Fracture and fragmentation of disordered solids

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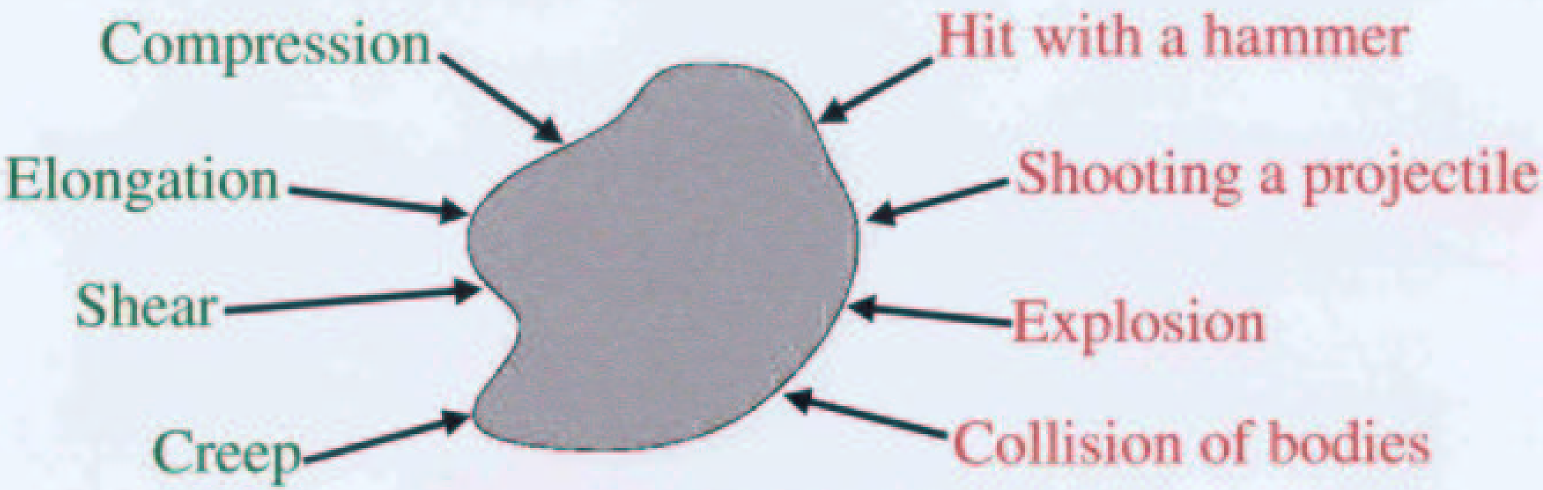
Loading of a disordered solid

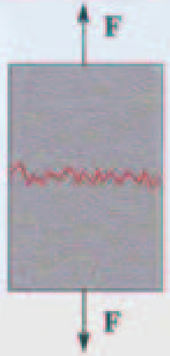
Quasistatic loading **Dynamic loading**



Disorder

- ➔ structural
- ➔ physical

Fracture



- ➔ Quasistatic process
- ➔ Stress localization
- ➔ One main crack
- ➔ Breaking into two pieces

Fragmentation

- ➔ Dynamic process
- ➔ Shock waves
- ➔ Many cracks
- ➔ Breaking into many pieces

  Fragments: $m_i \ll M_o$

Schedule

➔ Fragmentation of brittle solids (rocks, ceramics, concrete)

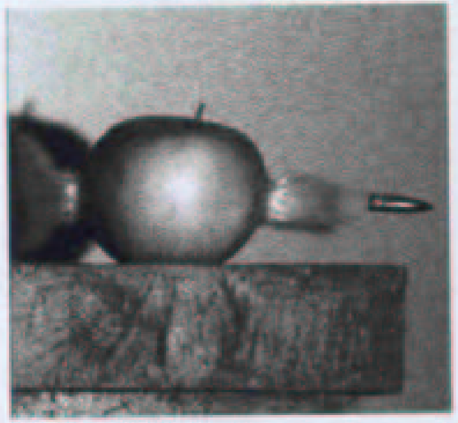
- Experiments
- Discrete element model
- Computer simulations

