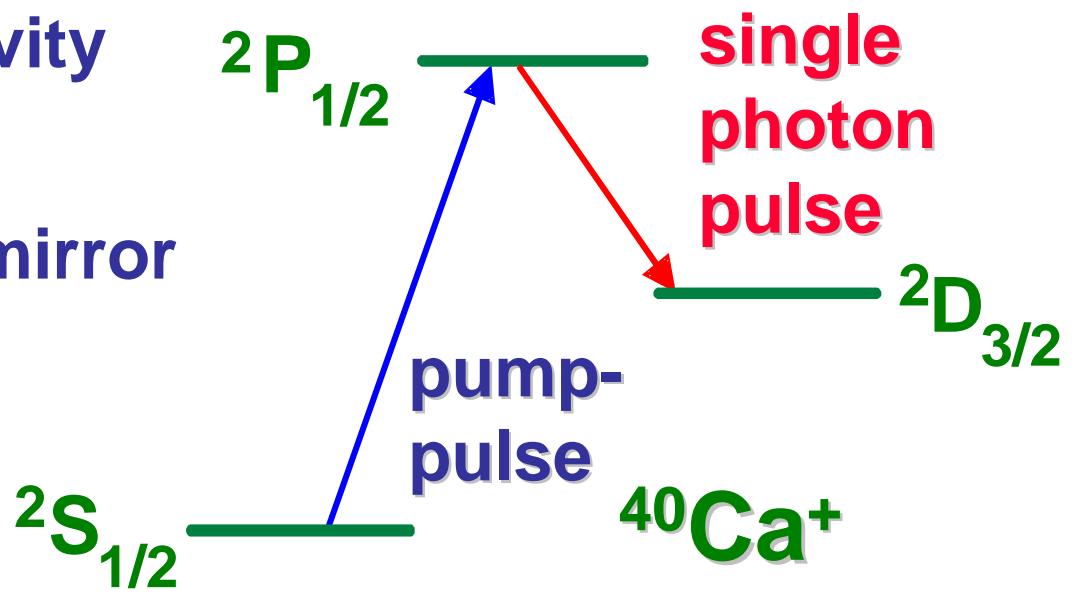
C. K. Law, H. J. Kimble, J. Mod. Opt. 44, 2067 (97)



- Single ion at a node of the cavity
- external pump pulse
- cavity with one leaky output mirror

