

Active detection of sound in the inner ear

QuickTime™ and a
PNG decompressor
are needed to see this pict

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Performance of human ear

Frequency analysis: responds selectively to frequencies in range **20–10,000 Hz**

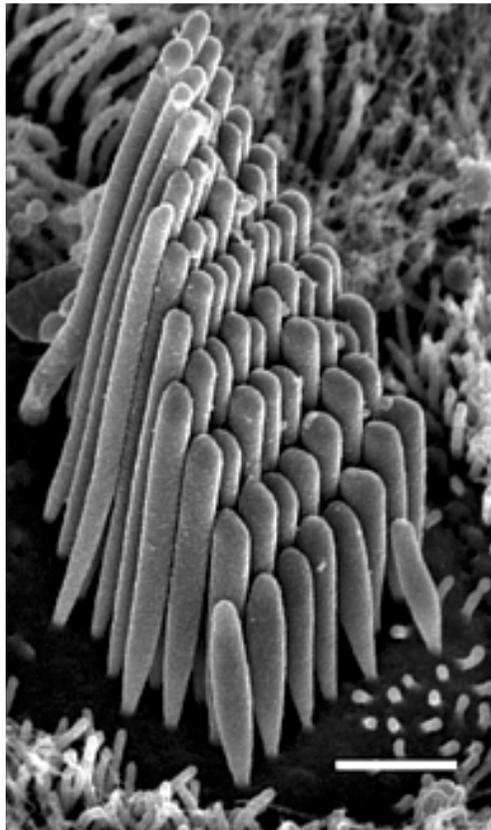
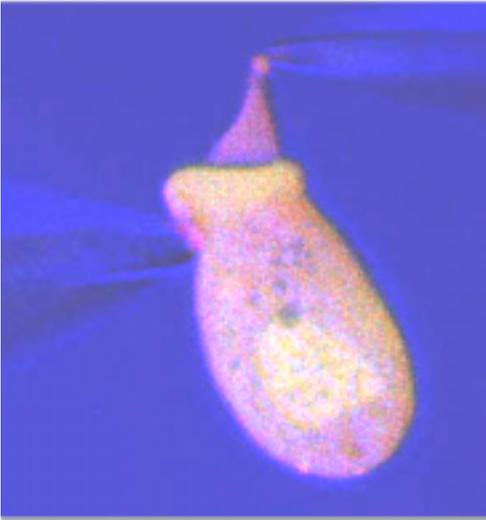
Sensitivity: faintest audible sounds impart no more energy than thermal noise: **4 zJ**

Dynamic range: responds and adapts over 7 orders of magnitude of pressure: **0–140 dB**

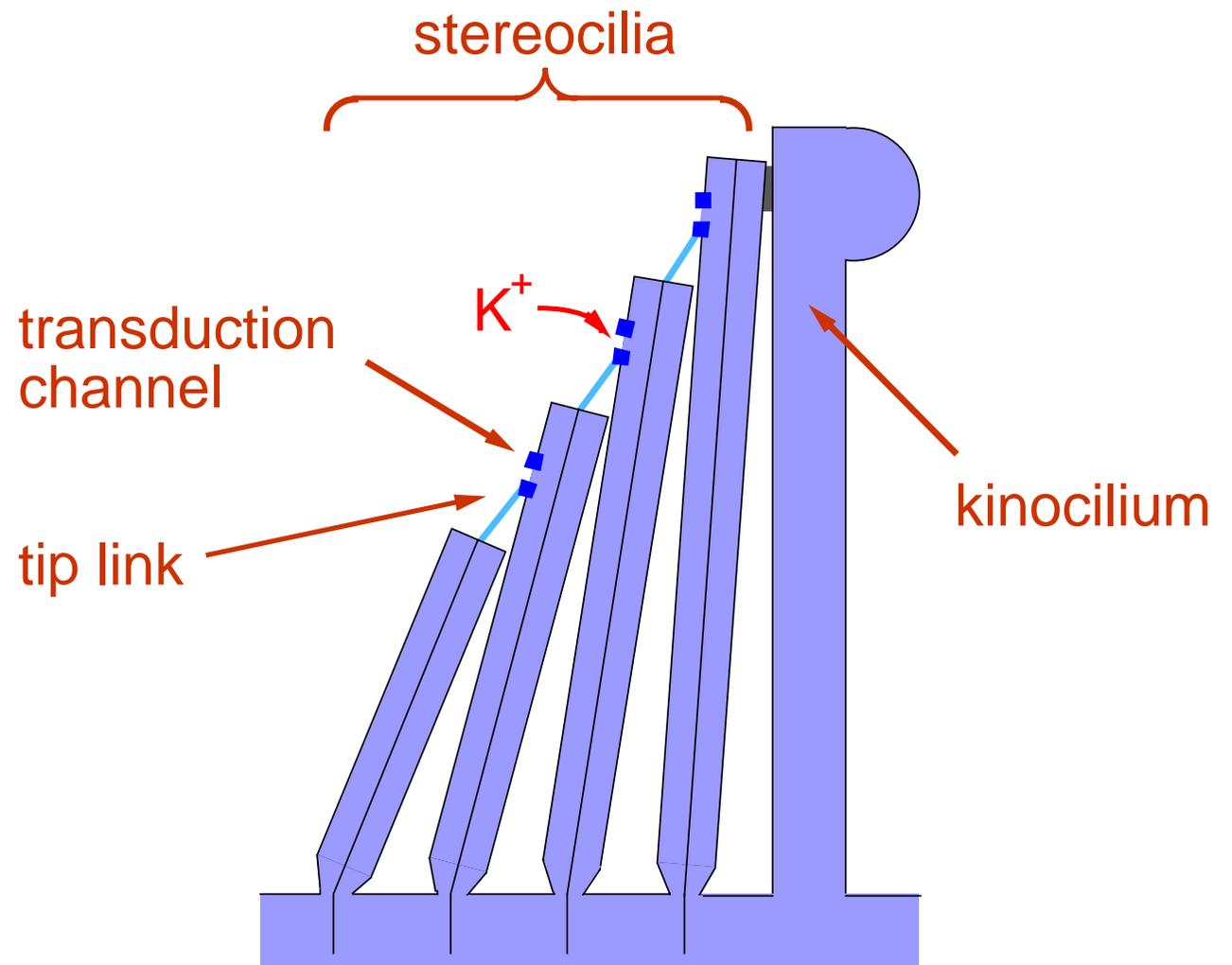
Detection apparatus

sources: Hudspeth, Hackney

hair cell (*frog*)



hair bundle (*turtle*)

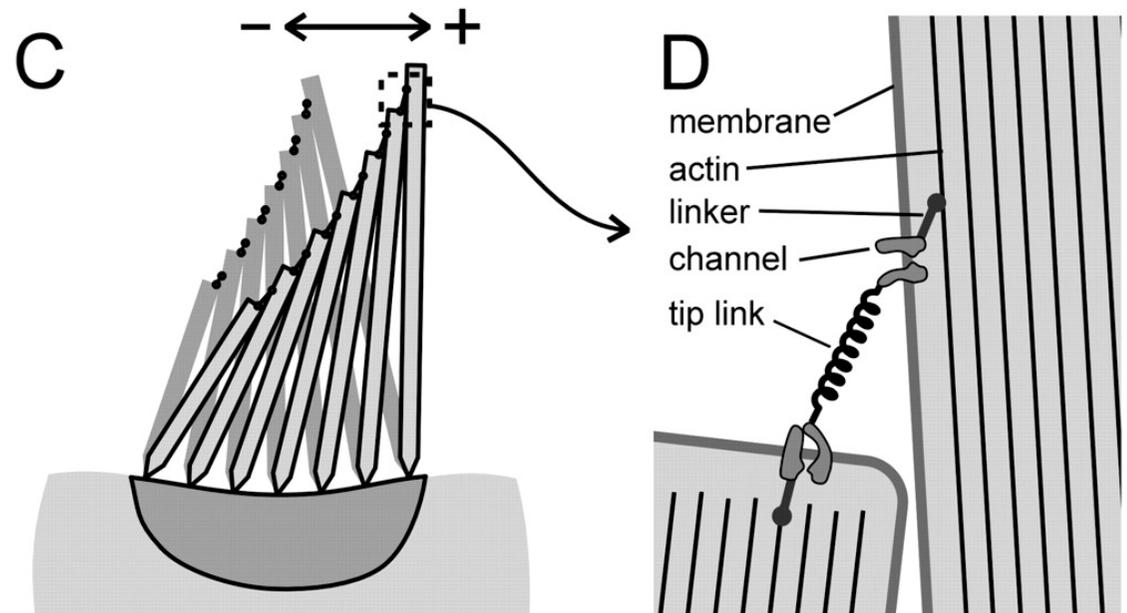
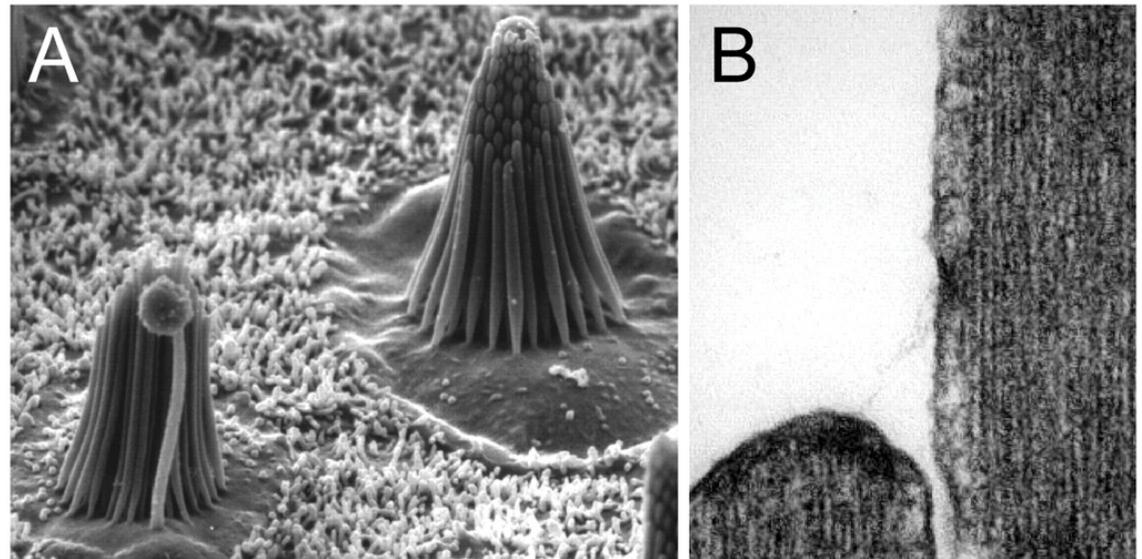


