

Non-linear Effects in Diffusion on Nanoscale

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Diffusion in nanomaterials

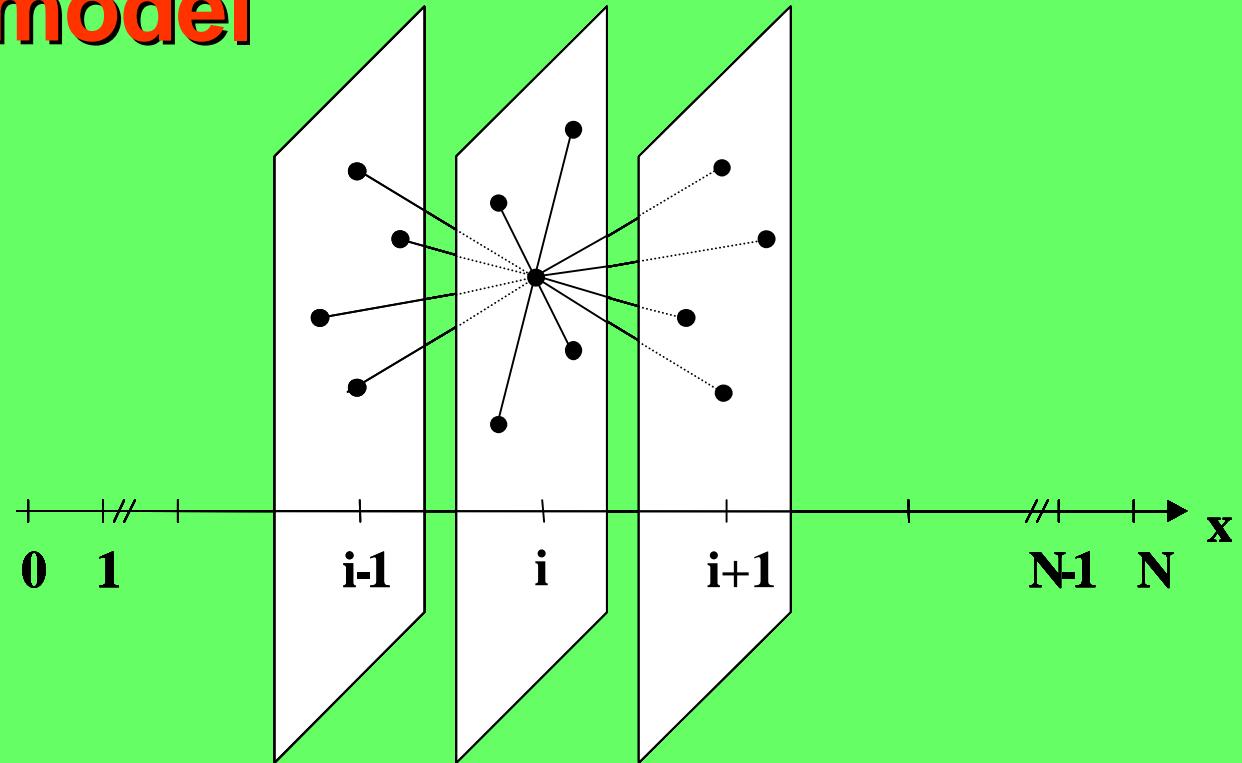
Two important features

- High numbers of grain- or phase boundaries (GB or PB) and dislocations
 - fast diffusion and solid st. reactions, segregation, etc.
- b) Principal problems (very short distances and preferably no structural defects)

Principal difficulties:

- Short diffusion distances $L \approx d$
(continuum description fails)
- Gradient energy corrections
- Stress effects

Discrete (Martin's) model



$$\begin{aligned} \frac{dc_i}{dt} = & -z_v [c_i(1-c_{i-1})\Gamma_{i,i-1} - (1-c_i)c_{i-1}\Gamma_{i-1,i} + \\ & + c_i(1-c_{i+1})\Gamma_{i,i+1} - c_{i+1}(1-c_i)\Gamma_{i+1,i}]. \quad \Gamma_{i,i+1} = v \exp(-E_{i,i+1}/kT) \end{aligned}$$

„Classical” Fick I.-II.

$$j_{i,i+1} = -D_i(\partial c / \partial x) / \Omega$$

$$\partial c_i / \partial t = \partial [D_i(\partial c / \partial x) / \Omega] / \partial x$$

$$E_{i,i+1} = E^o - \alpha_i + \varepsilon_i \quad \text{and} \quad E_{i+1,i} = E^o - \alpha_i - \varepsilon_i,$$