$$\kappa > g$$

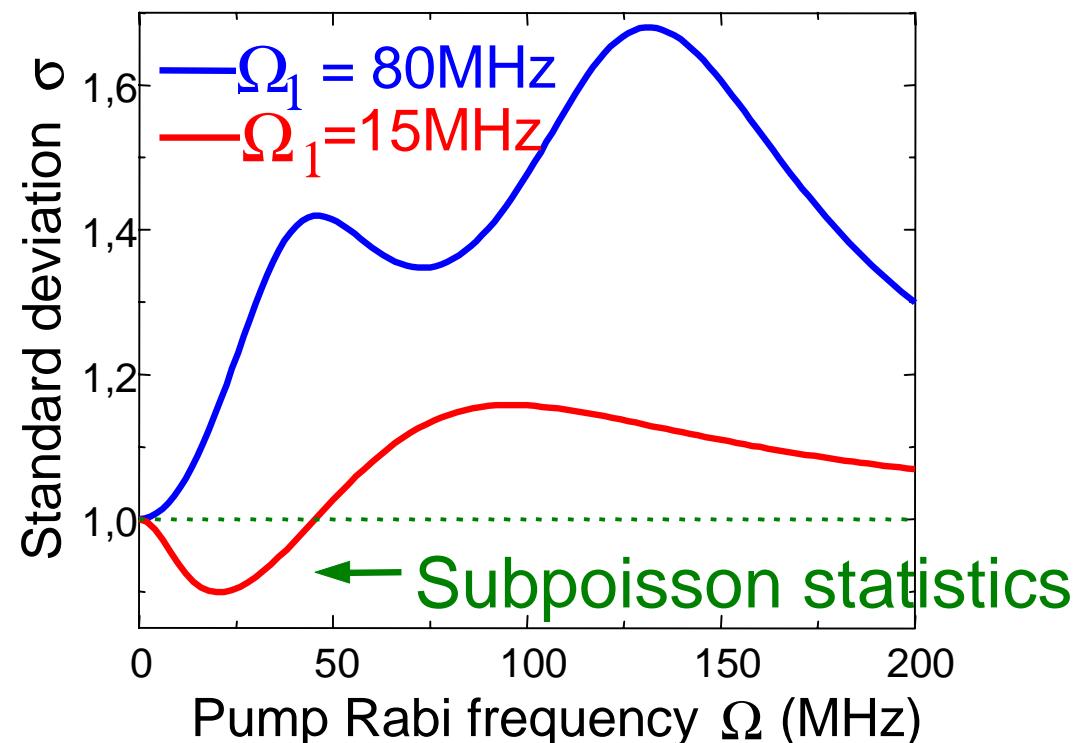
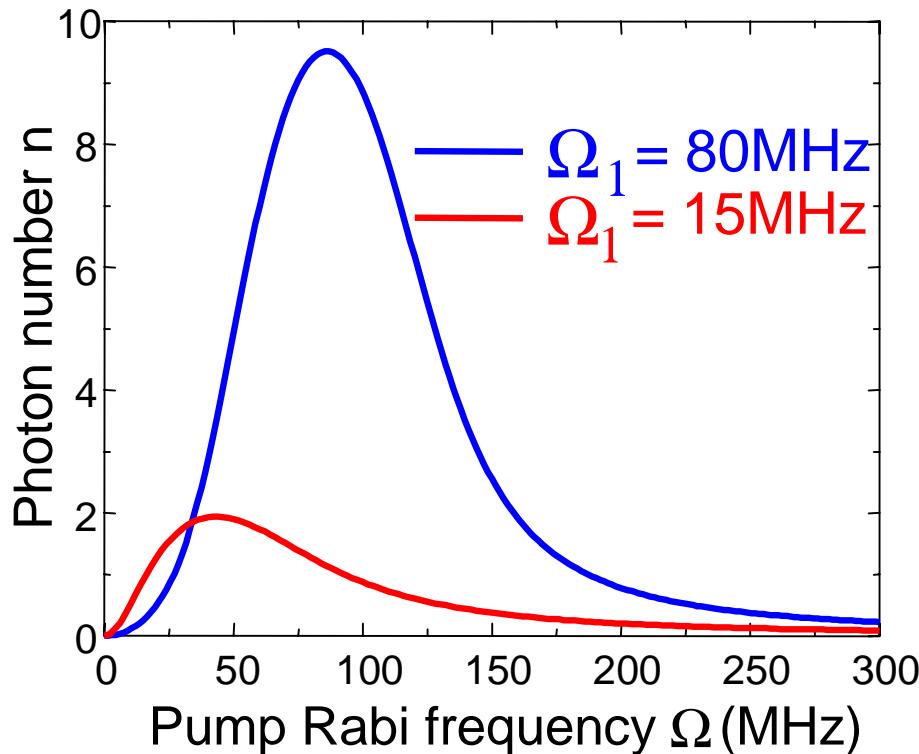
→ Single-photon pulse
at pre-determined time