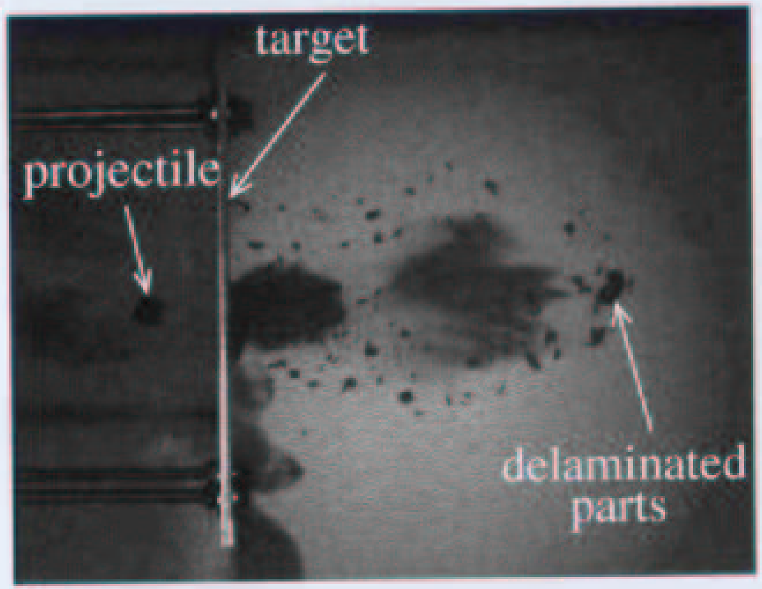
➔ Creep rupture of fiber reinforced composites

- Experiments
- Fiber bundle models
- Computer simulations

Shooting a projectile into a solid

High speed cameras



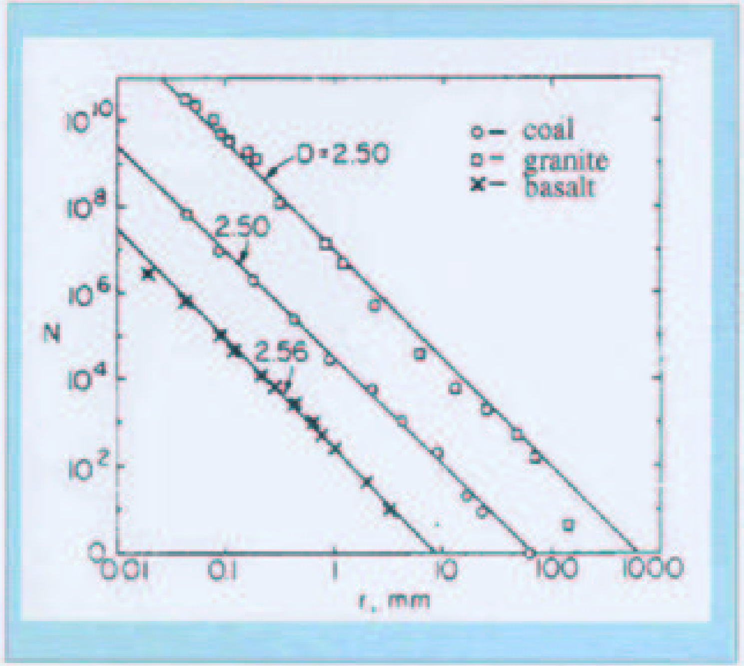


Measurables:

- ➔ Fragment size
- ➔ Fragment velocity

Experimental results

Different materials
–different way of fragmentation



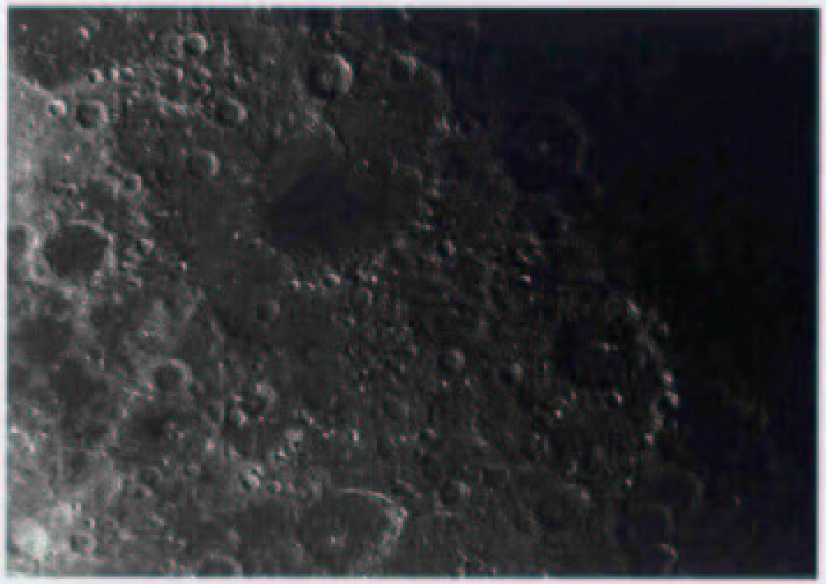
Power law size distribution of fragments independent of

- ➔ **Material**
- ➔ **Way of imparting energy**
- ➔ **Length scale**

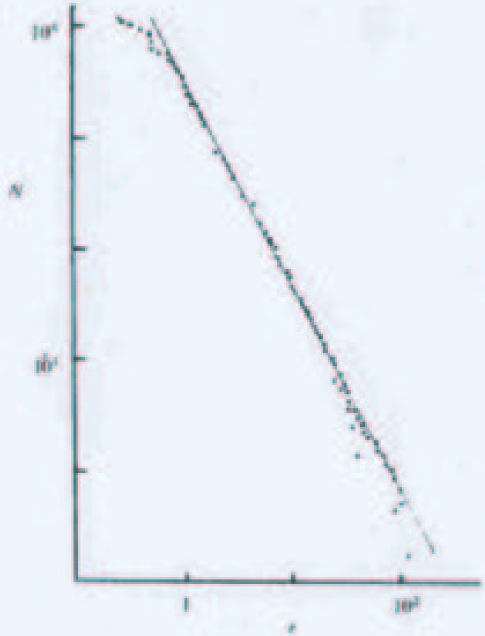
Universality

Craters of the Moon

Craters of various sizes



Size distribution



$\Rightarrow$ **Power law size distribution of asteroids**

Theoretical treatment

Disorder + Many interacting cracks

↓

Discrete element method

↓

Computer simulations

Monte Carlo & Molecular dynamics

Model construction

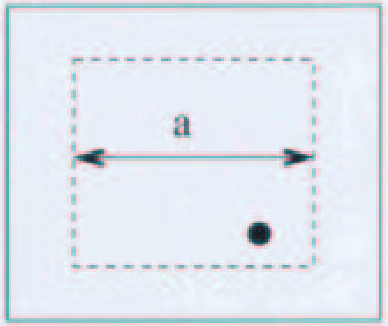
Two dimensional model of deformable, breakable granular solids

- **Granularity:**
  - –randomly shaped convex polygons model grains
  - –polygons can overlap each other
- **Elastic behavior:**
  - –restoring force between overlapping polygons
  - –polygons are connected by elastic beams
- **Breaking of the solid:**
  - –breaking criterion for beams

Granular structure: Voronoi construction

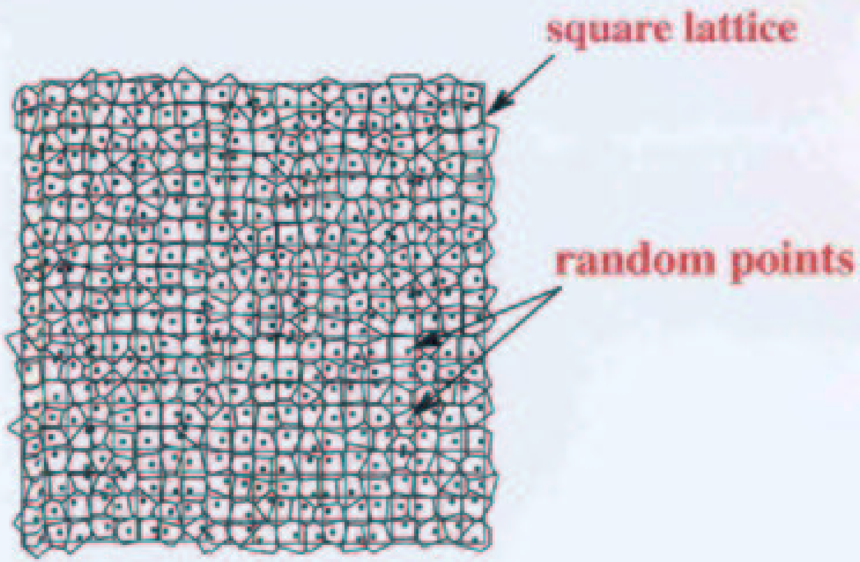
Voronoi tessellation with controllable disorder