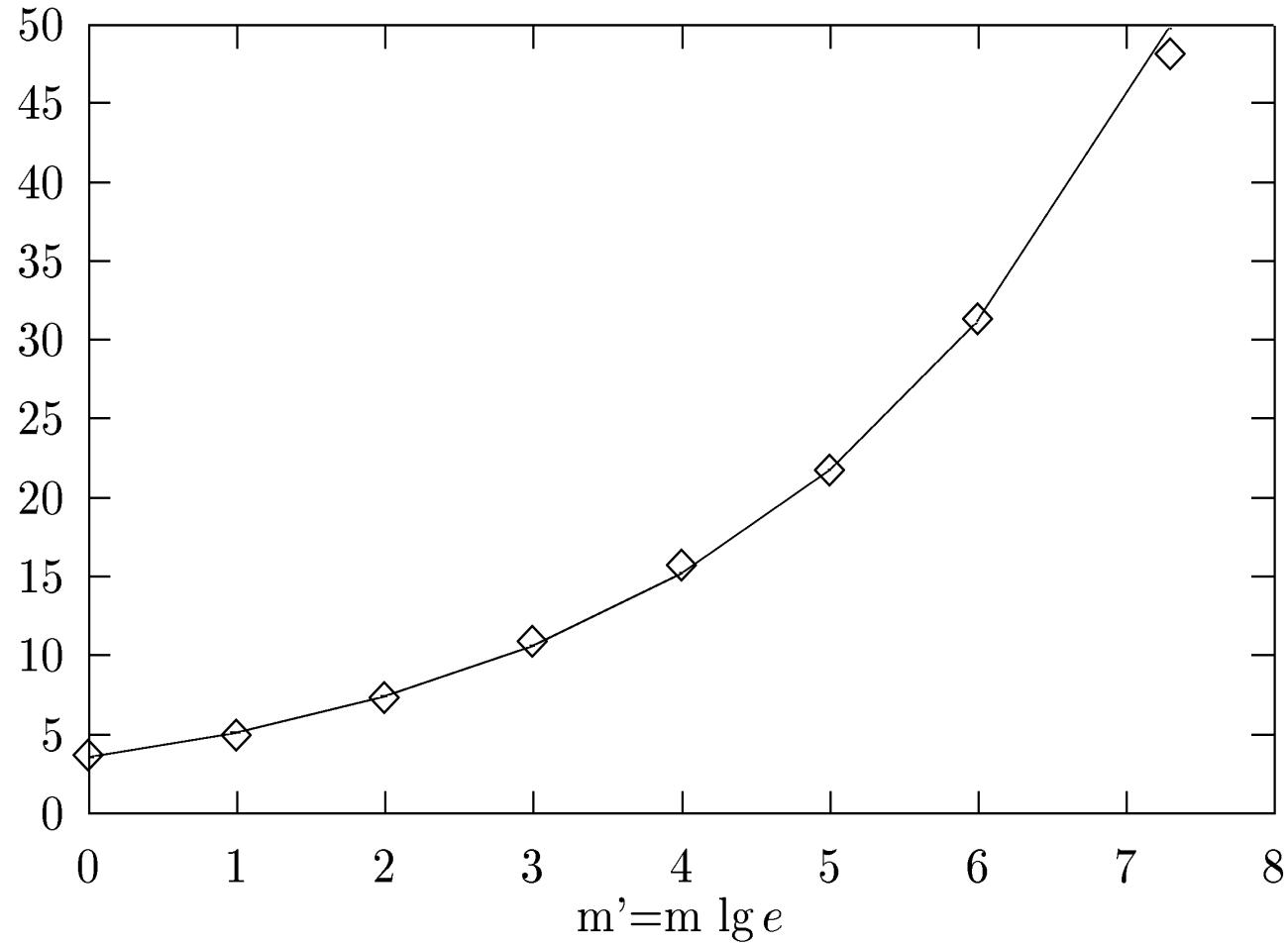
$$V = V_{AB} - (V_{AA} + V_{BB})/2$$

$$\alpha_i = [z_v(c_{i-1} + c_{i+1} + c_i + c_{i+2}) + z_l(c_i + c_{i+1})](V_{AA} - V_{BB})/2$$

$$\varepsilon_i \propto V.$$

Input parameters: $z_v, z_l, V_{AA} - V_{BB}, V, T$ ($Z = 2z_v + z_l$)

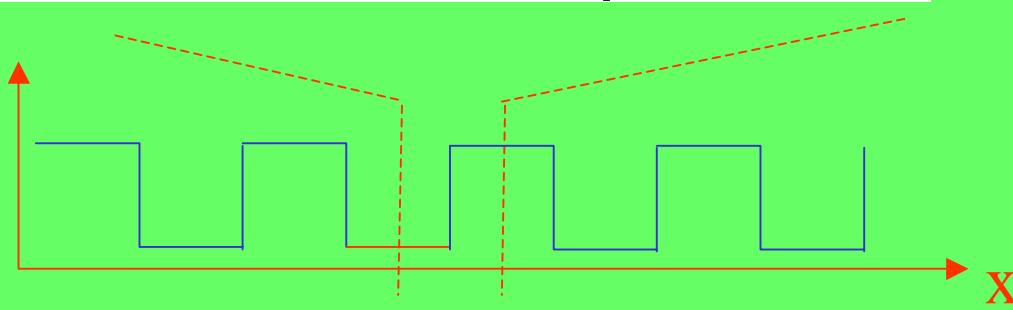
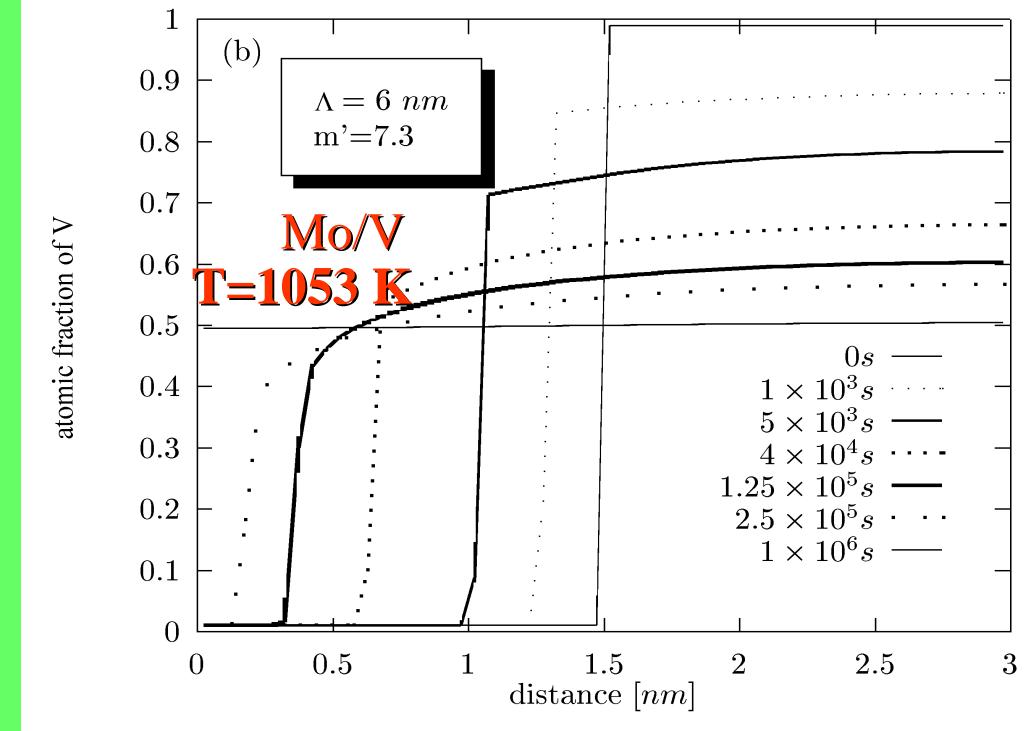
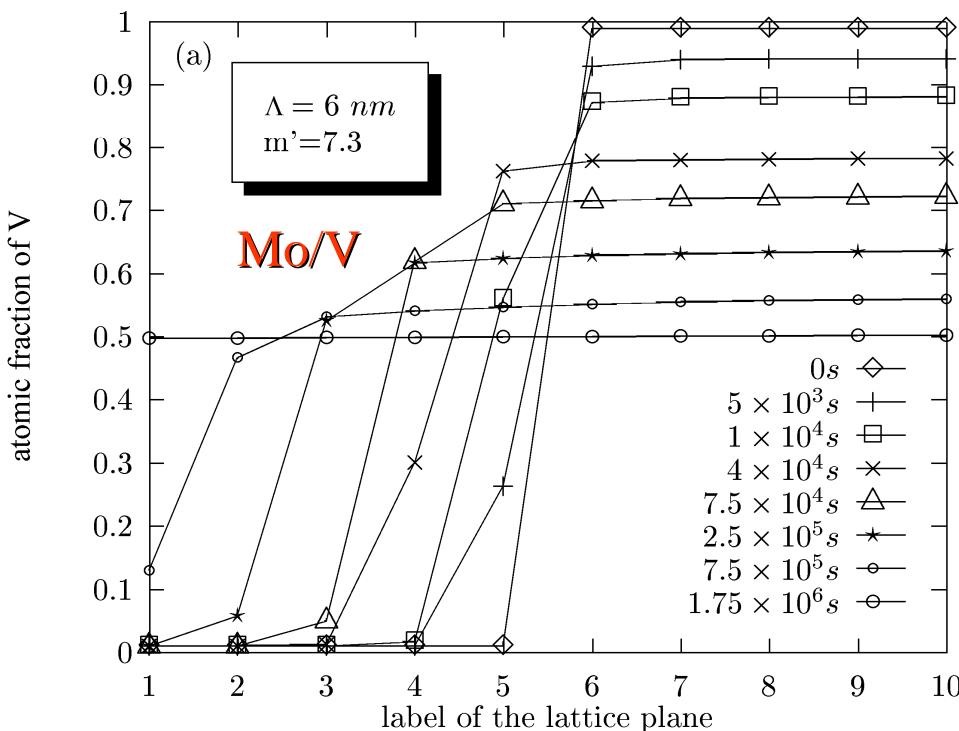
-validity limit



