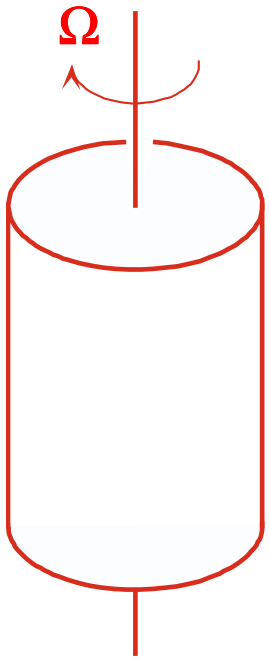


# Rotating Bose-Einstein condensates and Quantum Vortices

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ENS Paris

# Vortices in a rotating quantum fluid



In a condensate  $\psi(\vec{r}) = \sqrt{\rho(\vec{r})} e^{iS(\vec{r})}$

the velocity  $\vec{v} = \frac{\hbar}{m} \vec{\nabla} S$  is such that  $\int \vec{v} \cdot d\vec{r} = \frac{nh}{m}$

incompatible with rigid body rotation  $\vec{v} = \vec{\Omega} \times \vec{r}$

## Liquid superfluid helium

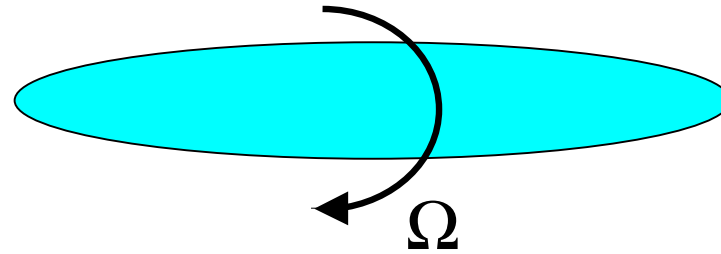
Below a critical rotation  $\Omega_c$ , no motion at all

Above  $\Omega_c$ , apparition of singular lines on which the density is zero and around which the circulation of the velocity is quantized

Onsager - Feynman

# Dynamics of a rotating Bose gas

Bose-Einstein condensate trapped in a harmonic potential with transverse frequency  $\omega_{\perp}$



The gas is stirred with a "spoon" rotating at frequency  $\Omega$

*vortex nucleation*

Single vortex preparation:

what is the shape of a single vortex line?

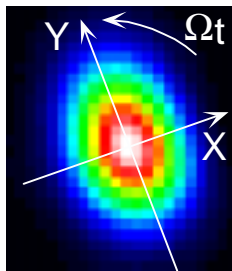
Single vortex dynamics: Kelvin modes of the line

**1.**

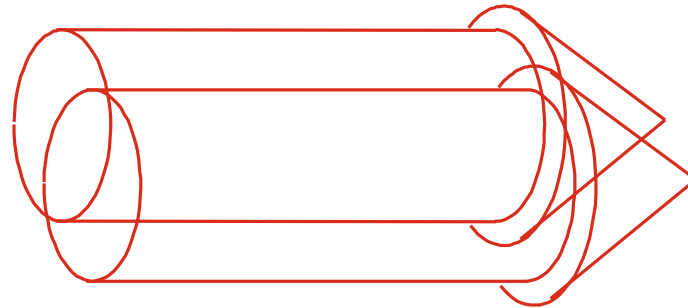
**Reliable production of a single vortex**

# The laser stirrer for a gaseous BEC

Use of a laser spoon to stir the condensate and produce vortices:  
ENS group, Phys. Rev. Lett. **84**, 806 (2000)



16  $\mu\text{m}$



stirring laser

small rotating perturbation  
of the magnetic potential  
( a few %)

Magnetically trapped  
Rb condensate



$$\omega_{\perp} / 2\pi = 97 \text{ Hz}$$

$$\omega_z / 2\pi = 12 \text{ Hz}$$

$$N \quad 5 \cdot 10^5 \text{ atoms}$$

$$\Omega \quad \frac{\omega_{\perp}}{\sqrt{2}} \quad \text{quadrupole resonance}$$