Single cell



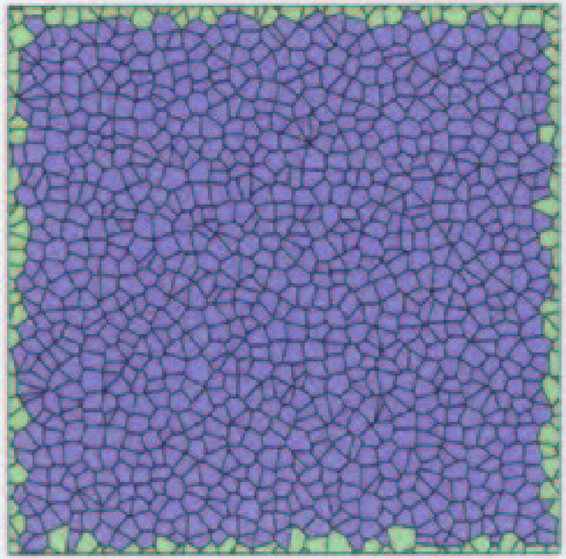
Parameter $0<a<1$ controls the amount of disorder

Tessellation of a 20x20 sample

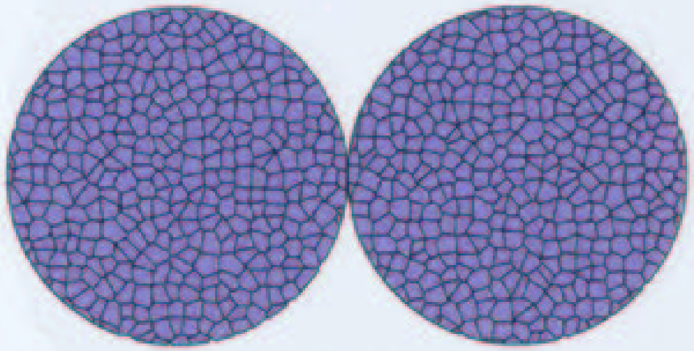


Cutting out various shapes

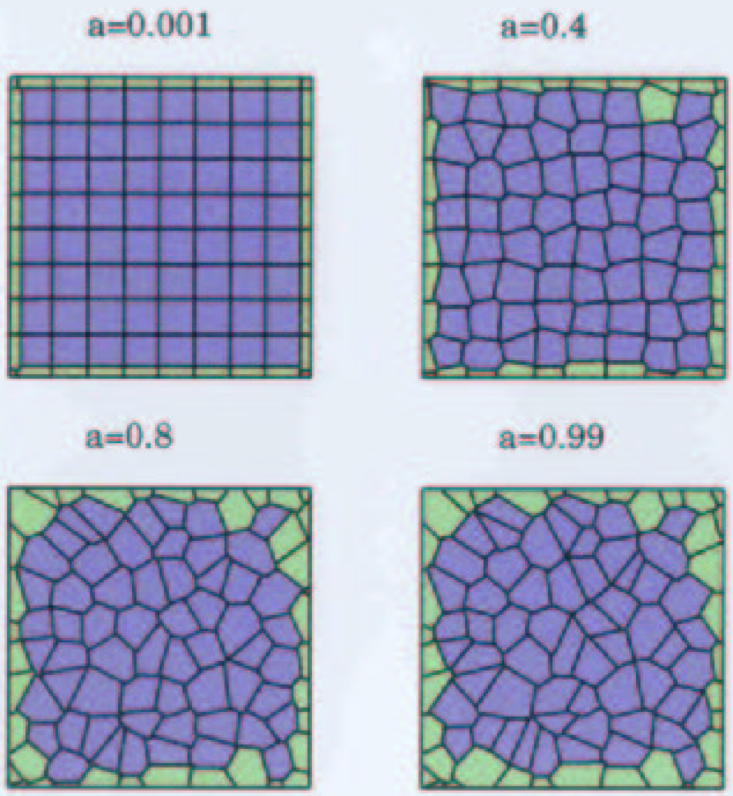
Square with a smooth surface



Circles cut out of a square



Controlling the amount of disorder