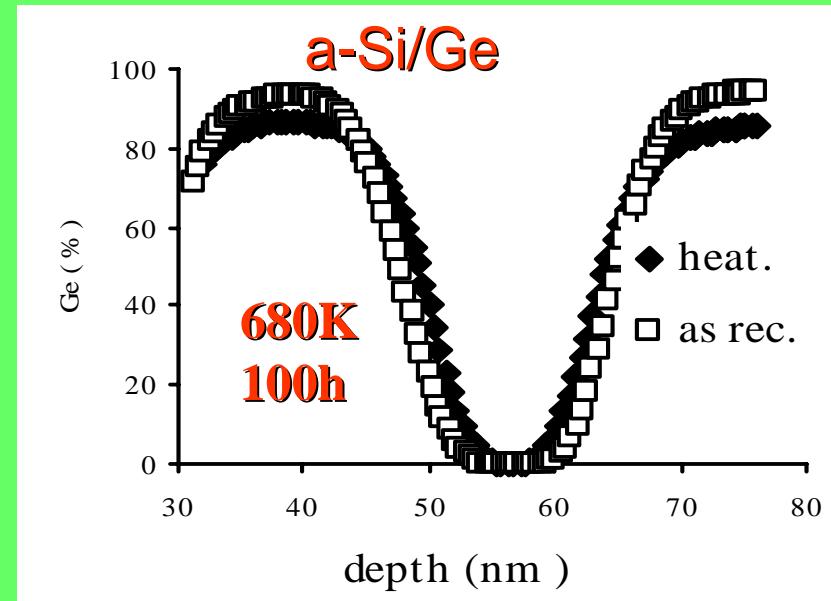
$$D_i^h = D(0) \exp(mc)$$

$$m = Z(V_{AA} - V_{BB})/kT$$

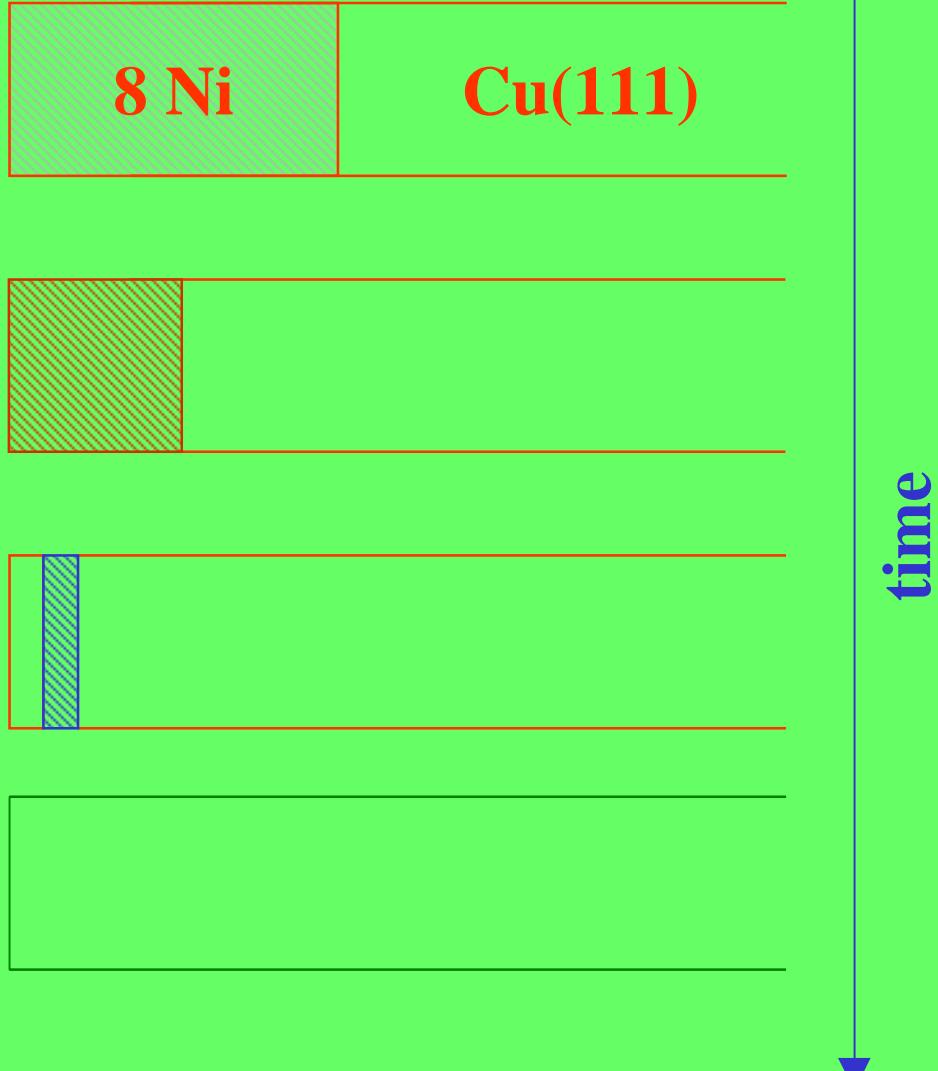
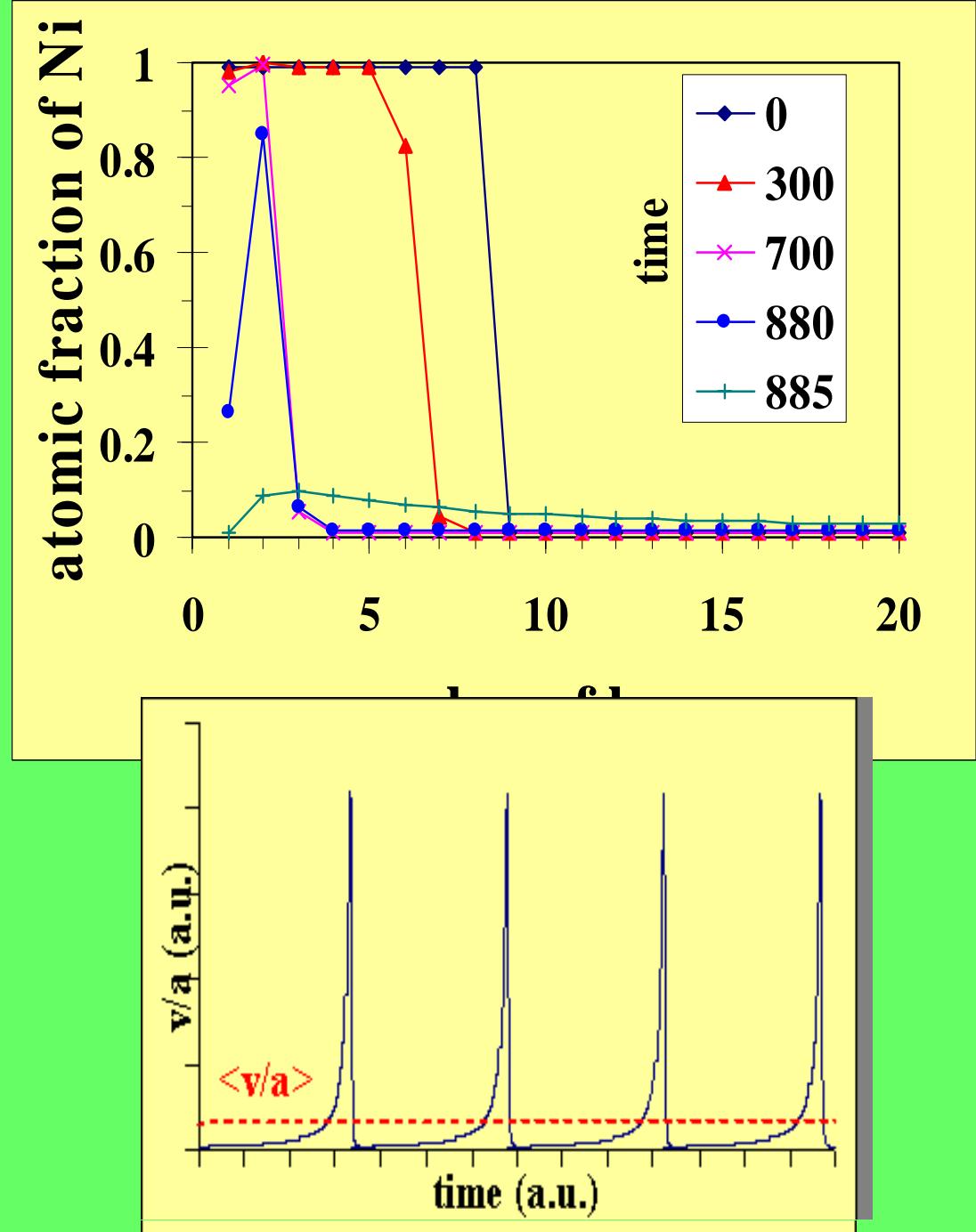
Effect of the strong concentration dependence of D