Mechano-chemo-electrical transduction

sources: Corey, Hudspeth

Tension in tip links
pulls open
transduction channels
& admits K^+

which depolarizes the
membrane & opens
voltage-gated channels
to nerve synapse



Spontaneous oscillations in the inner ear

Kemp '79

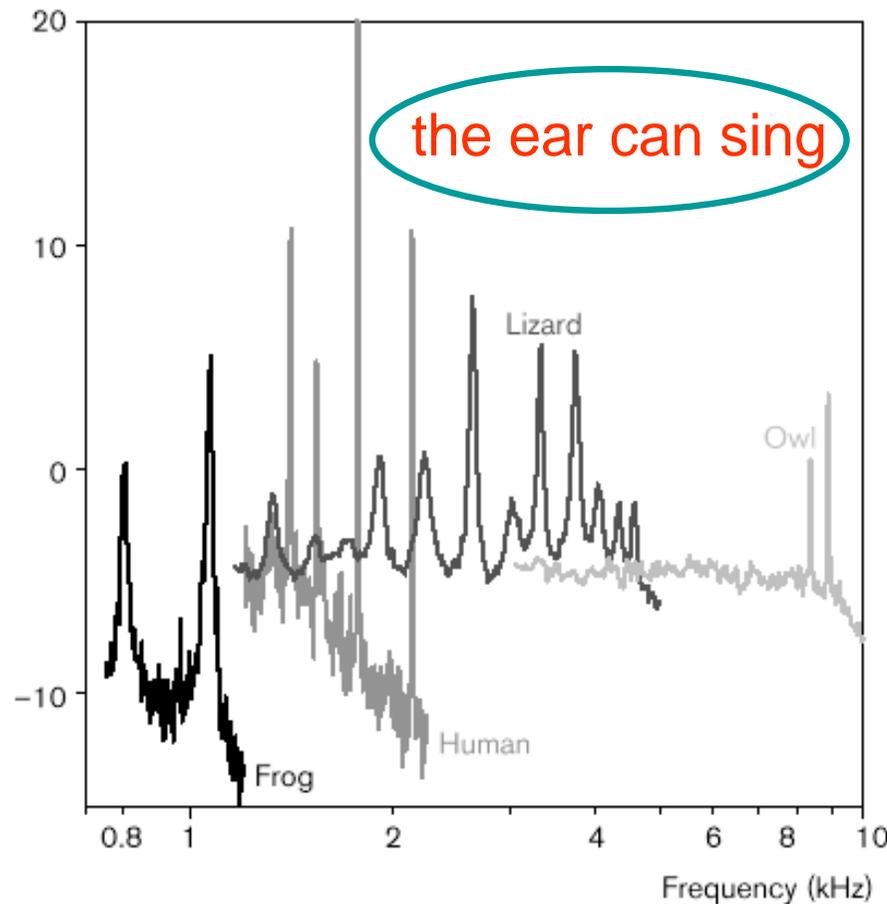
Manley & Koppl '98

Crawford & Fettiplace '86

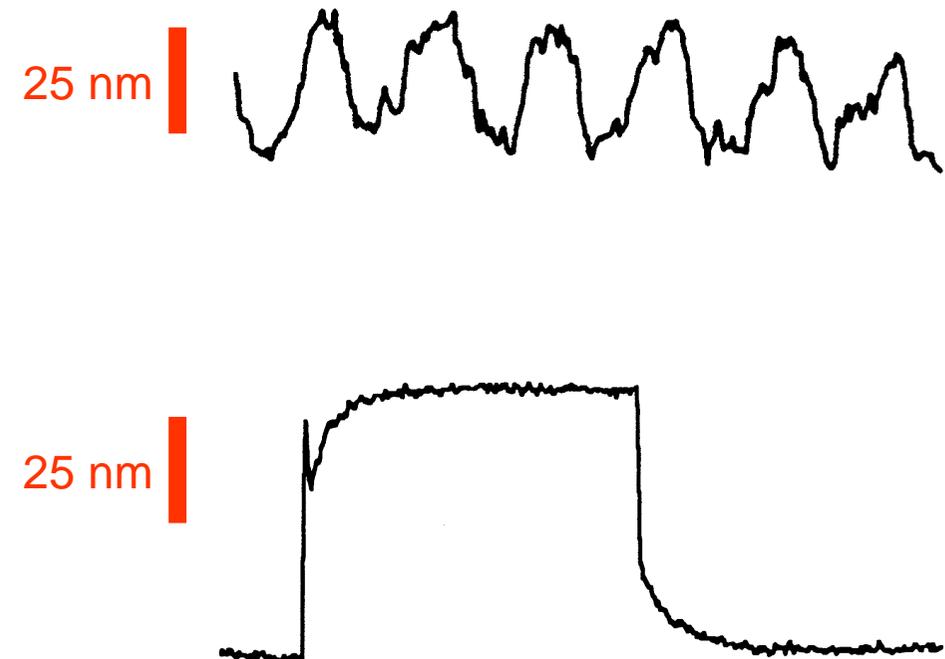
Howard & Hudspeth '87

- Otoacoustic emissions

Approximate
sound pressure
level (dB SPL)



- Active bundle movements

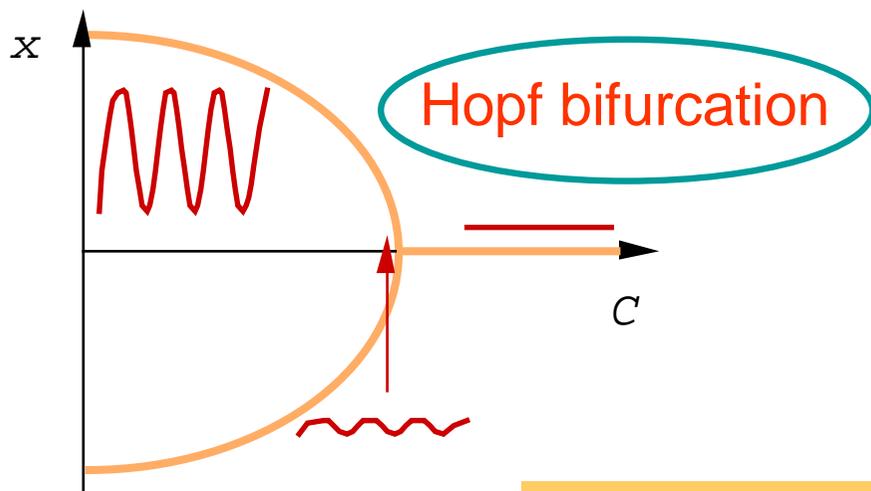


Self-tuned critical oscillators

Camalet, Duke, Jülicher & Prost '00

Active amplifiers: Ear contains a set of **nonlinear dynamical systems** each of which can generate **self-sustained oscillations** at a different **characteristic frequency**

Self-adjustment: **Feedback control mechanism** maintains each system **on the verge** of oscillating



→ remarkable response properties at critical point

Hopf resonance

force: $F(t) = \sum_{n=-\infty}^{\infty} F_n e^{in2\pi ft}$

displacement: $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{in2\pi ft}$

$$F_1 \simeq \mathcal{A}x_1 + \mathcal{B}|x_1|^2 x_1 + \dots$$

control parameter: C bifurcation point: $\mathcal{A}(f_c, C_c) = 0$

- stimulus at characteristic frequency:

$$|x_1| \simeq \frac{|F_1|^{1/3}}{|\mathcal{B}|^{1/3}}$$

gain diverges
for weak stimuli

Hopf resonance

force: $F(t) = \sum_{n=-\infty}^{\infty} F_n e^{in2\pi ft}$

$$F_1 \simeq \mathcal{A}x_1 + \mathcal{B}|x_1|^2 x_1 + \dots$$

displacement: $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{in2\pi ft}$

control parameter: C bifurcation point: $\mathcal{A}(f_c, C_c) = 0$

• stimulus at different frequency: $\mathcal{A}(f, C_c) = \alpha(f - f_c) + \dots$