a=0 ⇒ Square lattice

anisotropic

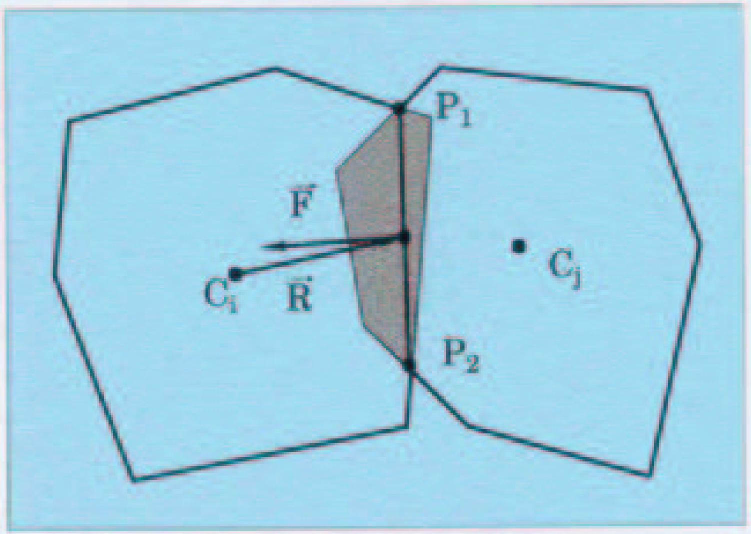
Increasing disorder

a=1 ⇒ Random lattice

isotropic

Contact force

Overlapping particles



Overlap represents local deformation of contacting particles

Contact force

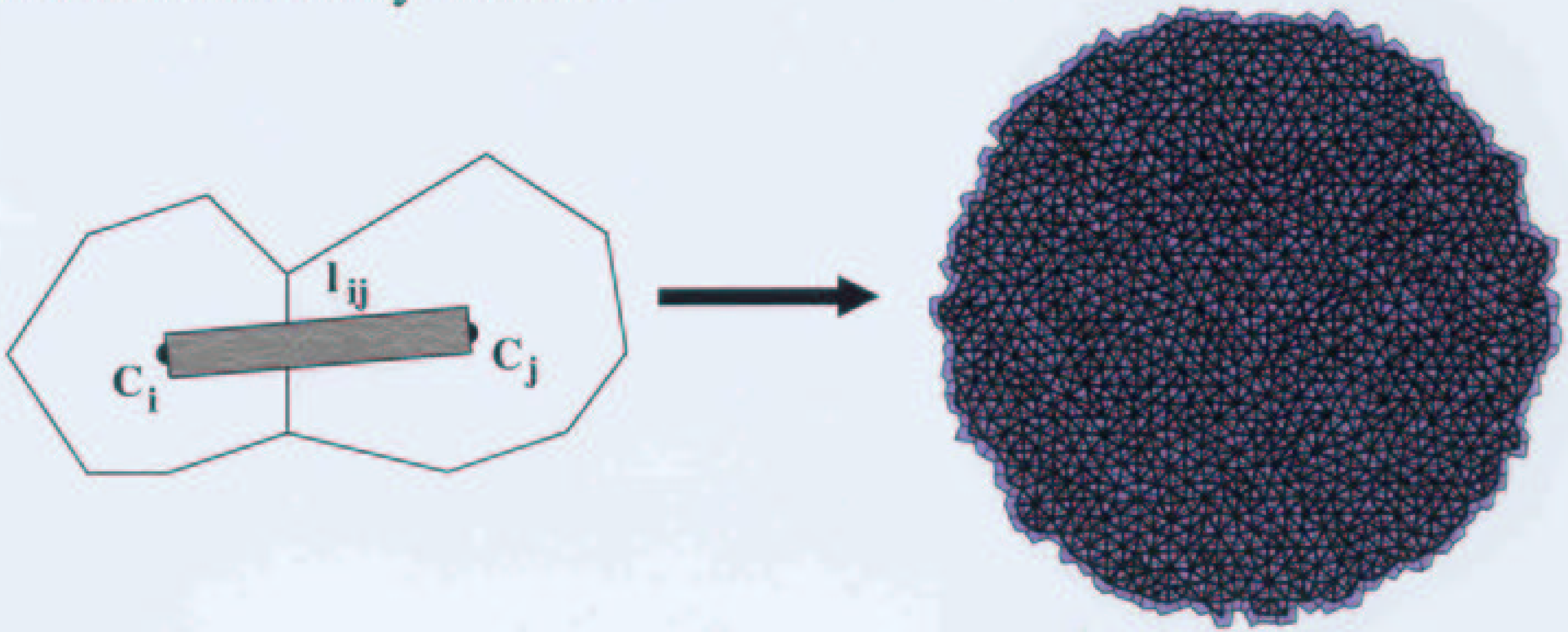
$$\vec{F}_{ij} = -\frac{Y\,A}{L_c}\vec{n}$$