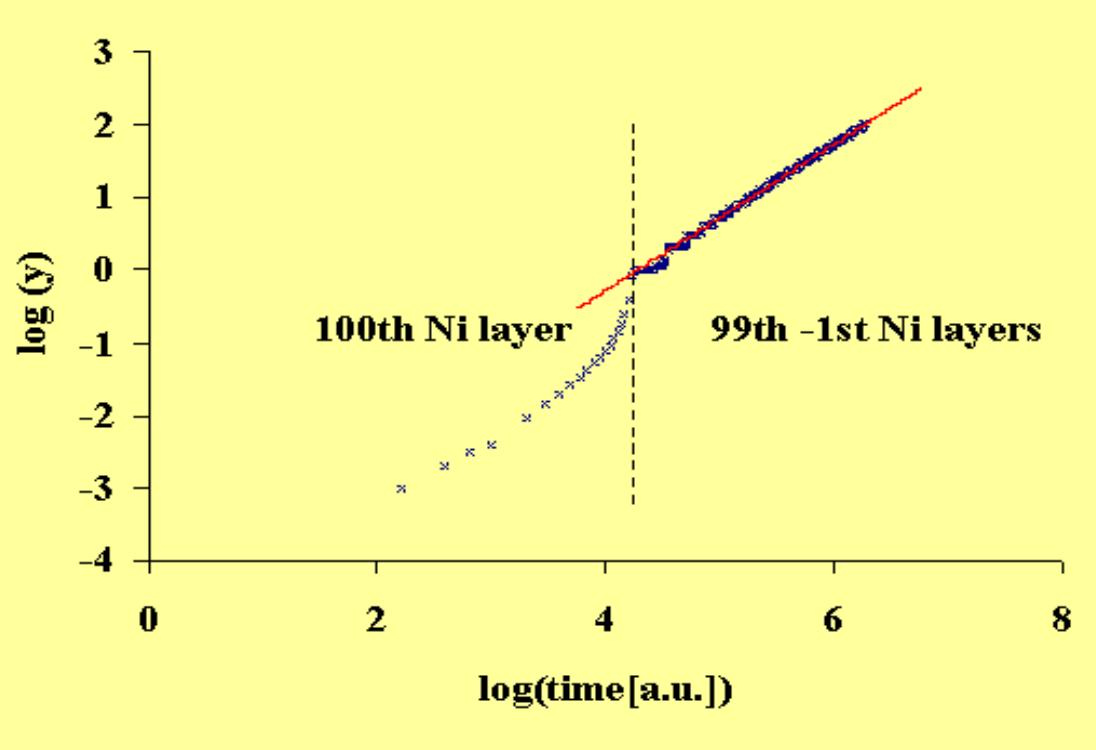
The interface remains sharp and shifts!