(also at MIT, Boulder, Oxford)

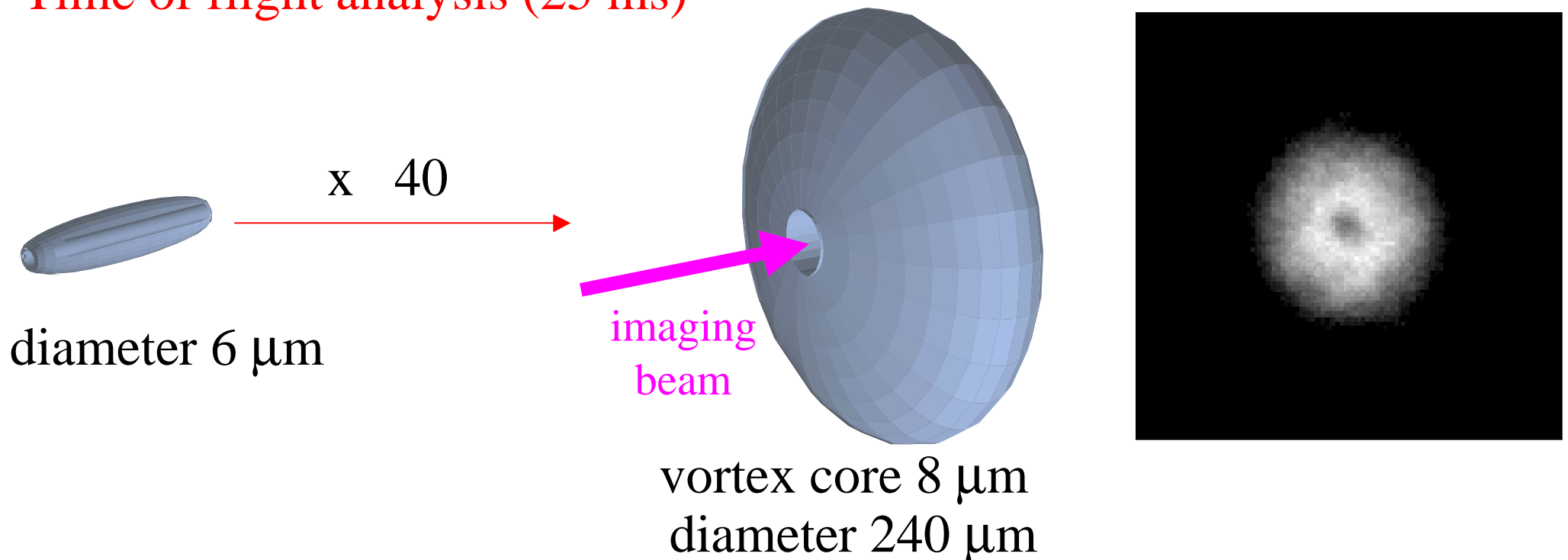
# Diagnostic of the rotating quantum gas