A : overlap area
Y : Young modulus
L_c : characteristic length

Beams attached to the polygons

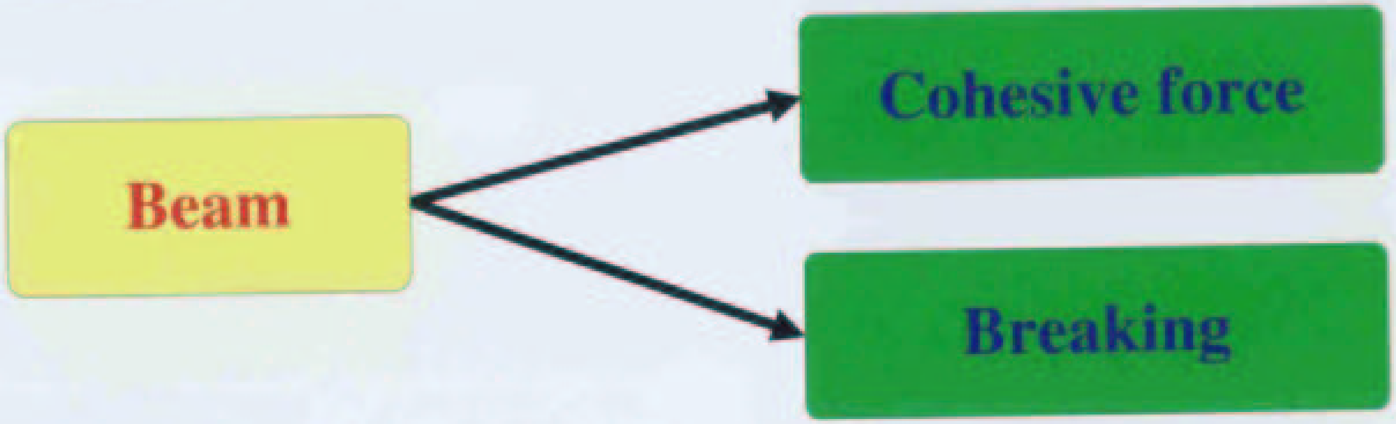
Center of mass of polygons are connected by beams

Polygon–beam system

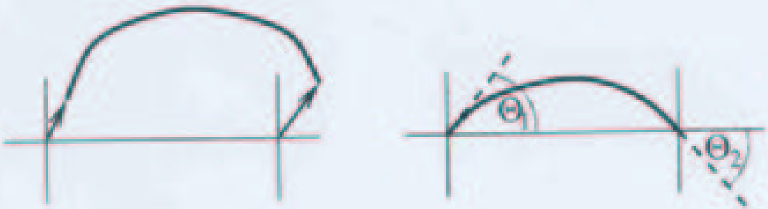


Random lattice of beams

The beam model



Deformation of a beam



Elongation, compression, shear, bending

Beam equations

$$
\begin{aligned}
F_x^1 &= A(x_1 - x_2) \\
F_y^1 &= B(y_1 - y_2) + \frac{Bl}{2}(\Theta_1 + \Theta_2) \\
M_z^1 &= \frac{Bl}{2}(y_1 - y_2 + l\Theta_2) + Dl^2(\Theta_1 - \Theta_2)
\end{aligned}
$$