Dissolution in ideal systems: Ni into Cu

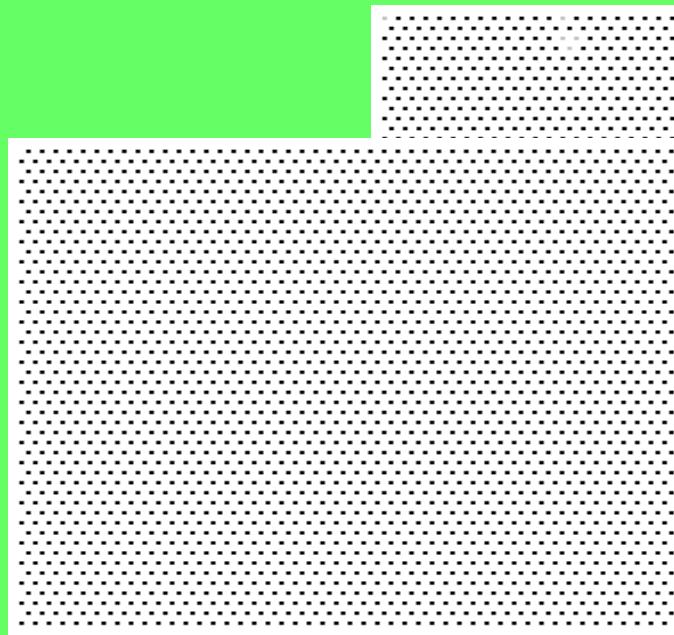


$D(\text{in Ni}) \ll D(\text{in Cu})$

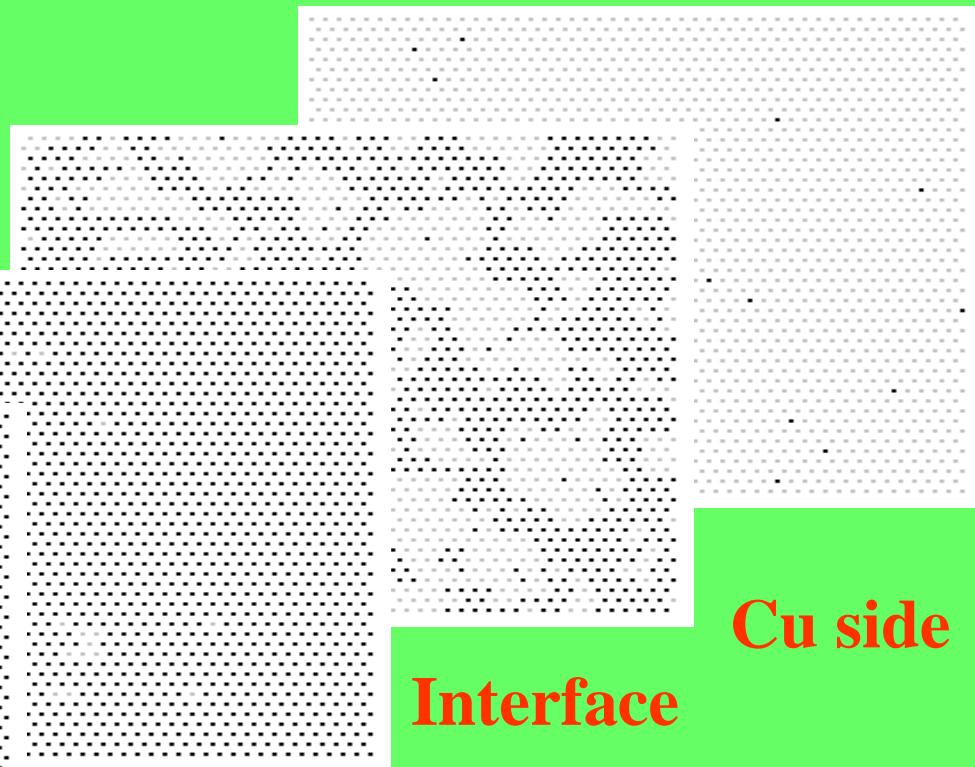


Linear kinetics !!!