A single vortex is not visible *in situ*

$$\xi = \frac{1}{\sqrt{8\pi\rho a}} = 0.2\mu\text{m}$$

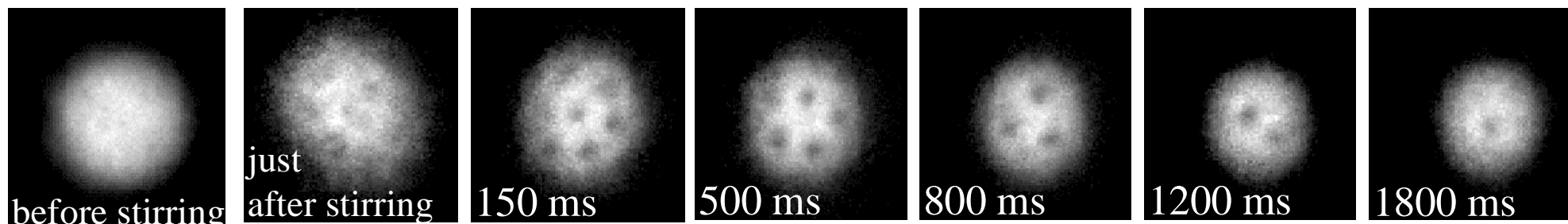
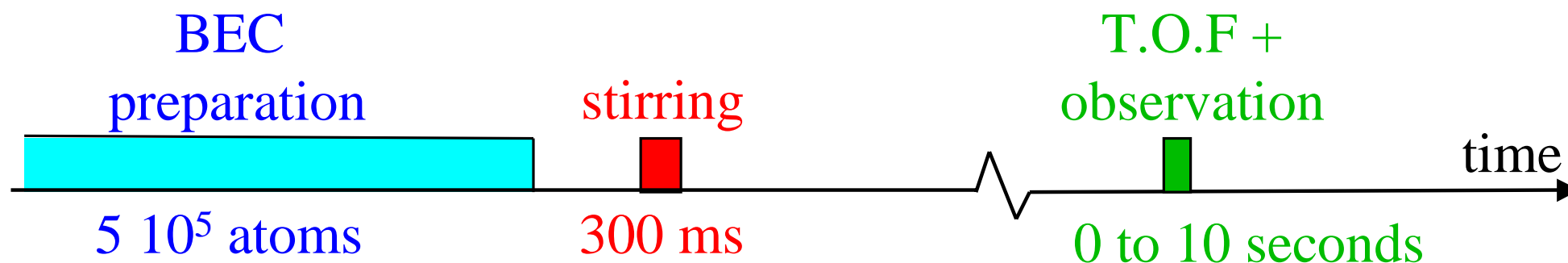
$$\rho = 2 \cdot 10^{14} \text{ cm}^{-3}$$
$$a = 5.5 \text{ nm}$$

Time of flight analysis (25 ms)



# Reliable preparation of a single vortex

Nucleation of a small vortex array which rapidly decays in a single, well-centered vortex



Different time scales for the decay of the array and of the last vortex (also at MIT)

**2.**

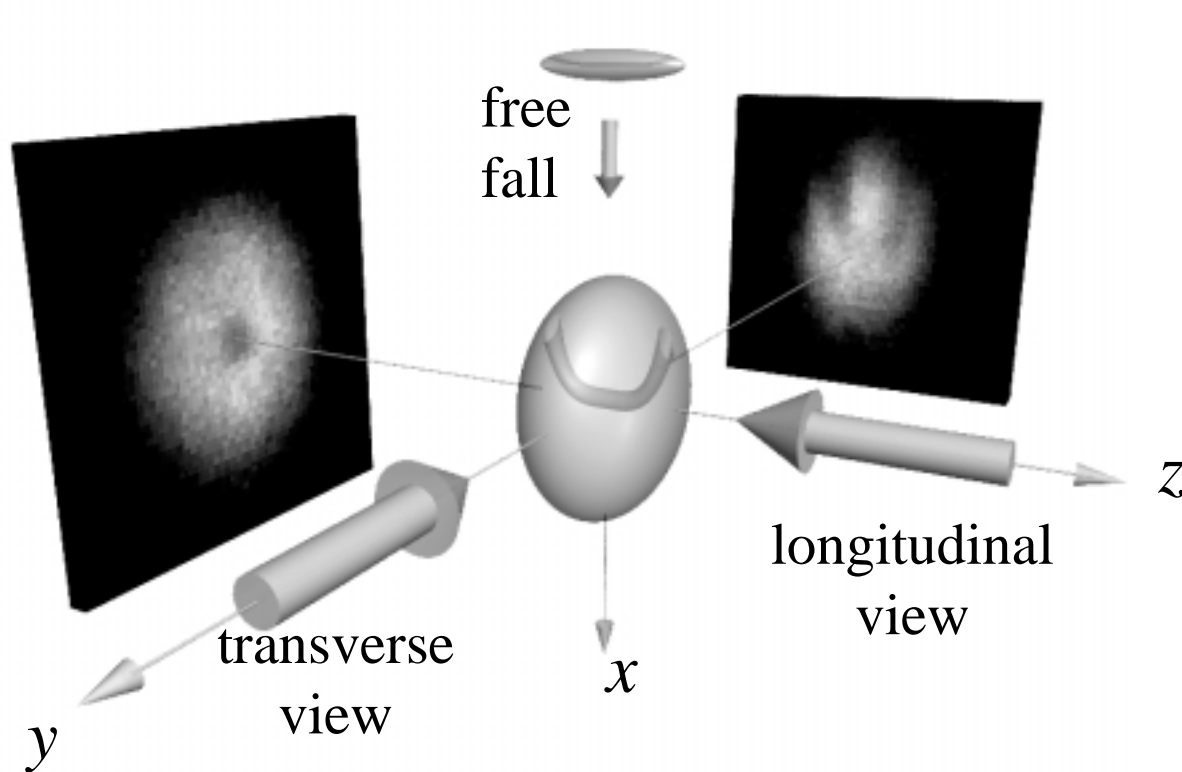
## **The shape of a single vortex line**