→ if $|f - f_c| \ll$

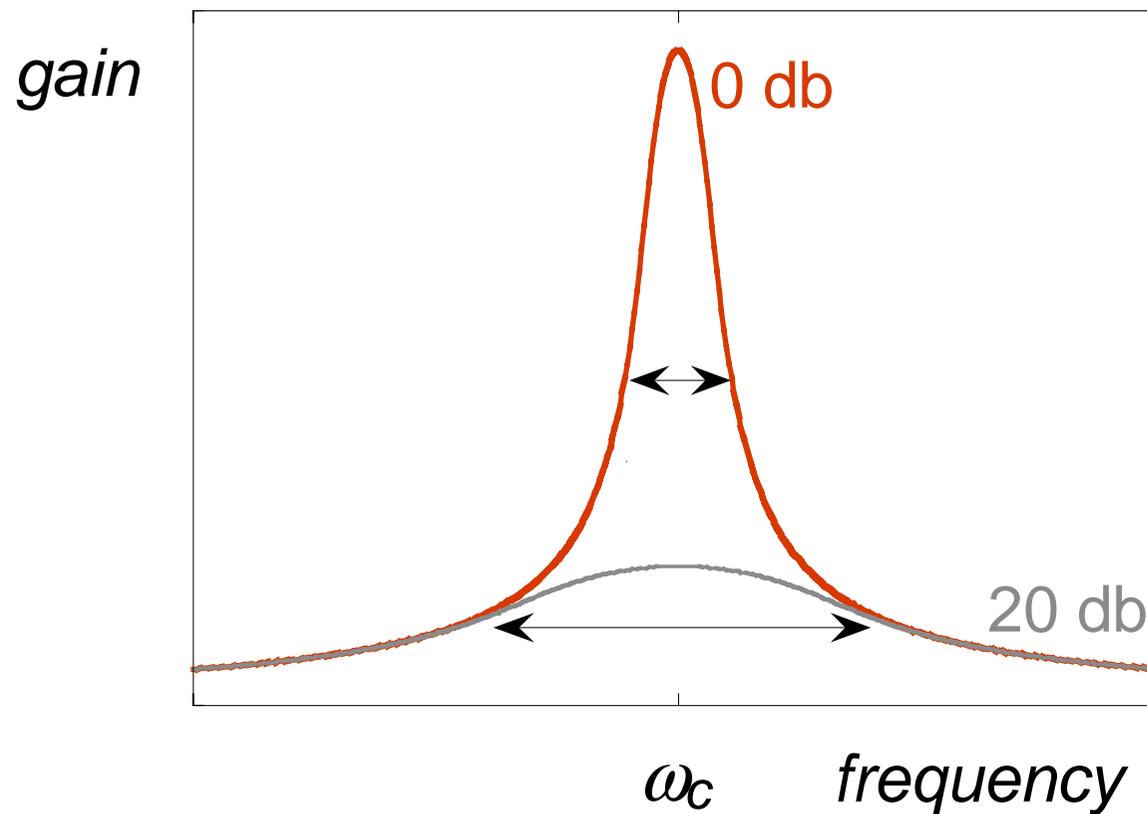
$$\Delta f_a \equiv \frac{|\mathcal{B}|^{1/3}}{|\alpha|} |F_1|^{2/3}$$

$$|x_1| \simeq \frac{|F_1|}{|(f - f_c)\alpha|}$$

active bandwidth

critical Hopf resonance
single tone response

Gain and active bandwidth depend on level of stimulus



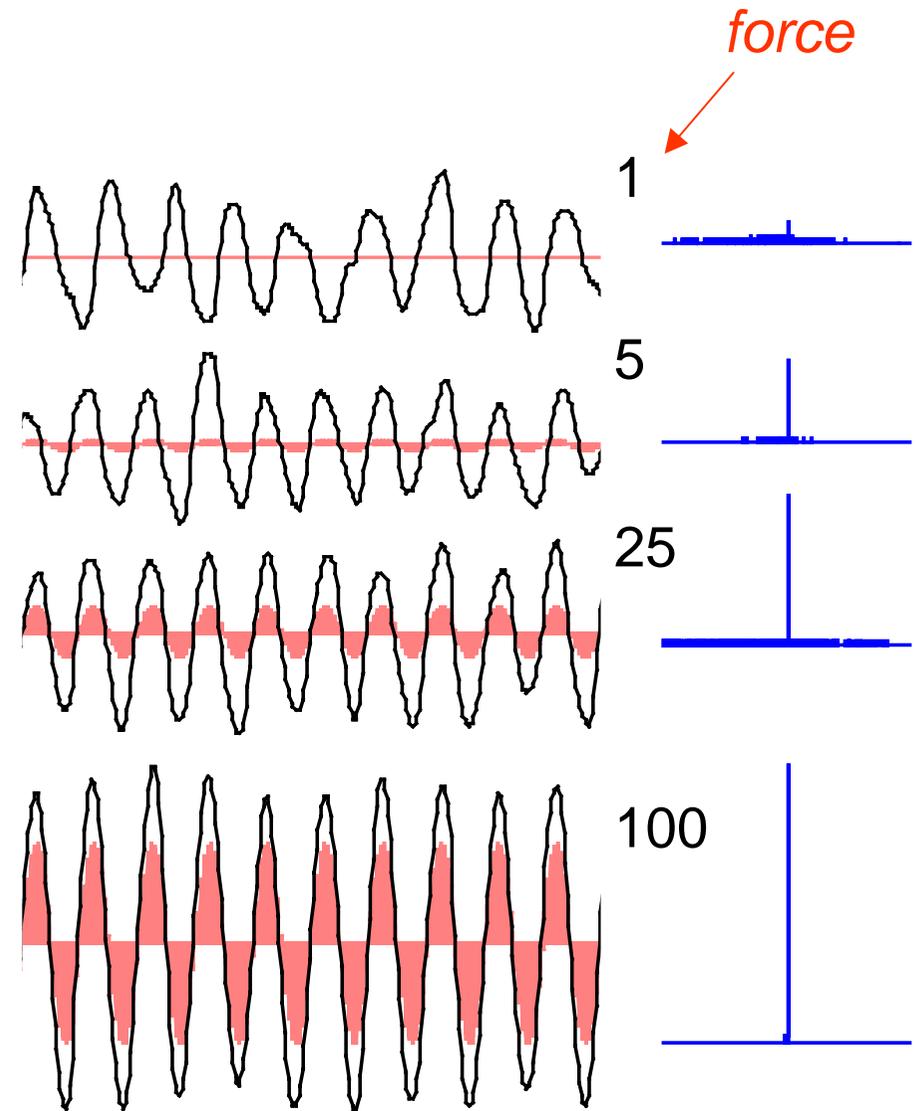
critical Hopf response

effect of noise

Camalet et al. '00

Response to a tone

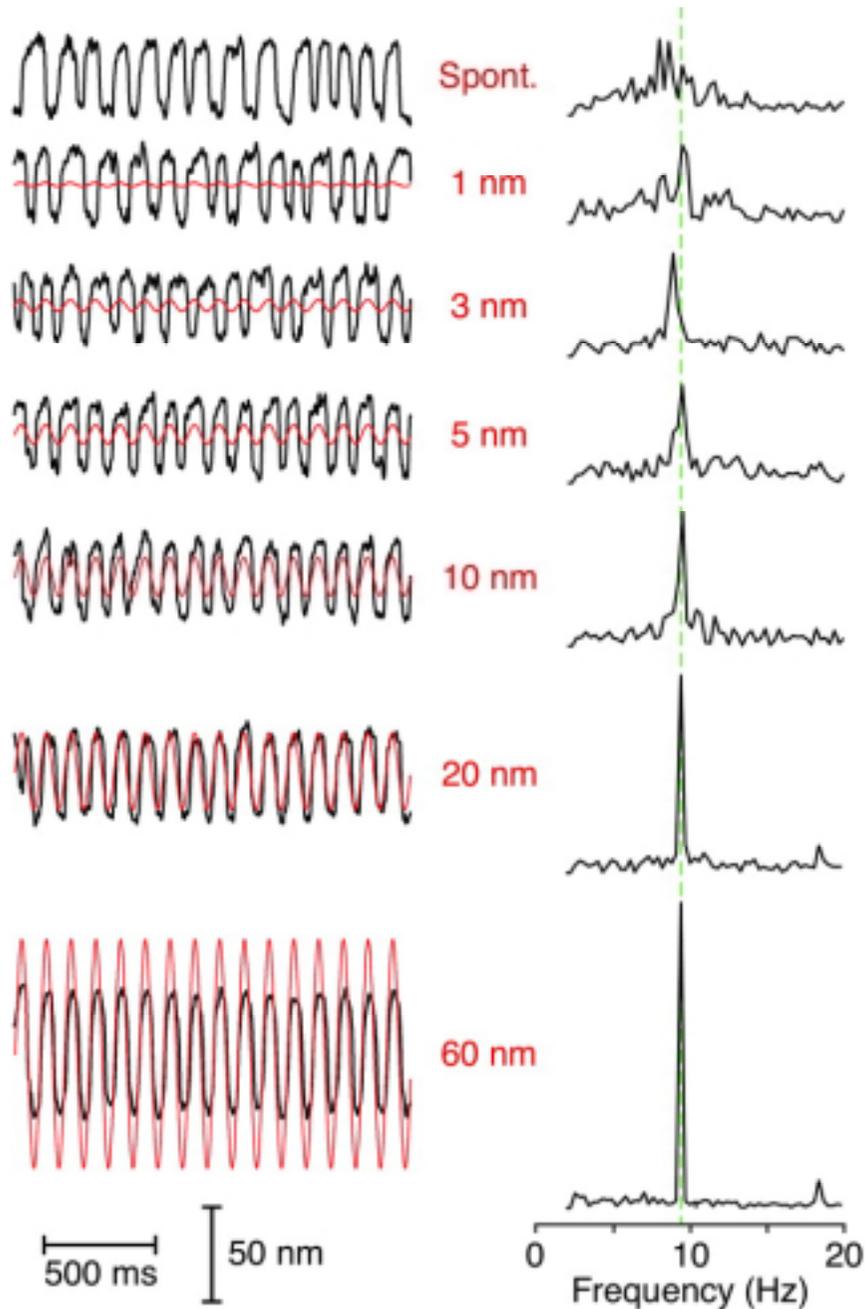
- spontaneous critical oscillations are **incoherent**
- stimulus at characteristic frequency gives rise to **phase-locking**