Monte Carlo



Ni side

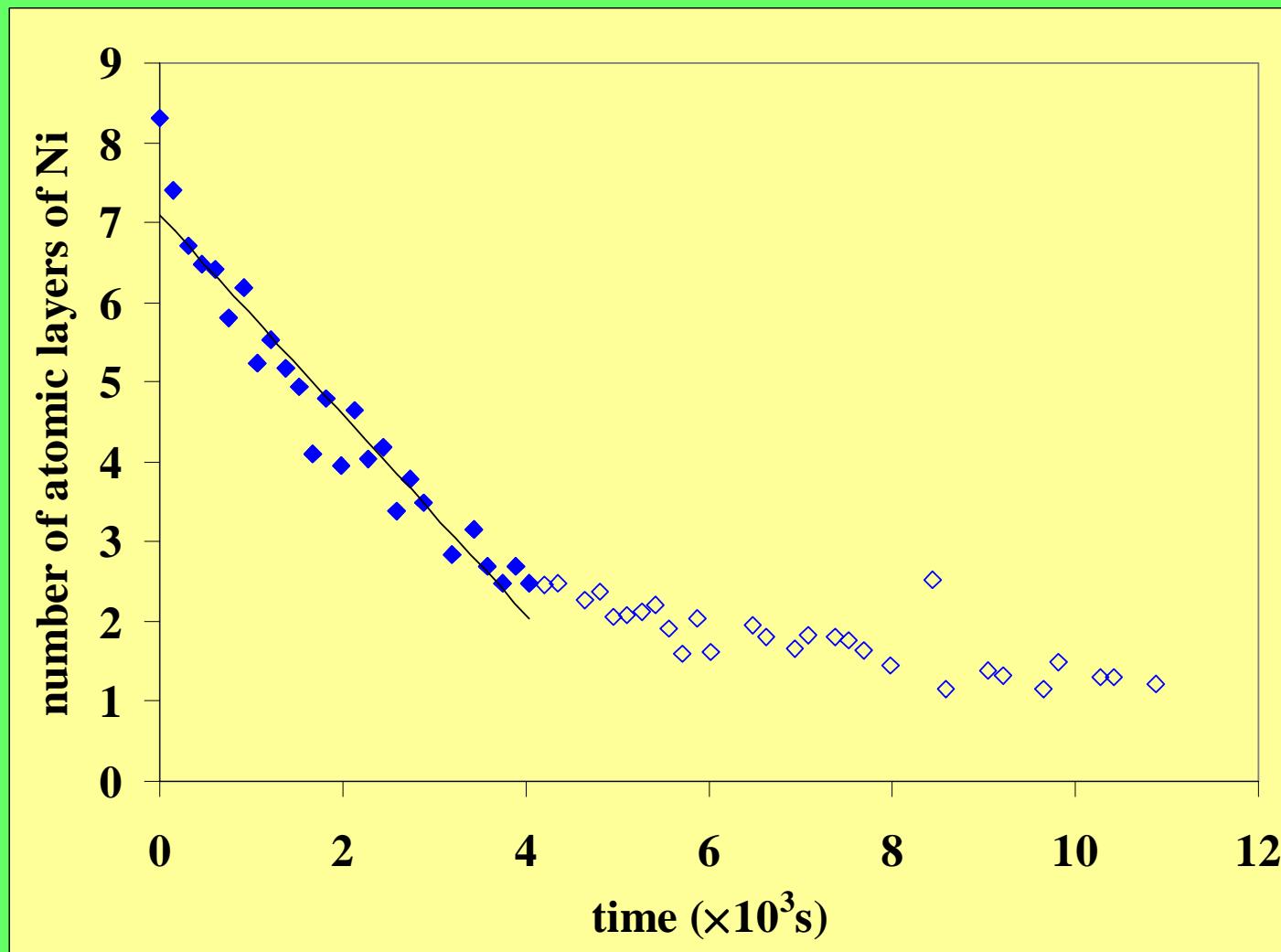


Cu side

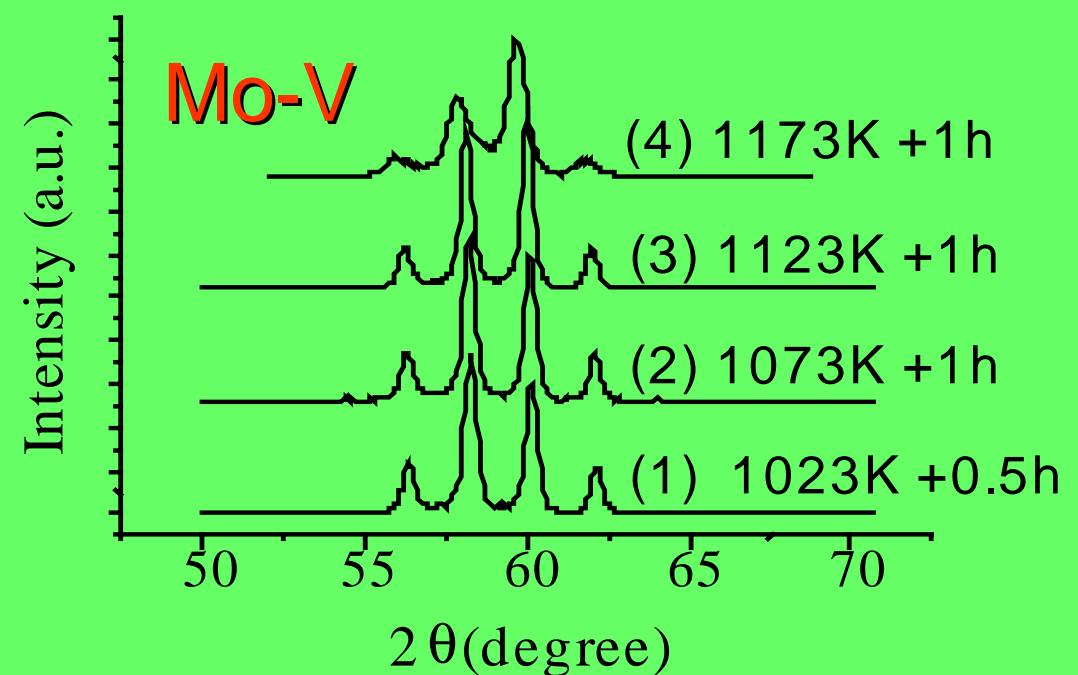
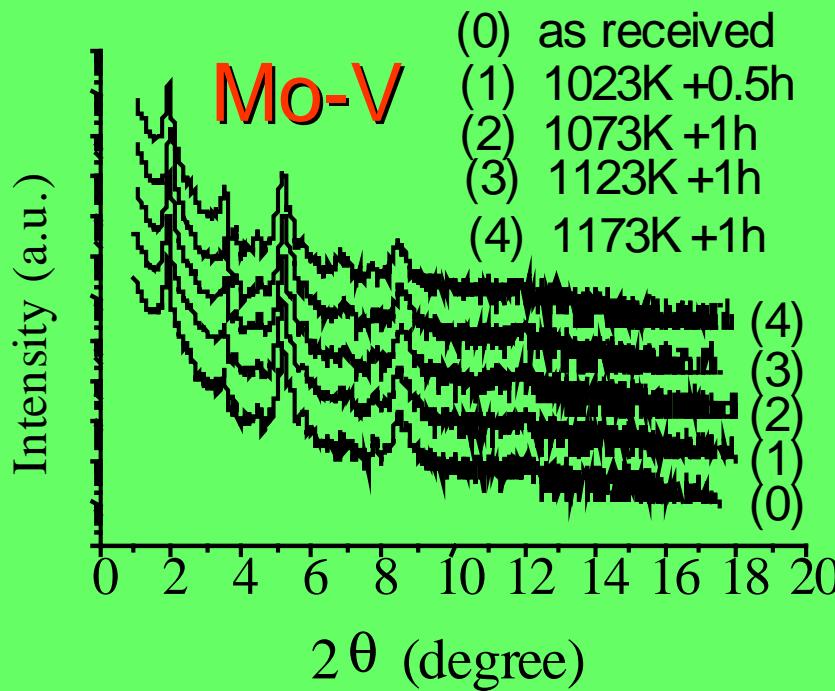
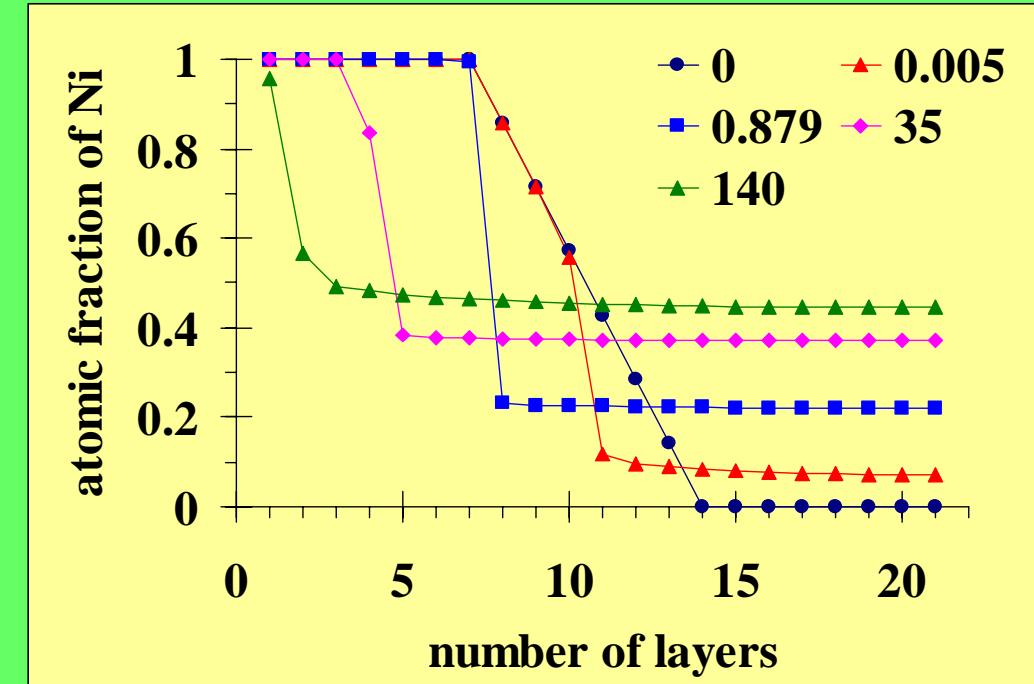
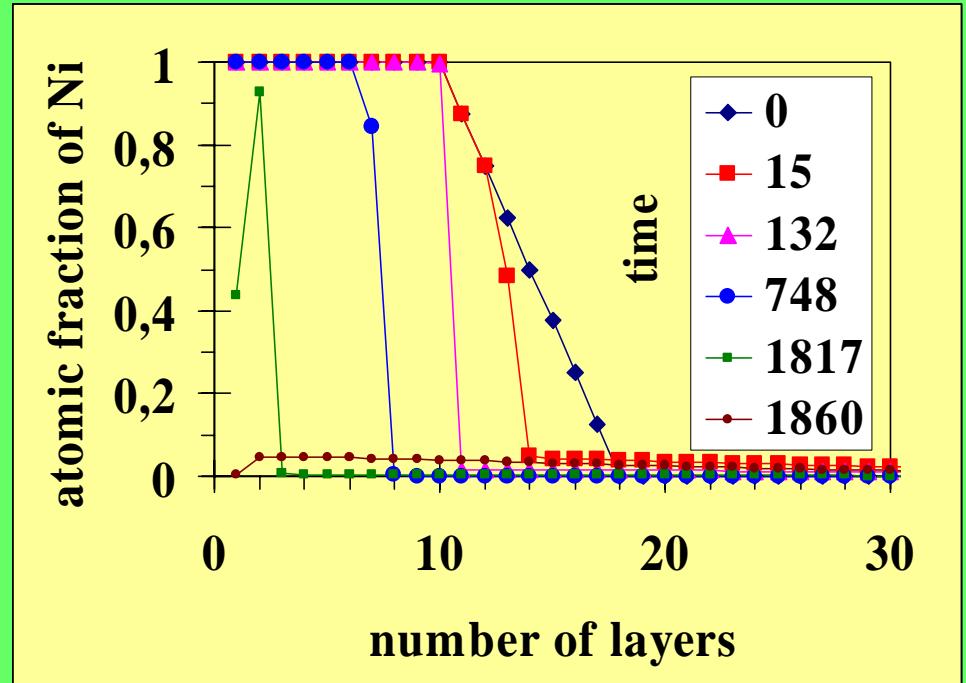
Interface

Experiment: AES from the top of Ni on Cu(111)