P. Rosenbusch, V. Bretin, J. Dalibard  
cond-mat/0206511



# Observation of the complete vortex line

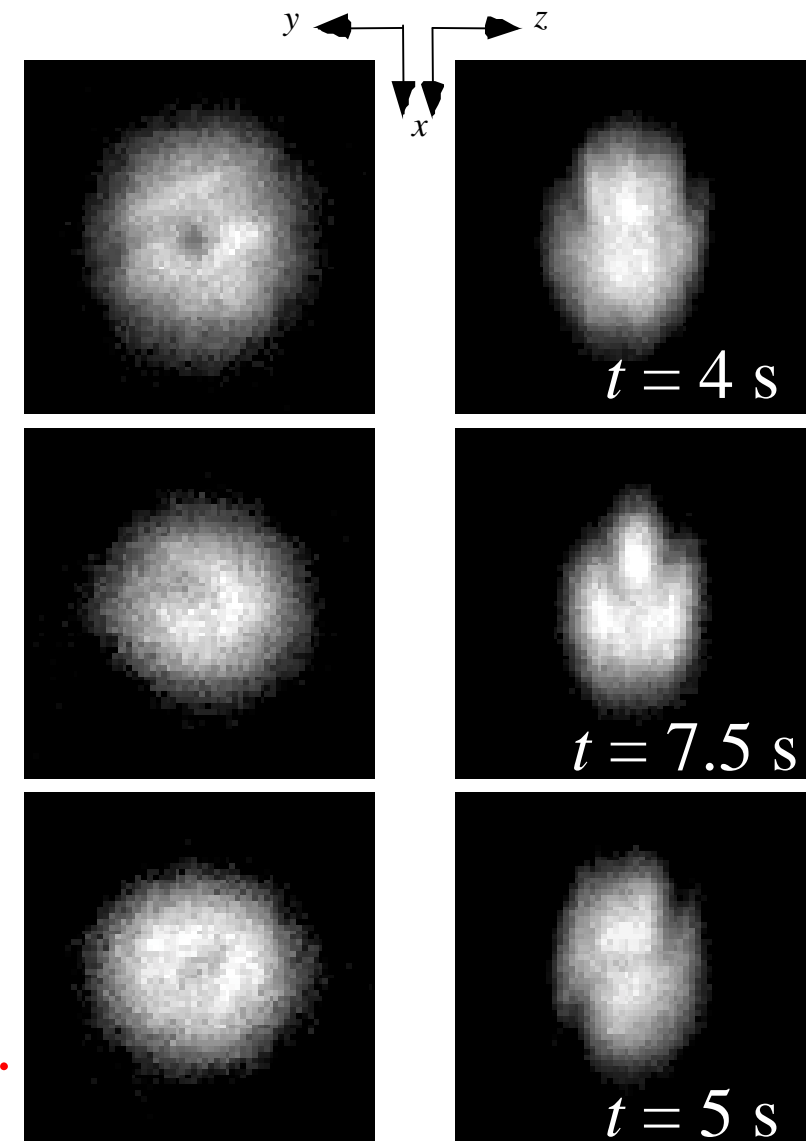
## Simultaneous longitudinal and transverse imaging



similar transverse imaging developed  
in Boulder and Oxford for a vortex array

A single vortex line is generally not straight...