Hair bundle response

Martin & Hudspeth '01

Response of a frog hair bundle forced by a microneedle



Questions

- What is the physical basis of the force-generating dynamical system ?
- How is the self-tuning realised ?

We might expect that different organisms use different apparatus to implement the same general strategy



Model for *non-mammalian vertebrates*

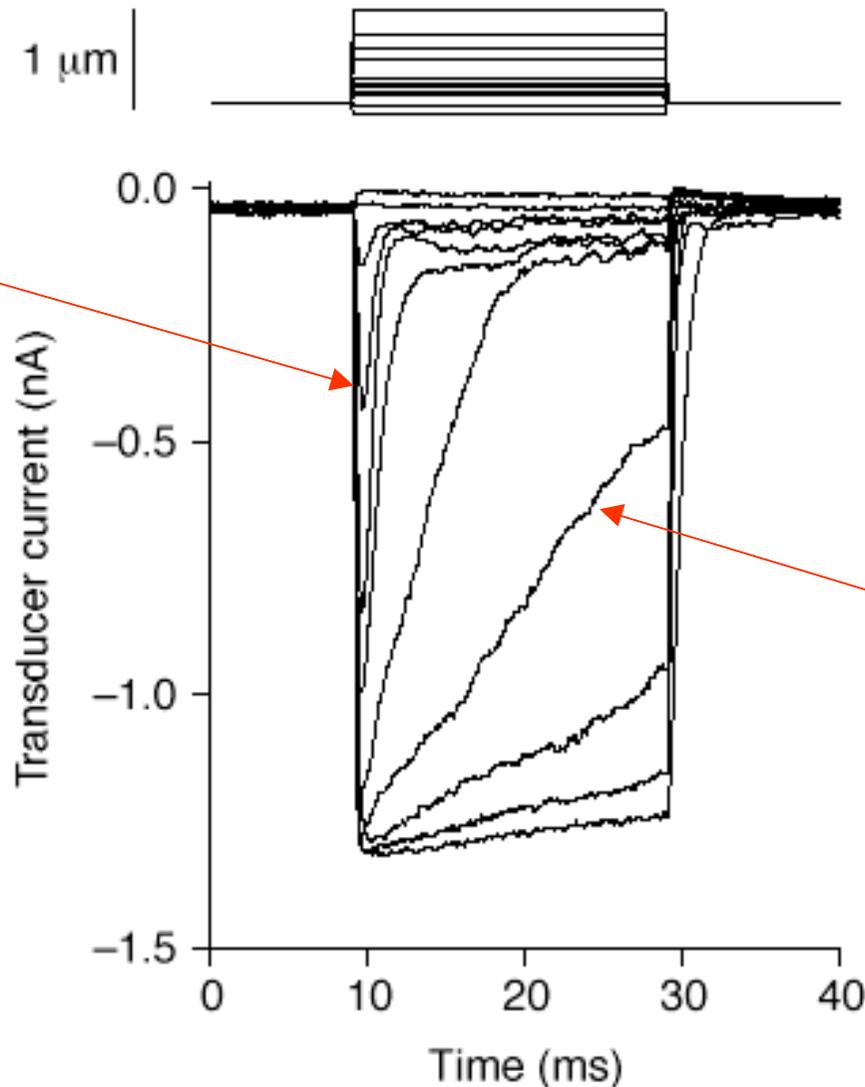
Two adaptation mechanisms

Fettiplace et al. '01

Fast process

Ca²⁺ binding to
transduction channel

~ 1 ms



Slow process

movement of
myosin-1C motors

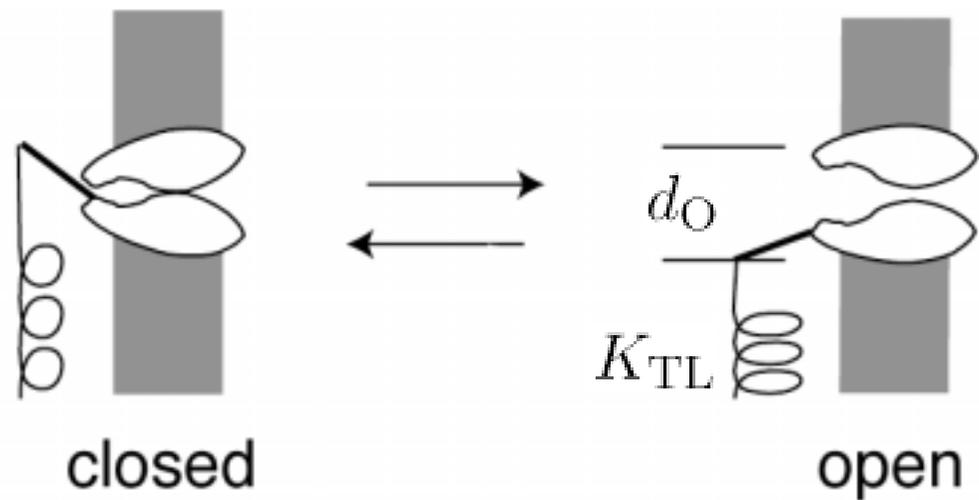
~ 100 ms

Channel gating compliance

Howard & Hudspeth '88; Martin, Mehta & Hudspeth '00

Suppose channel incorporates a **lever arm**

→ opening of channel can substantially reduce the tension in the tip link



- **negative elasticity** if

$$K_{TL}d_0^2 > 4k_B T$$

$$K_{TL} \approx 0.5 \text{ pN/nm}$$

Physical basis of self-tuned critical oscillators