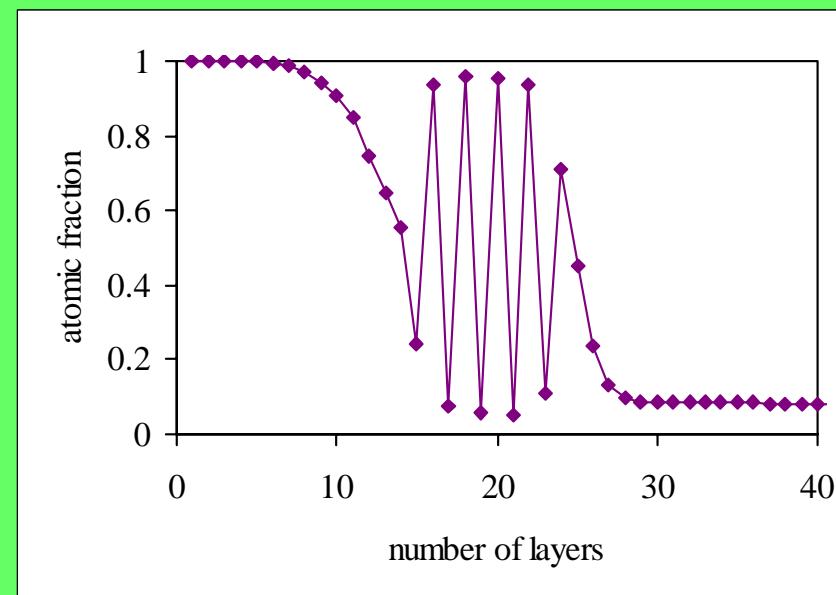
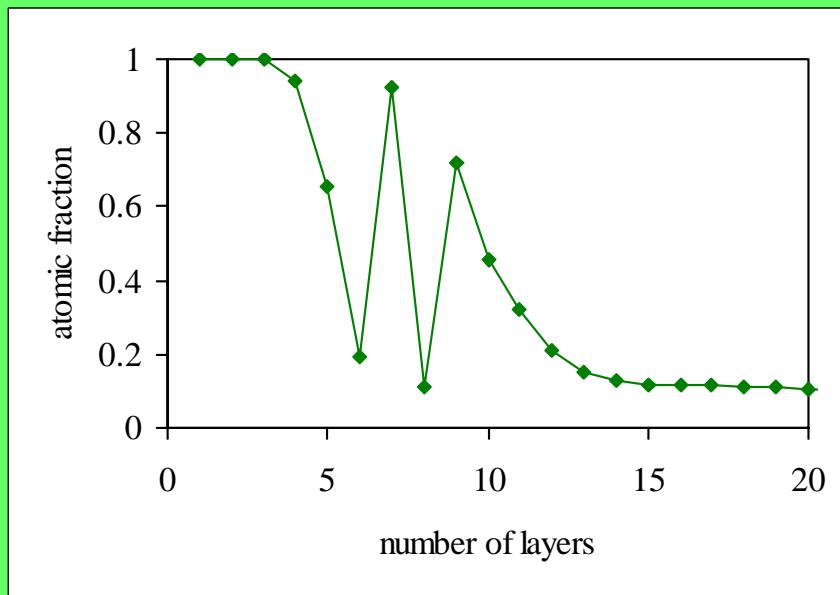
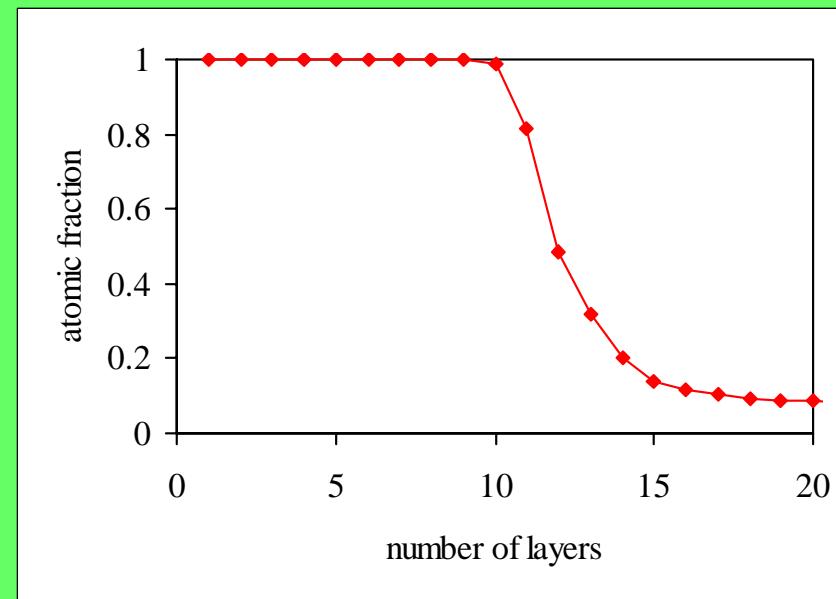
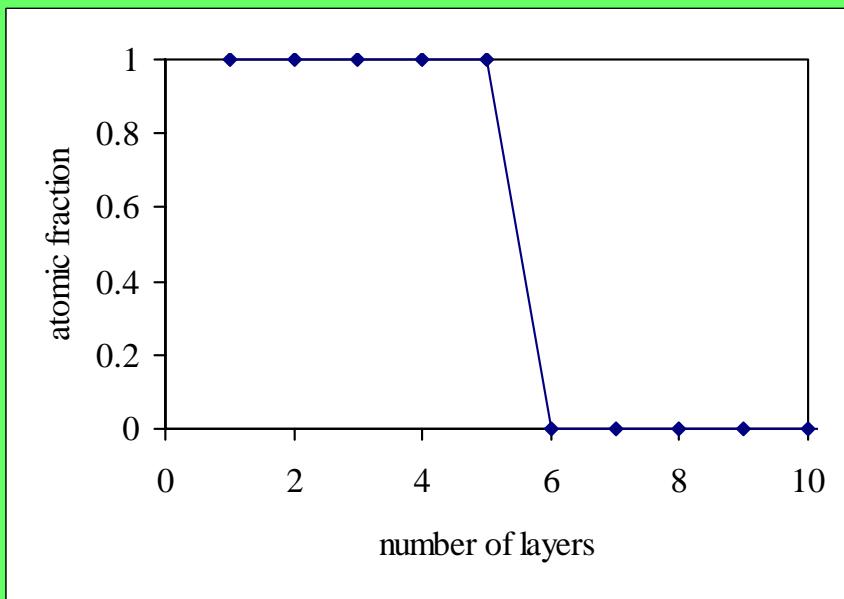
$T=679K$