U shaped or N shaped



# Why is a single vortex line curved?

**Precursor:** for cigar shaped condensates, anomalous modes of the state with one straight vortex (Fetter-Svidzinsky 2000, Feder *et al.* )

Garcia-Ripoll & Perez-Garcia (2001)

symmetry breaking effect,  
which occurs even for  $\varepsilon=0$



See also Aftalion *et al.* (2001)

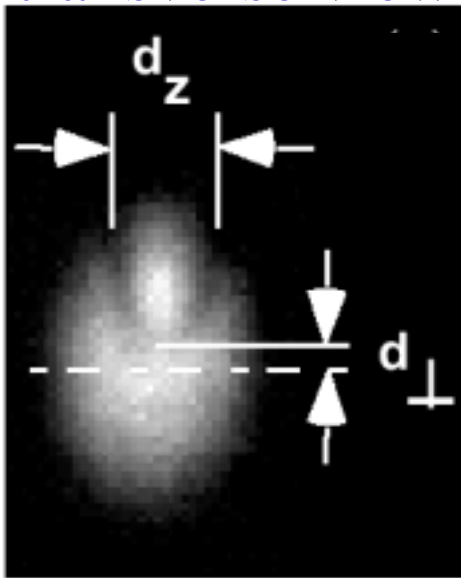
Modugno, Pricoupenko, Castin (2002)

Interpretation in terms of a series of 2D problems

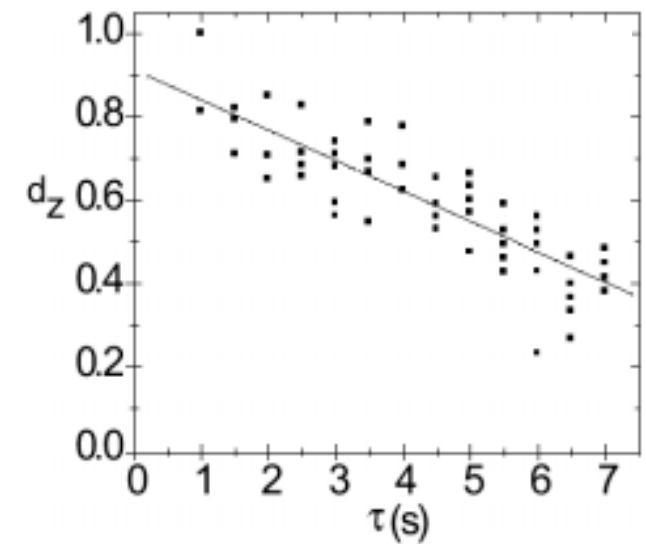
# The decay of a vortex line

The trap is not perfectly axisymmetric...