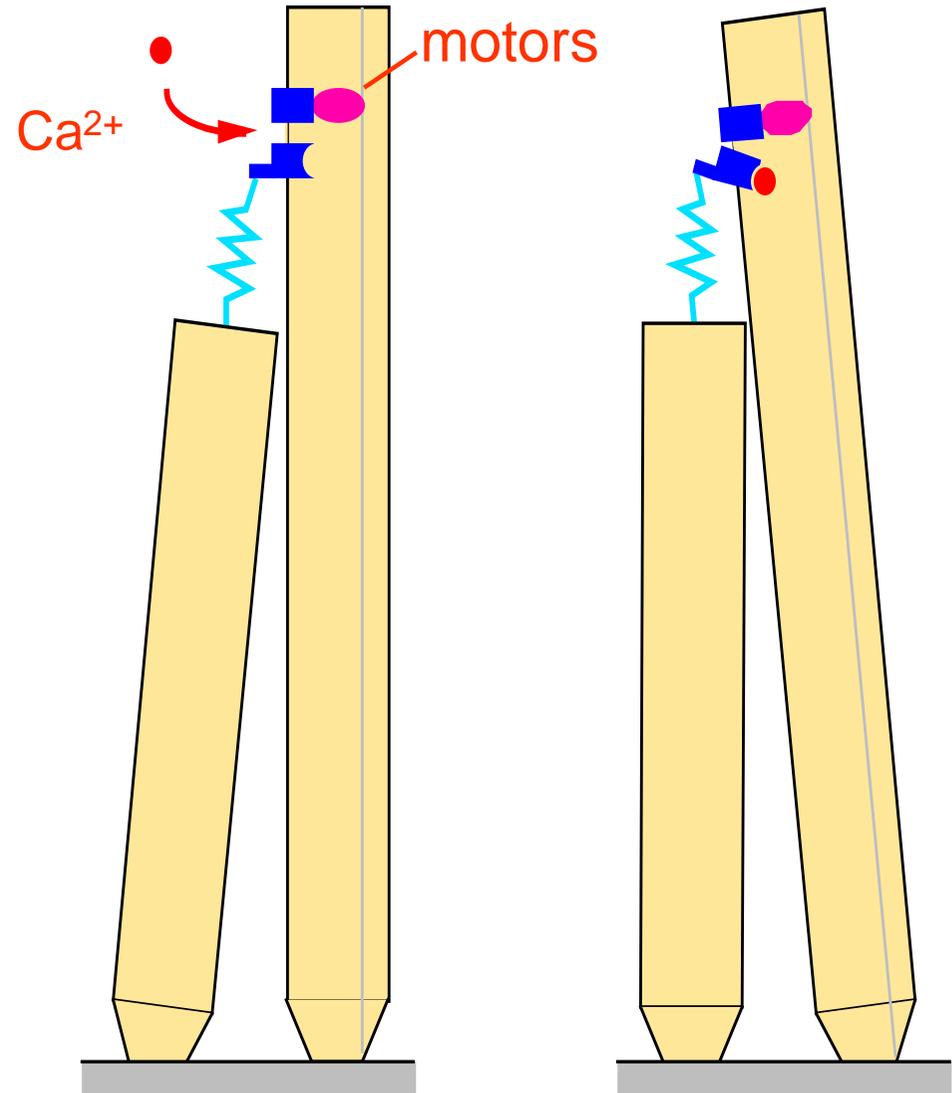
Vilfan & Duke

- Oscillations generated by interaction of Ca^{2+} with transduction channels

frequency $\omega_c \approx \sqrt{\frac{1}{\tau_{mech}\tau_{ca}}}$

depends on bundle geometry

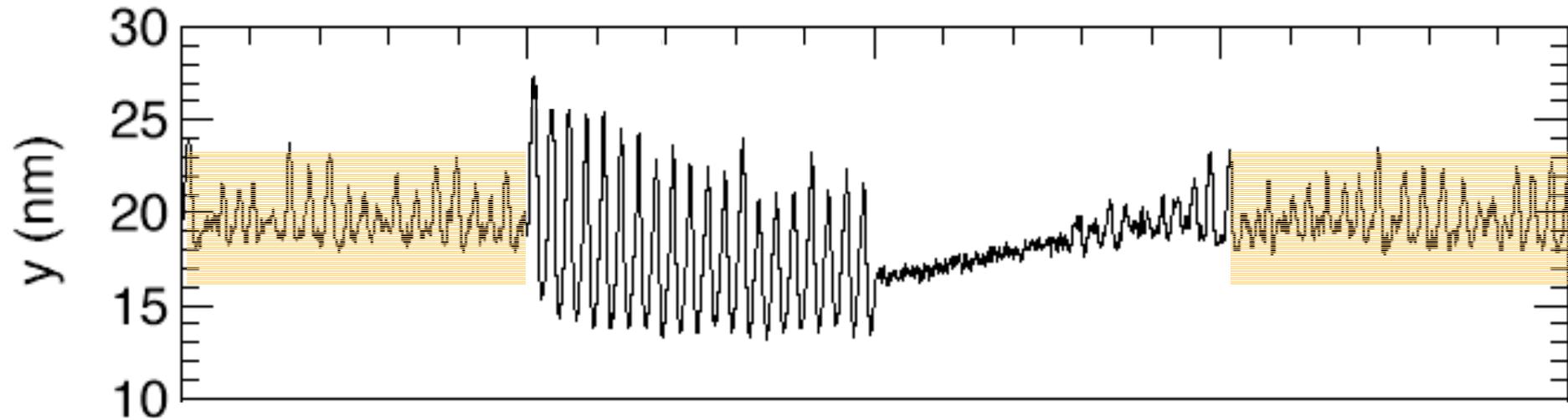
- Self-tuning accomplished by movement of **molecular motors**, regulated by Ca^{2+}



hair bundle model

self-tuned critical oscillations

stimulus



Nonlinearities due to active amplification

Self-tuned Hopf bifurcation is ideal for detecting a single tone ...

... but it causes tones of **different frequency** to **interfere**

Response to two tones:

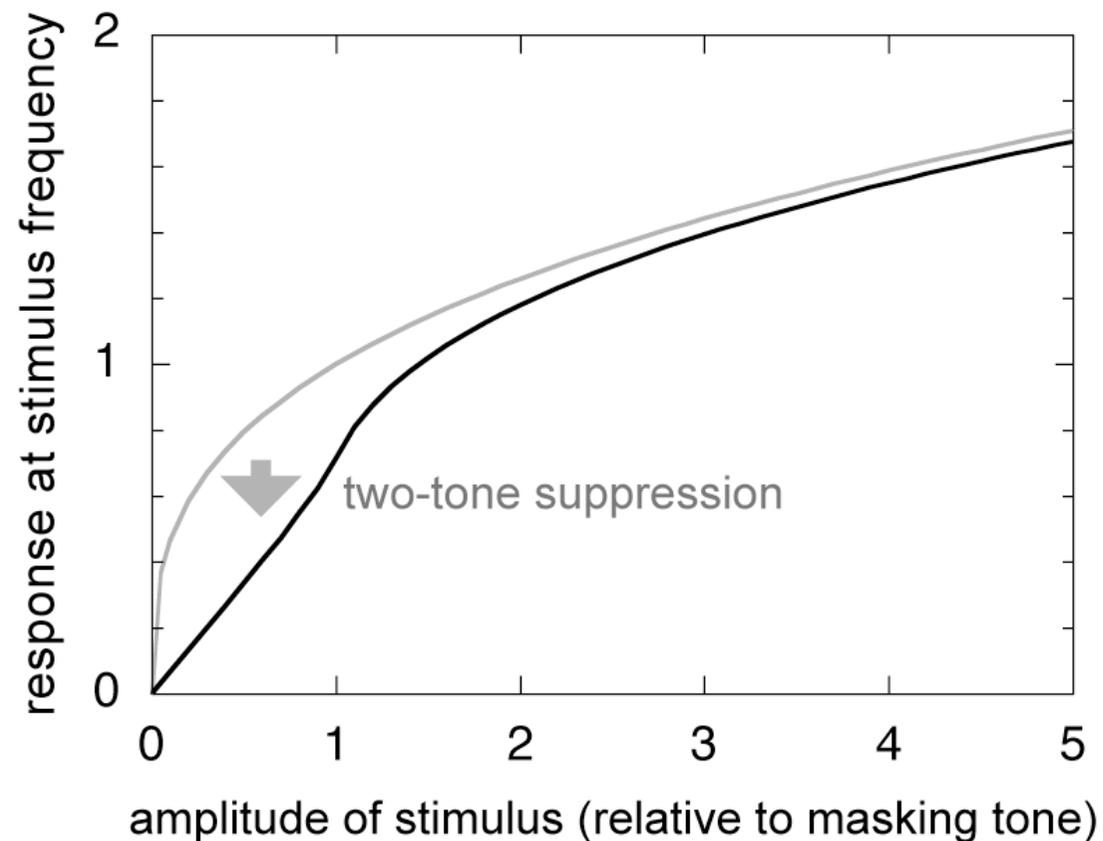
$$F_{f_k} = \mathcal{A}(f_k)X_{f_k} + \sum_{mn} \mathcal{B}(f_k, f_m, f_n)X_{f_k-f_m-f_n}X_{f_m}X_{f_n} + \dots$$

Two-tone suppression

Presence of second tone can extinguish the nonlinear amplification

$$F_{f_1} = A(f_1)X_{f_1} + B(f_1)|X_{f_1}|^2X_{f_1} + \bar{B}(f_1, f_2)|X_{f_2}|^2X_{f_1}$$