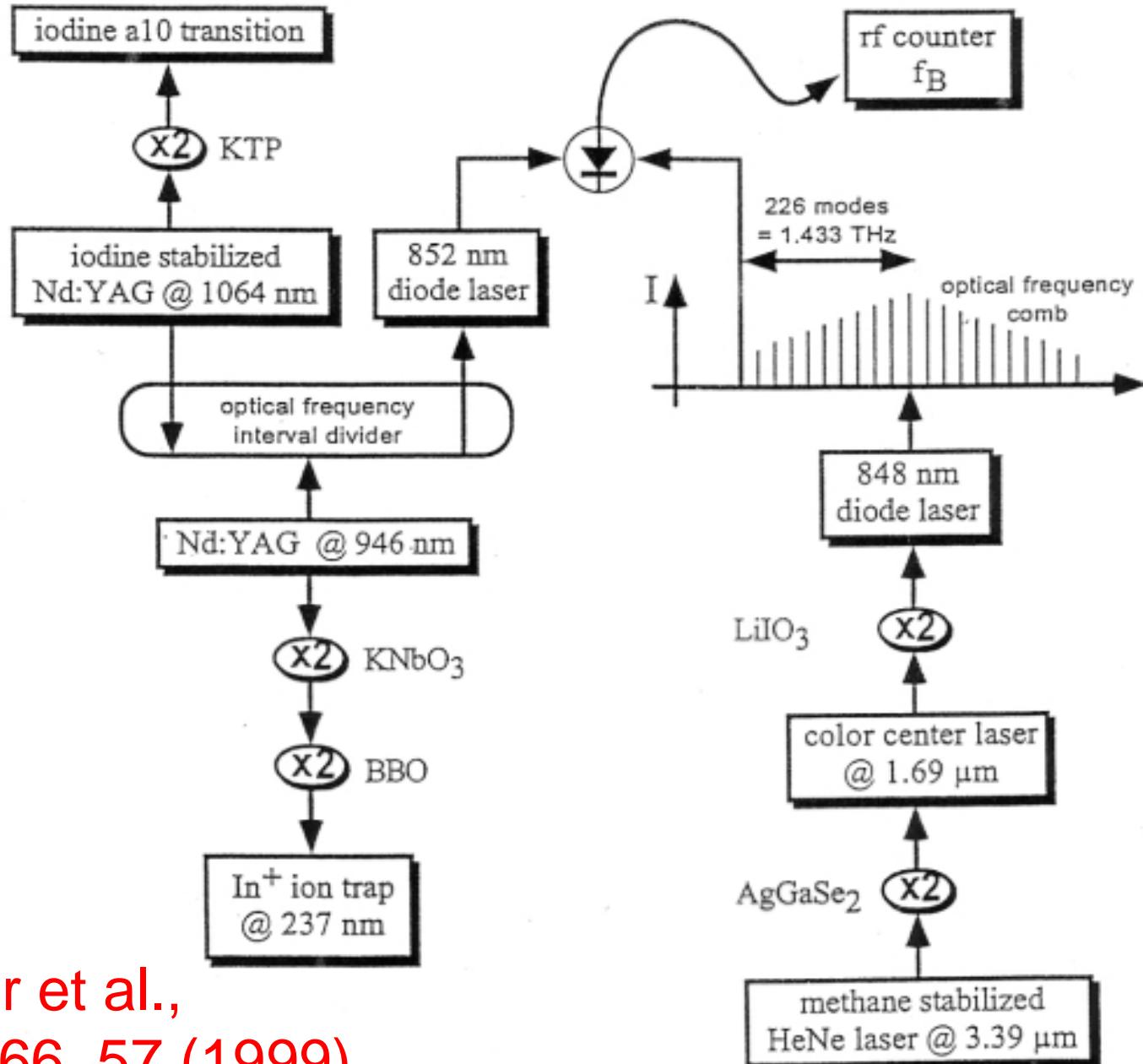
Single-Ion Laser

- stationary field of up to 10 photons in the cavity
- 10 MHz output rate
- non-classical characteristics

$$\begin{aligned}g/2\pi &= 2.3 \text{ MHz} \\ \kappa/2\pi &= 0.08 \text{ MHz} \\ \gamma/2\pi &= 1.7 \text{ MHz}\end{aligned}$$



Optical Frequency Measurement



J. von Zanthier et al.,
 Opt. Comm. 166, 57 (1999)