transverse view



We normalize  $d_z$   
by the length of  
the condensate

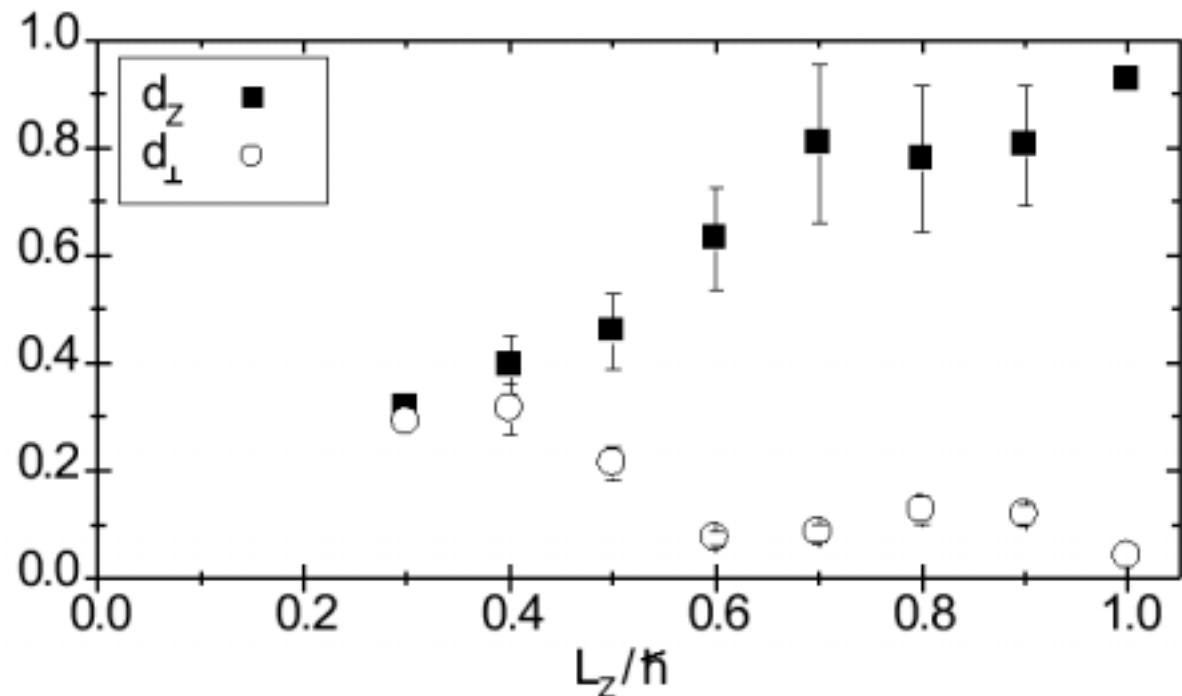
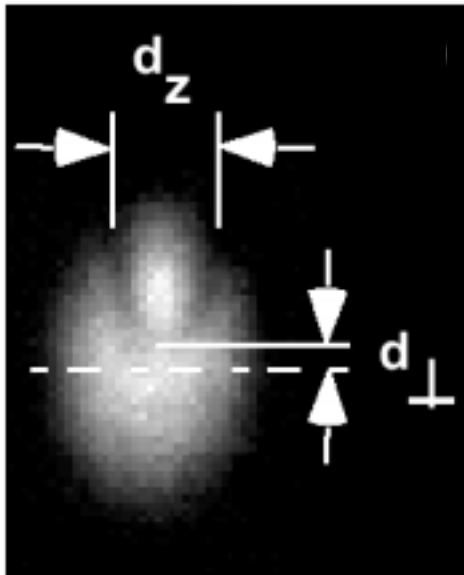


# Relation between the shape of the vortex line and the angular momentum

Measurement of the angular momentum using the frequency of the two quadrupole modes  $m = \pm 2$

Theory: Zambelli and Stringari, PRL **81**, 1754 (1998)

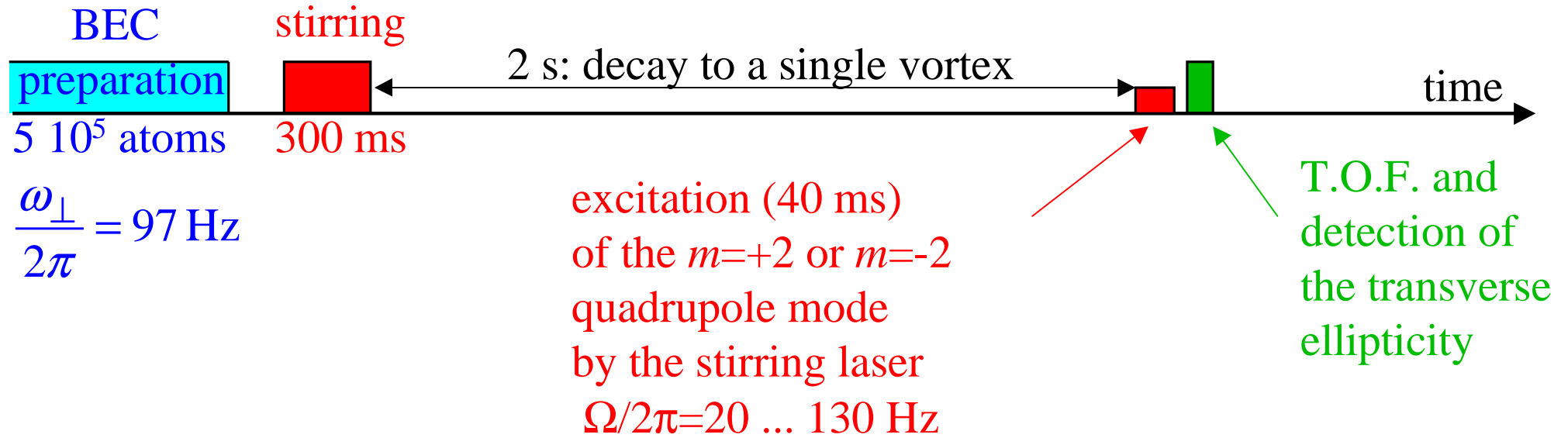
Exp. for a well centered vortex: ENS group, PRL **85**, 2223 (2000)