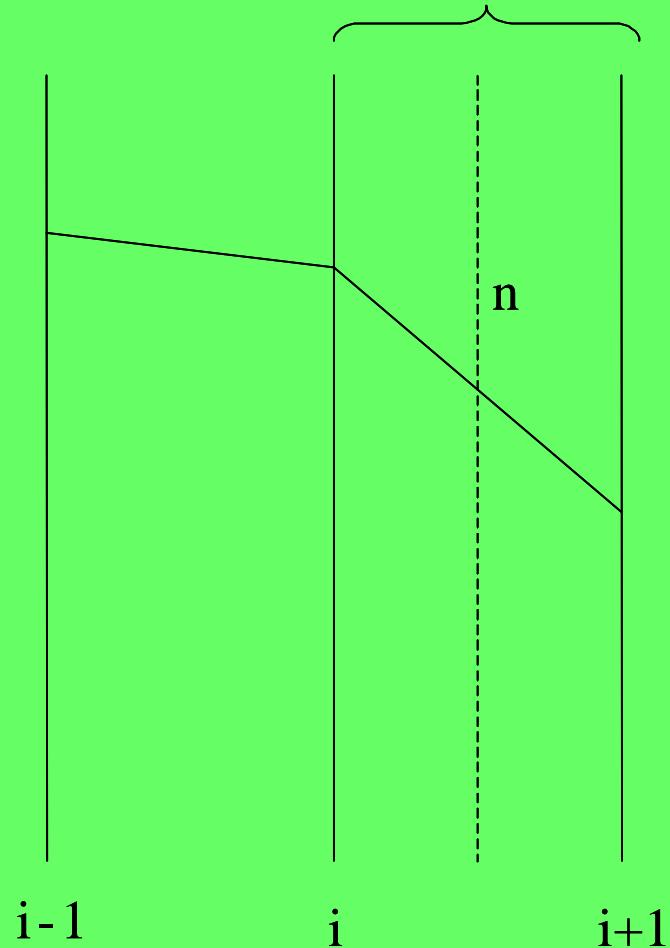
Sharpening of a wide interface ($T=1000\text{K}$, $m'=9$)



Growth of intermetallic layer:



$$nd = \rho = c/\Omega$$



$$n_i = n - \frac{d}{2} \frac{\partial n}{\partial x} + \frac{(d/2)^2}{2!} \frac{\partial^2 n}{\partial x^2} - \frac{(d/2)^3}{3!} \frac{\partial^3 n}{\partial x^3} + \dots$$

$$\frac{\partial c}{\partial x} = \frac{c_{i+1} - c_i}{d}$$

$$\frac{\partial^2 c}{\partial x^2} = \frac{\frac{c_{i+1} - c_i}{d} - \frac{c_i - c_{i-1}}{d}}{d}$$

$$\text{e.g. } \alpha_i = (V_{AA} - V_{BB}) \{ cZ + (z_v + Z/4) d^2 \partial^2 c / \partial x^2 + \dots \} = cZ(V_{AA} - V_{BB}) + \alpha'_i + \dots$$

Introducing $\Gamma_i = v \exp[-(E^\circ - \alpha_i)/kT] = \Gamma_i^h \exp[\alpha'_i/kT]$.

and if $\varepsilon_i/kT \ll 1$ (i.e. $\exp[\varepsilon_i/kT] \approx 1 + \varepsilon_i/kT$

$$j_{i,i+1} = J_{i,i+1}/q = J_{i,i+1} d/\Omega = -D_i (\partial c / \partial x) / \Omega + \\ + D_i [2\kappa/f_o'' - d^2/24] (\partial^3 c / \partial x^3) / \Omega + \dots$$

$$D_i = z_v d^2 \Gamma_i \theta \quad \kappa \sim V \quad \text{Fick I.}$$

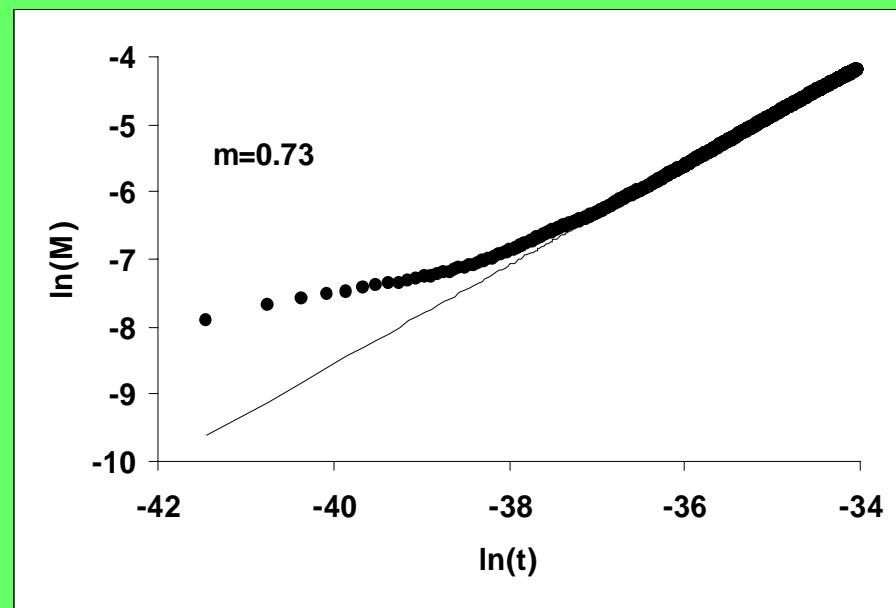
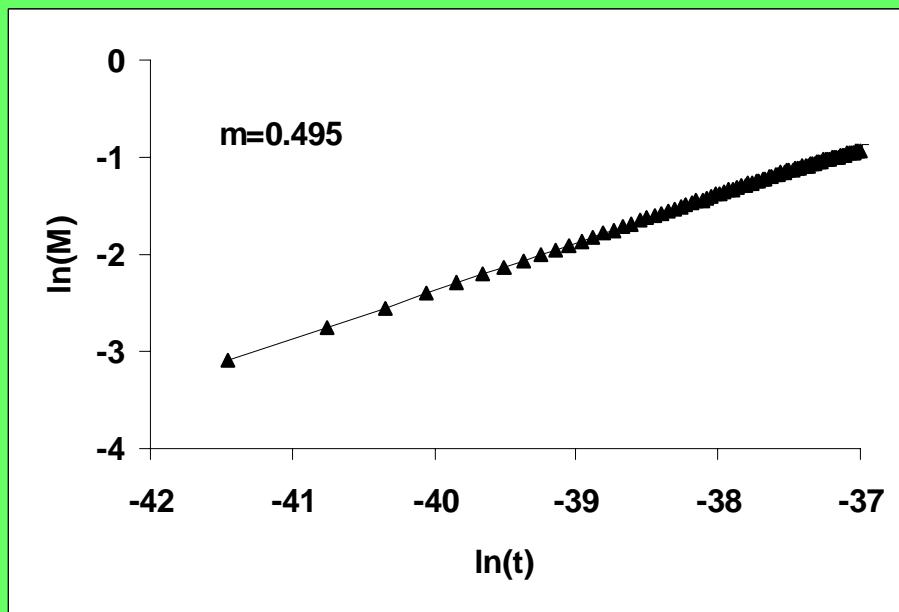
Amorphous systems?, Stress effects...

Constant concentration at the surface:

$X_o=0$



$$D(c) = D(0) e^{m' c}$$



Conclusions (ideal systems):

- At short distances the continuum descriptions fails and this strongly depends on the concentration dependence of D (non-linearity)
- Non-linearity leads to *shift of a sharp interface*
- The non-linearity leads to
 - a linear shift of a sharp interface*
 - sharpening of an originally wide interface*
- Gradient energy corrections are important not only in the currents but also in the mobilities

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