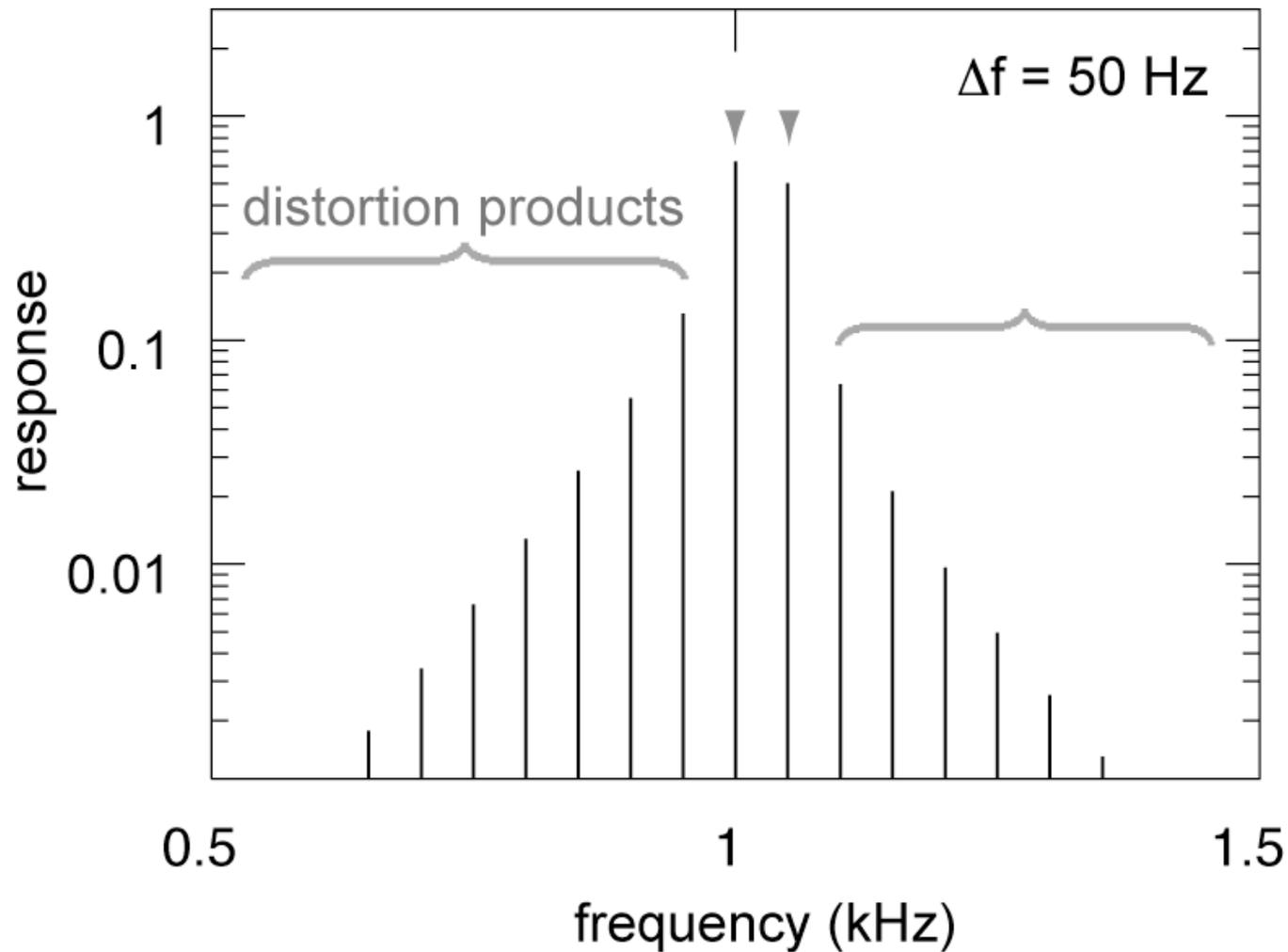
↑ = 0 ↑ 0



Distortion products

Julicher, Andor & Duke '01

Nonlinearities create a characteristic spectrum of distortion products



distortion products analysis

Responses at f_1 and f_2 couple to frequency $2f_1 - f_2$

$$0 = A X_{2f_1 - f_2} + B |X_{2f_1 - f_2}|^2 X_{2f_1 - f_2} + C X_{f_1}^2 X_{f_2}^*$$

... which in turn excites a hierarchy of further distortion products:

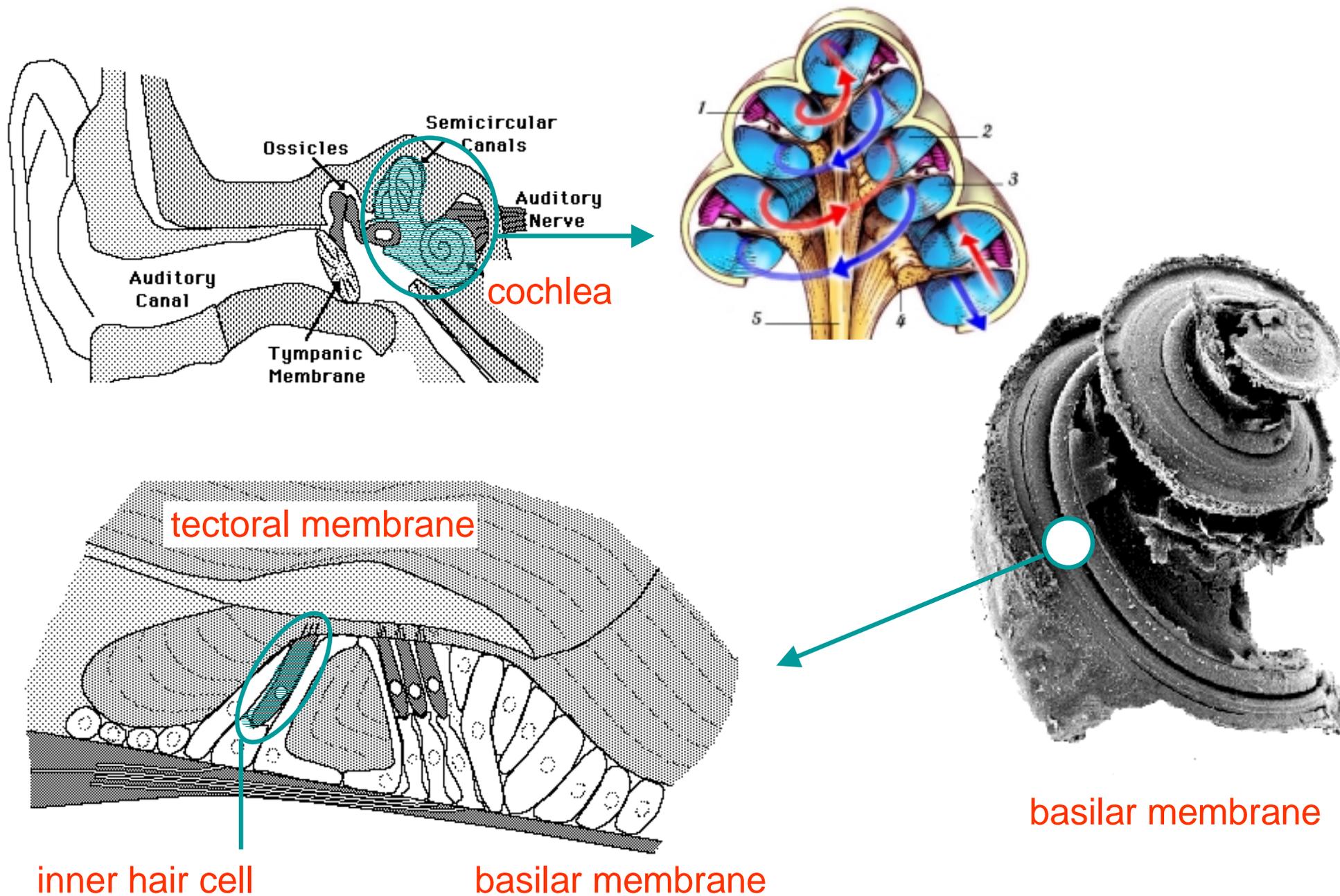
$$f_k = f_1 + (k - 1) \Delta f$$

Spectrum:

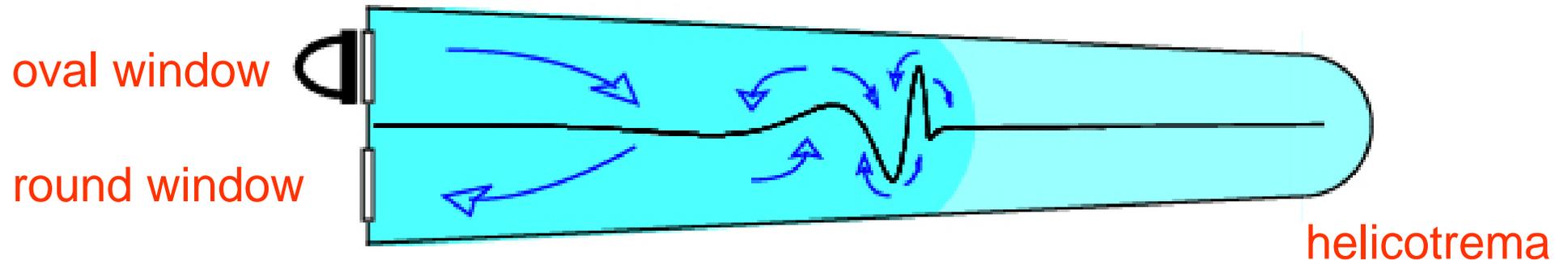
$$|X_{f_k}| \sim \left(\frac{\Delta f}{\Delta f_a} \right)^{-|k - \frac{3}{2}|}, \quad \Delta f \gg \Delta f_a$$

$$|X_{f_k}| \sim \left| k - \frac{3}{2} \right|^{-4/3}, \quad \Delta f \ll \Delta f_a$$

Mammalian cochlea



Cochlear travelling wave

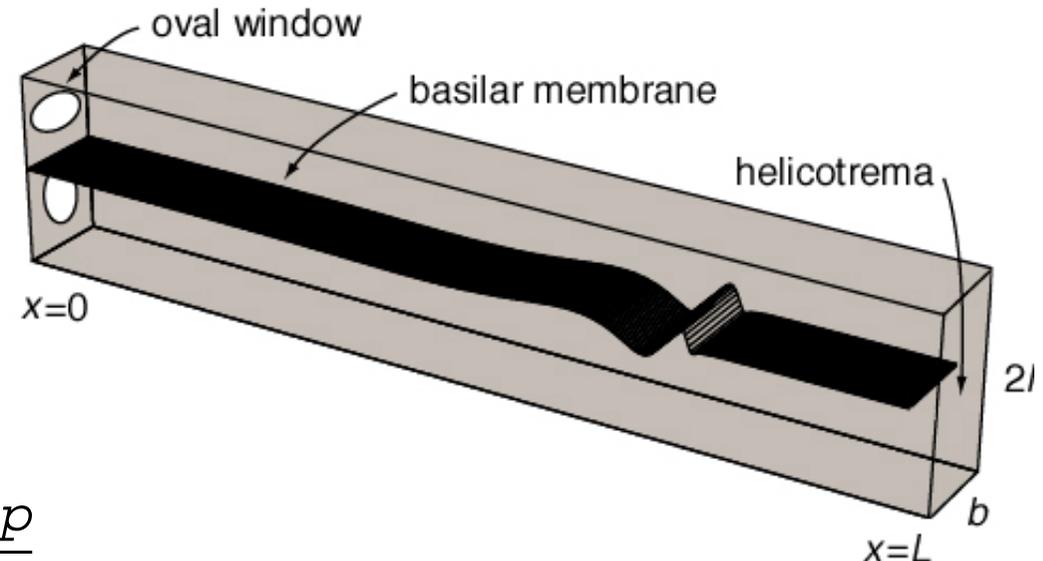


- sound sets fluid into motion
- variation in flow rate is accommodated by movement of membrane
- membrane acceleration is caused by difference in fluid pressure

travelling wave one-dimensional model

Zwislocki '48

membrane displacement h
 pressure difference $p = P_1 - P_2$
 difference in flows $j = J_1 - J_2$