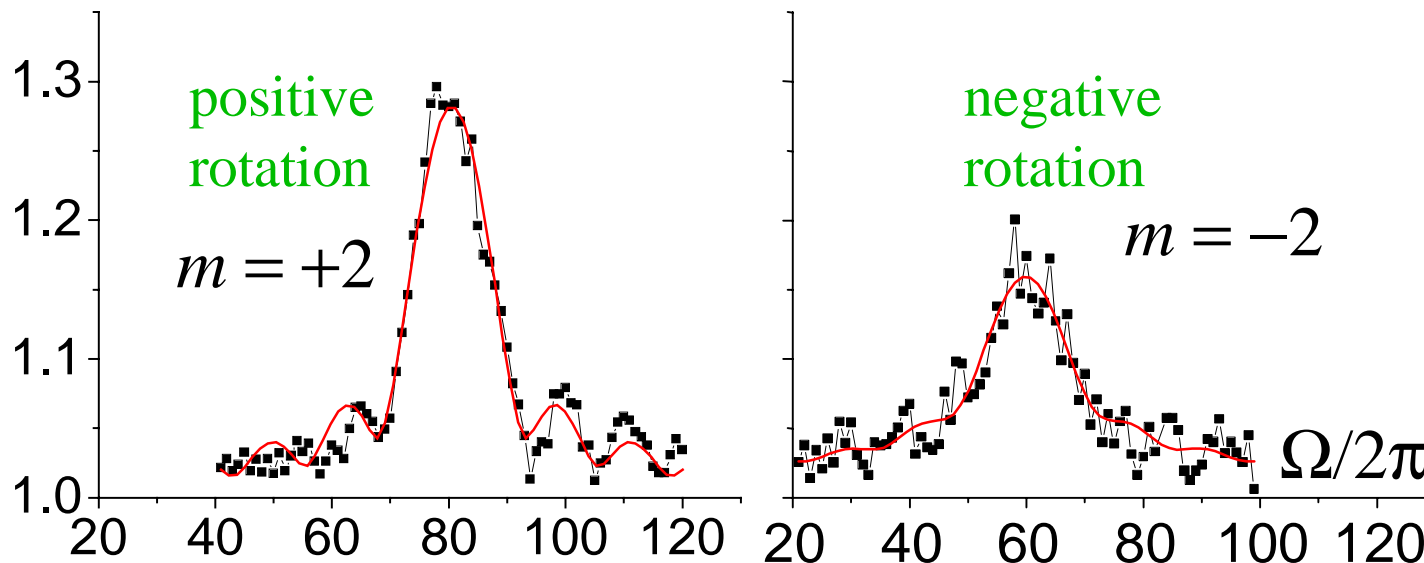
**3.**

**Excitations of a single vortex line: the Kelvin mode**

# Quadrupole excitation of a condensate with a single vortex line



ellipticity



center + : 80.5 Hz

center - : 59.9 Hz

→ angular momentum  $h$  (also in Boulder)

lifetime + : 42 ms

lifetime - : 18 ms

## Possible explanation (s) for the rapid decay of the $m = -2$ mode

- After the vortex nucleation phase, the uncondensed part is rotating in the positive sense, as the vortex itself.