- fluid flow

$$\rho \frac{\partial j}{\partial t} = -b l \frac{\partial p}{\partial x}$$

- incompressibility

$$2b \frac{\partial h}{\partial t} - \frac{\partial j}{\partial x} = 0$$

- membrane response

$$p(x, t) = K(x) h(x, t)$$

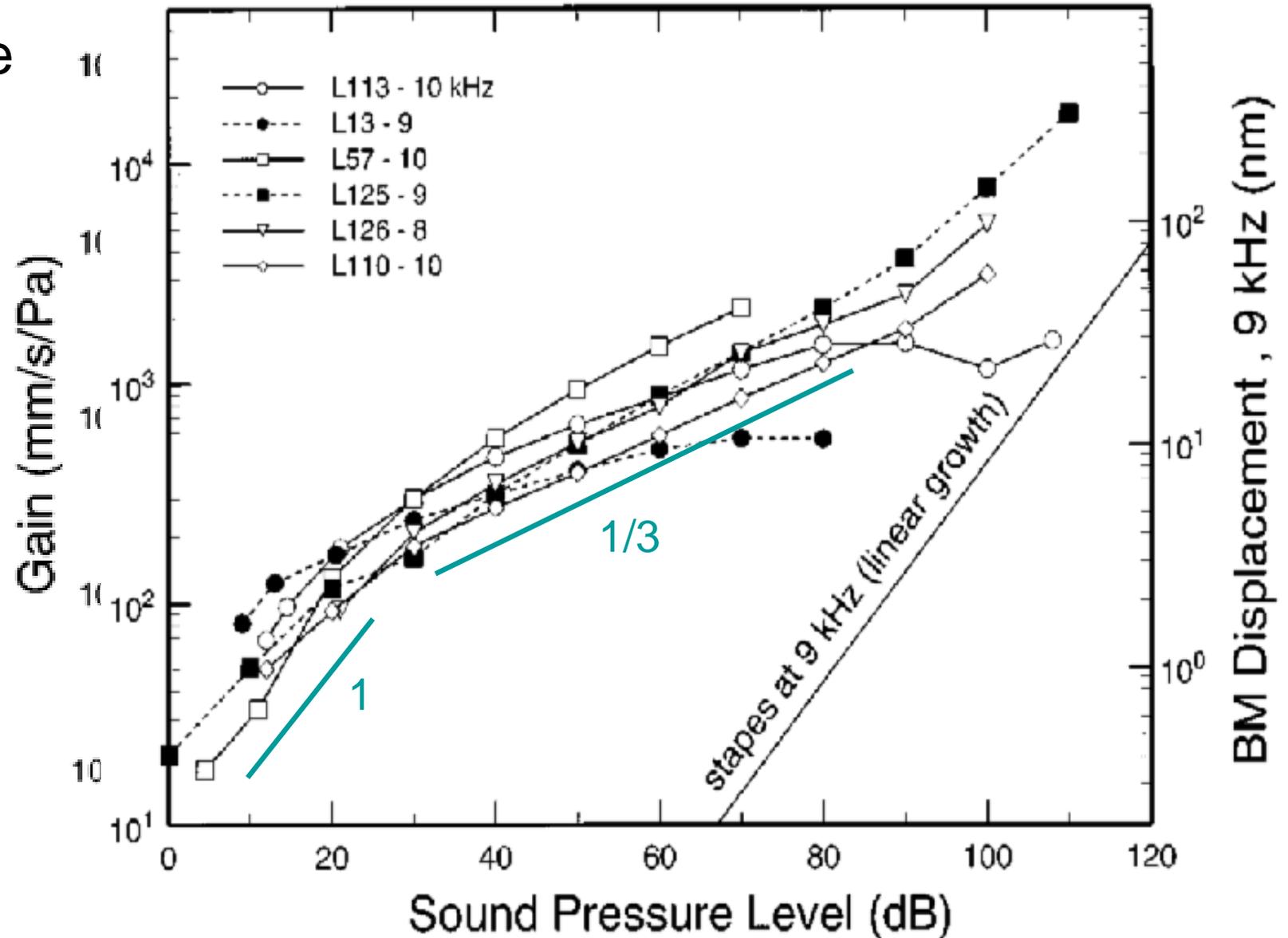
→ wave velocity

$$c(x) = \sqrt{\frac{K(x)l}{2\rho}}$$

Basilar membrane motion

Rhode '71; Ruggero et al. '97

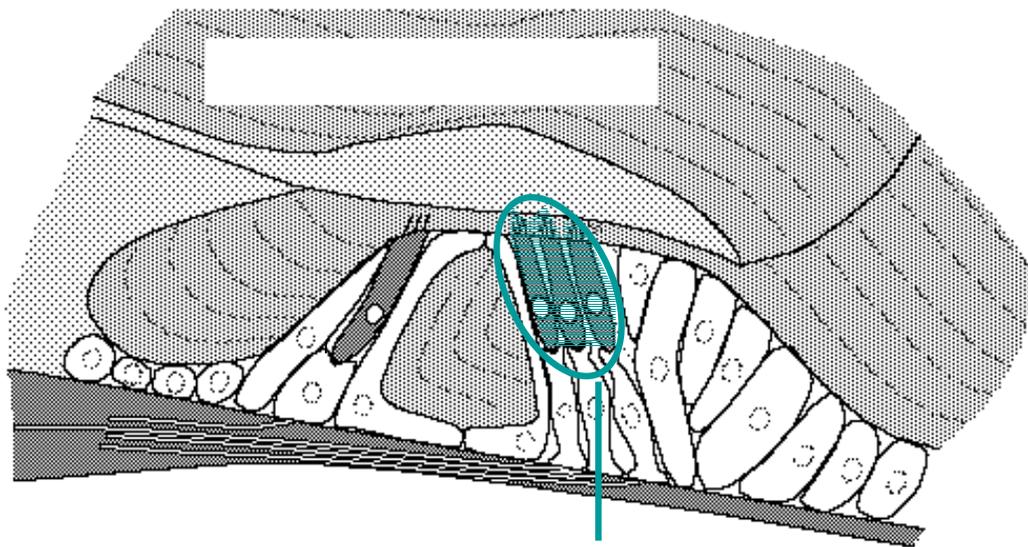
BM response
is **nonlinear**



Outer hair cell motor

Brownell '85; Ashmore '87

Outer hair cells are **electromotile**



outer hair cells



prestin

Dallos et al. '99



Active basilar membrane

Duke & Jülicher

Critical oscillators ranged along basilar membrane

characteristic frequencies span audible range:

$$\omega_c(x) = \omega_0 e^{-x/d}$$

→ membrane is an excitable medium with a **nonlinear active response**

$$\bar{p}(\omega) = A(x, \omega) \bar{h} + B |\bar{h}|^2 \bar{h}$$

$$A(x, \omega) = \alpha (\omega_c(x) - \omega)$$

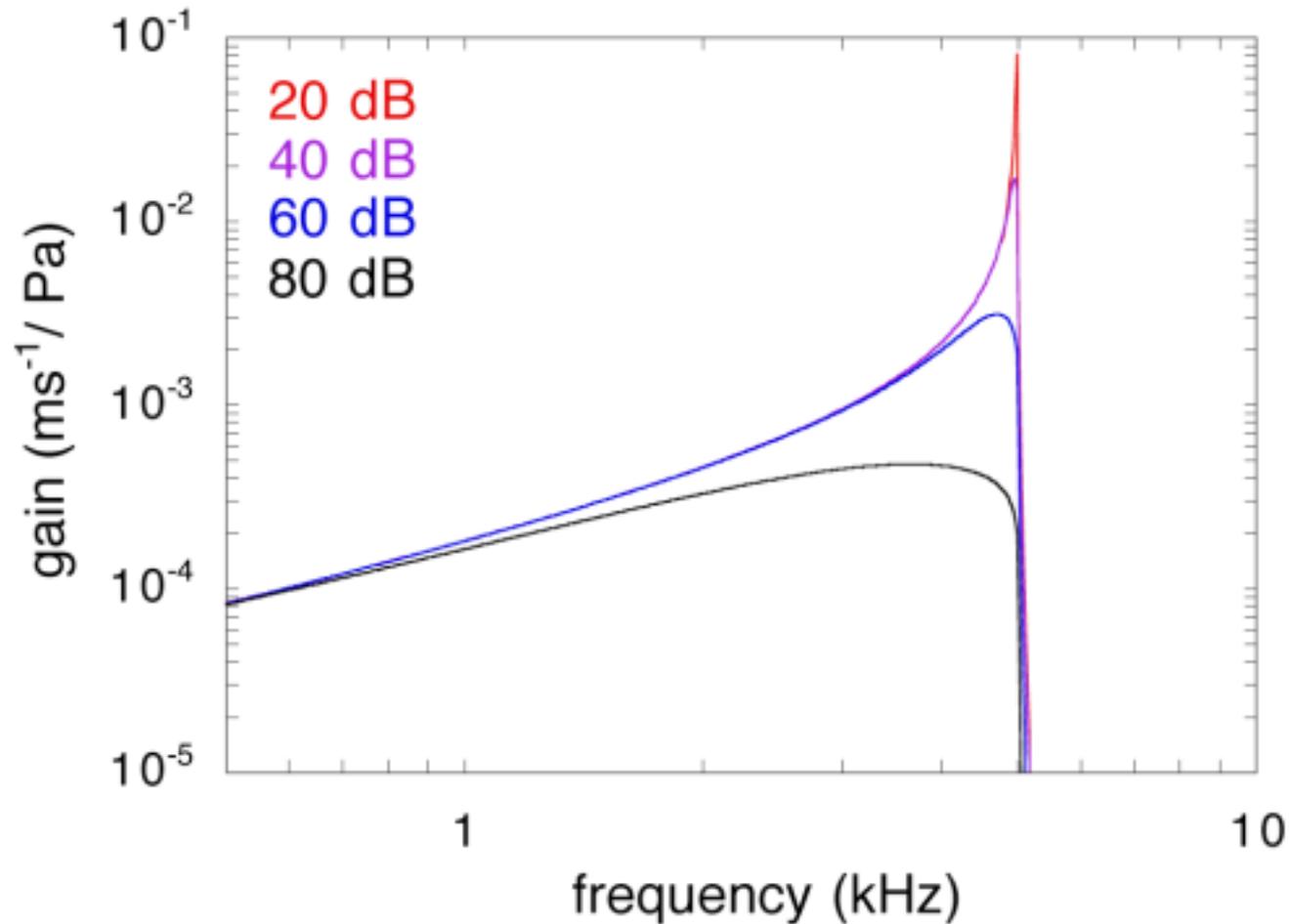
$$B = i\beta$$

captures essence of active membrane

$$A(x, \omega_c(x)) = 0$$

$$K(x) = A(x, 0) = \alpha \omega_c(x)$$

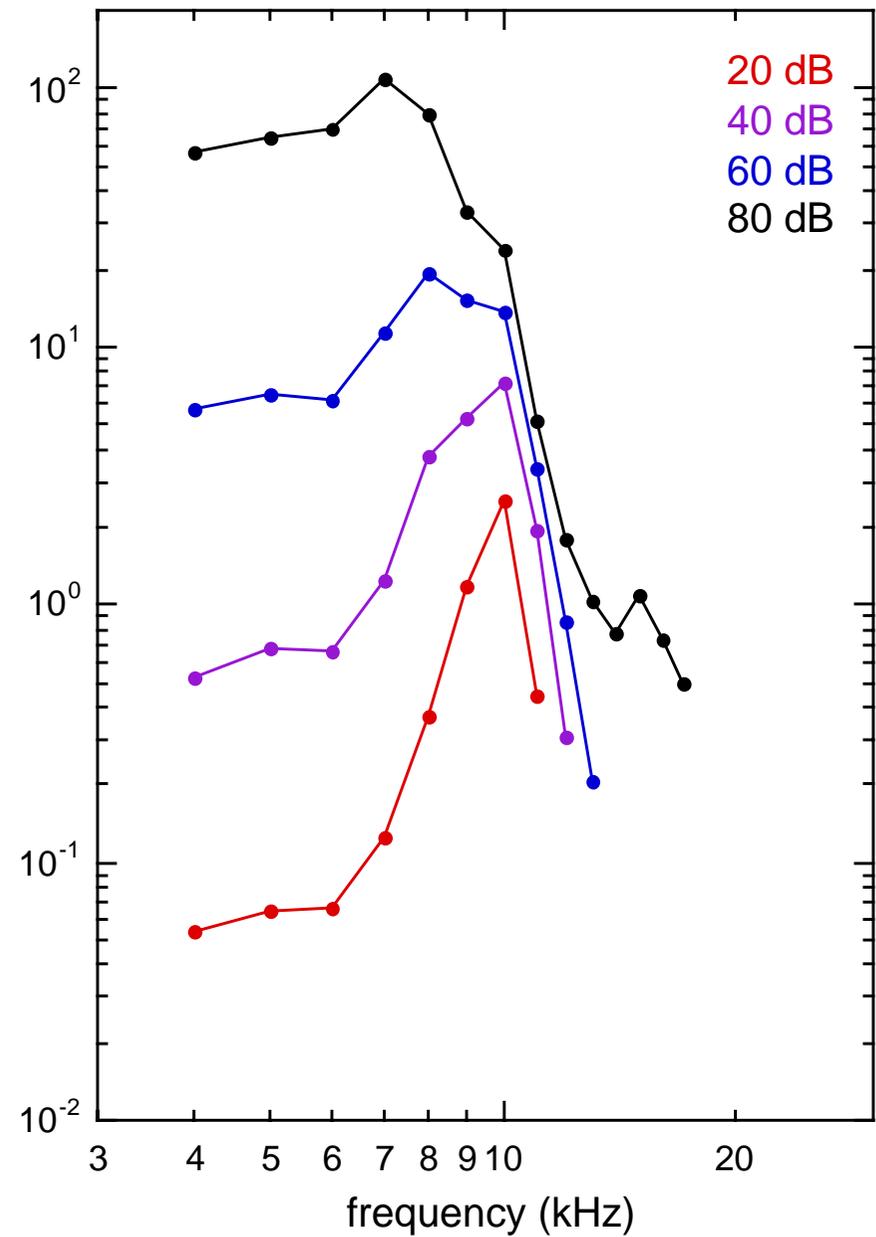
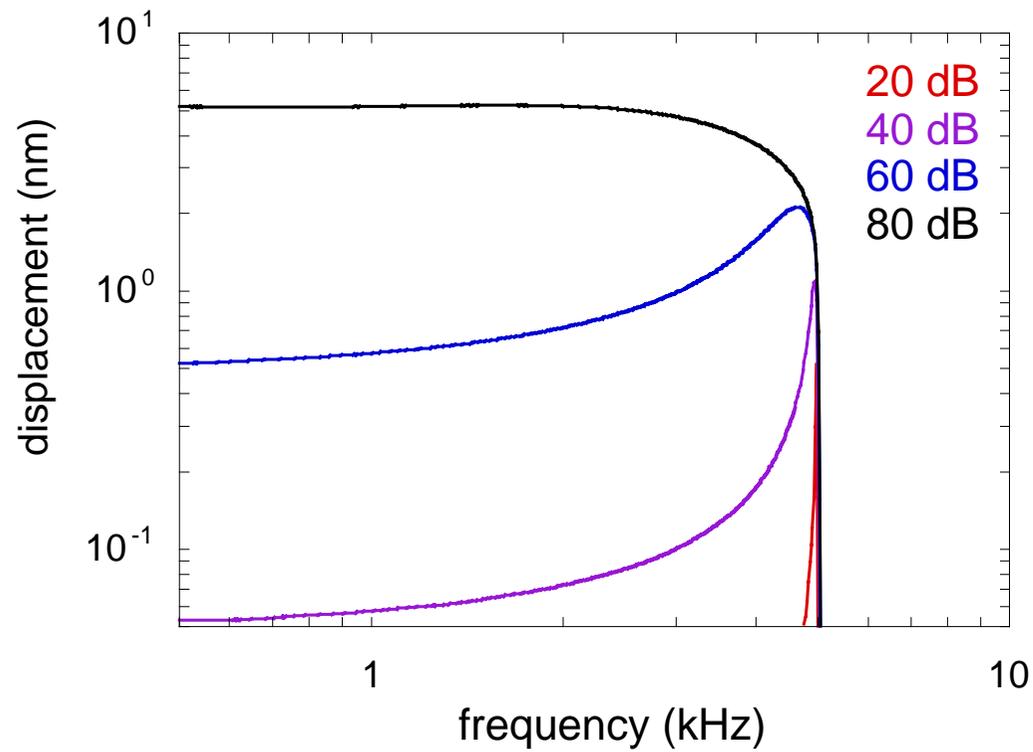
active travelling wave cochlear tuning curve



Precipitous fall-off on high frequency side owing to **critical-layer absorption**

active travelling wave cochlear tuning curve

Ruggero et al. '97



Summary

- Active amplification by critical oscillators is ideally suited to the ears needs:
frequency selectivity, exquisite sensitivity, dynamic range
- Spontaneous hair-bundle oscillations may be generated by transduction channels and regulated by molecular motors
- Critical oscillators that pump the basilar membrane give rise to an active travelling wave with a sharp peak
- Many psychoacoustic phenomena may be related to the nonlinearities caused by active amplification