The  $m = -2$  mode is more damped than the  $m = +2$

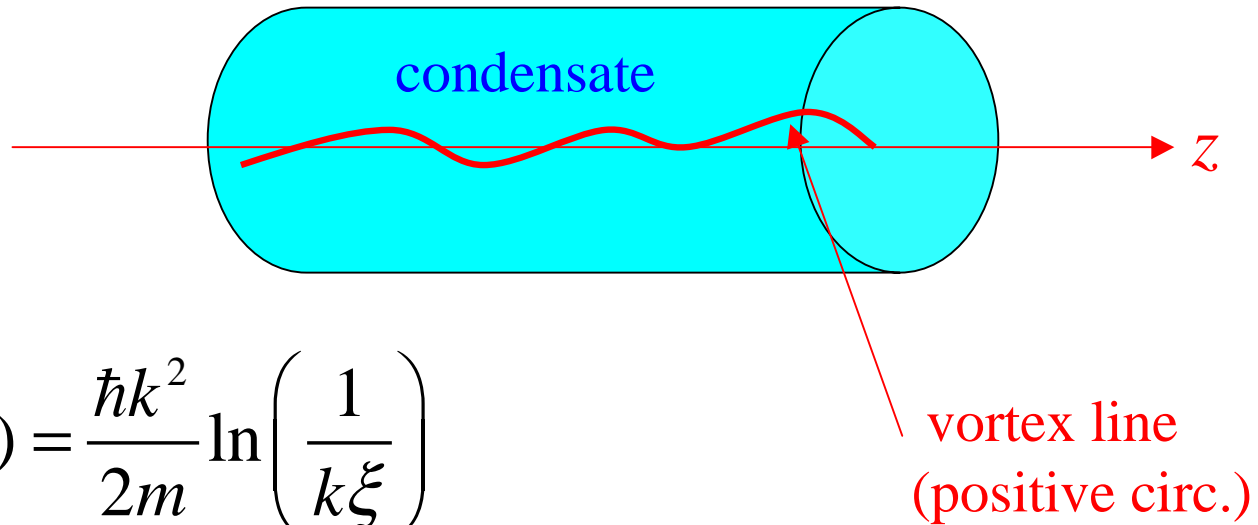
Williams, Zaremba, Jackson, Nikuni, Griffin, P.R.L. 2002

- Conversion of the  $m=-2$  quadrupole mode into a pair of "kelvons"

Excitations of the vortex line

Shlyapnikov, private com.

# The Kelvin modes of a vortex line



Wave vector along  $z$  :  $k$

Dispersion relation: 
$$\omega(k) = \frac{\hbar k^2}{2m} \ln \left( \frac{1}{k\xi} \right)$$

Angular momentum along  $z$  :  $-\hbar$

Possible mechanism for the decay of the quadrupole  $m=-2$  mode:

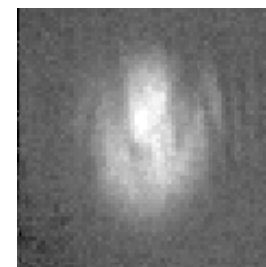
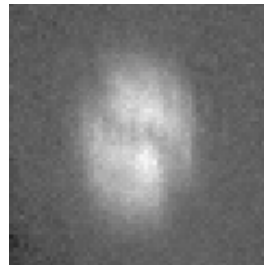
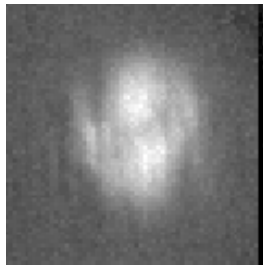
$$\text{Quadrupole, } \omega_{-2} \longrightarrow \text{Kelvon } k, \frac{\omega_{-2}}{2} + \text{Kelvon } -k, \frac{\omega_{-2}}{2}$$



# Conclusions

Are the Kelvin modes directly observable?

transverse pictures of the condensate after the excitation of the  $m=-2$  mode:



no equivalent "fringes" after excitation of the  $m=+2$  mode

Universal character of vortices as a way to set a macroscopic quantum object into motion:

superfluid liquid helium, (type II) superconductors, neutron stars

Gaseous BECs offer unique possibilities for a direct observation of these objects and of their dynamics