A, B, D: material dependent constants

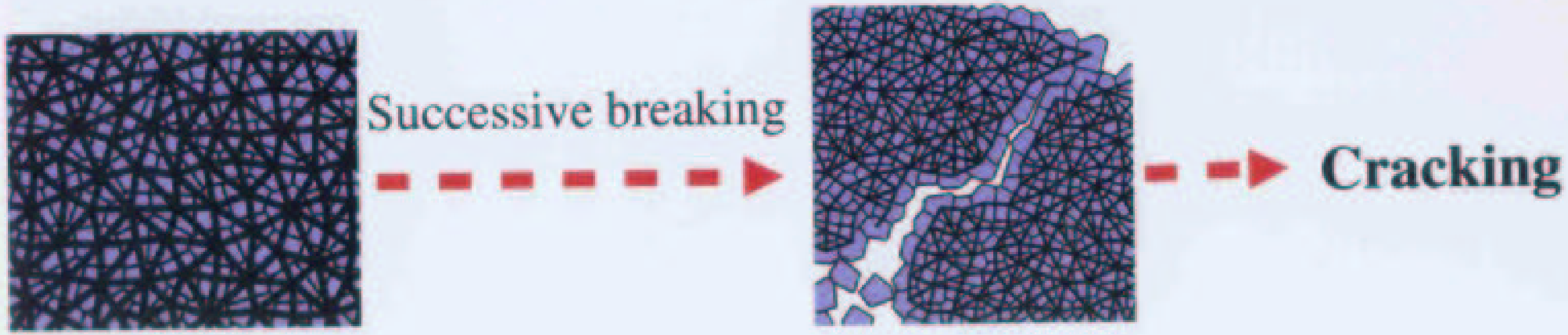
Beam breaking → crack formation

Overstressed beams break

Breaking criterion:

$$\left(\frac{\epsilon}{t_\epsilon}\right)^2 + \frac{max(|\Theta_1|, |\Theta_2|)}{t_\Theta} \geq 1$$

t_ε, t_θ: breaking thresholds



Molecular dynamic simulations

Equation of motion of polygons in two dimensions

$$m_i \ddot{\vec{r}}_i = \sum_j \vec{F}_{ij}$$

$$I_i \ddot{\Theta}_i = \sum_j M^z_{ij}$$

Numerical solution:

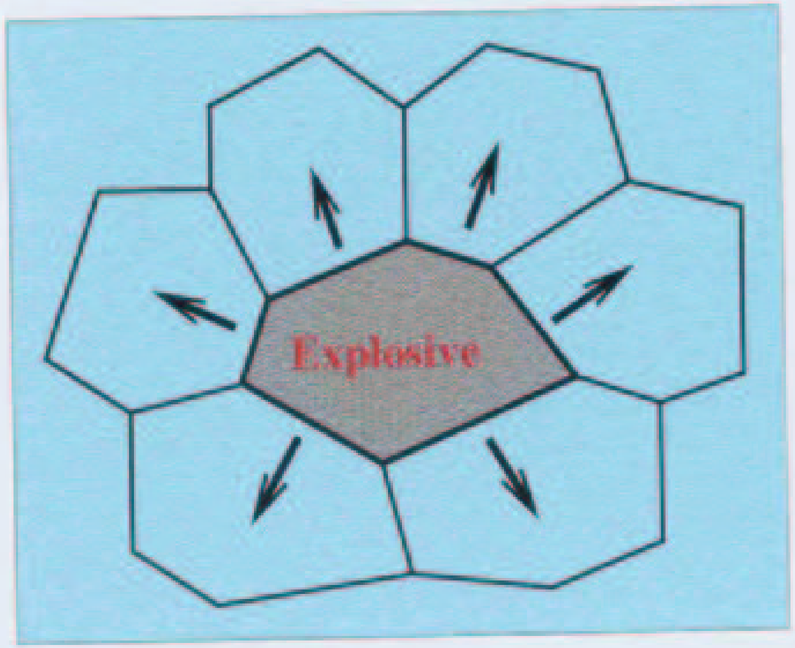
5^{th} order Predictor–Corrector scheme

Simulations:

- Initial condition
- Boundary condition
- Stopping condition

Explosion of a disc–shaped solid

Initial condition



Initial velocities

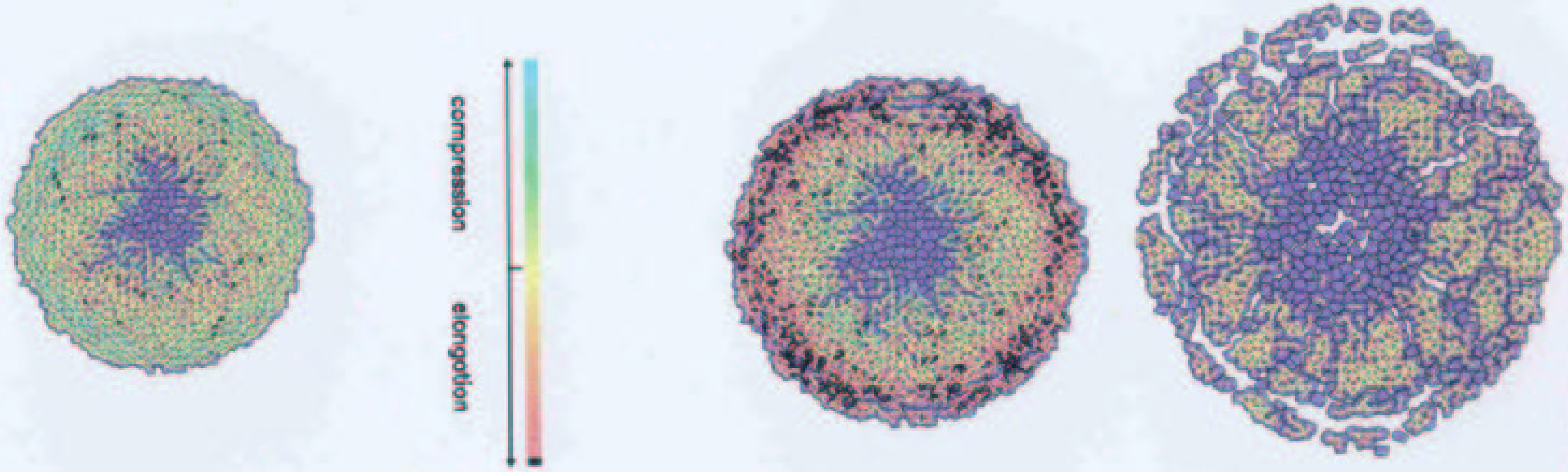
Circular symmetry

$$\sum_i \vec{p}_i = 0$$

Outgoing compression wave

Time evolution of explosion

Snapshots of the process

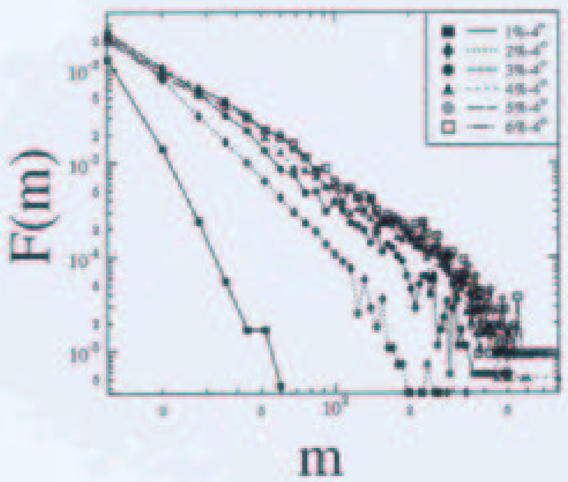


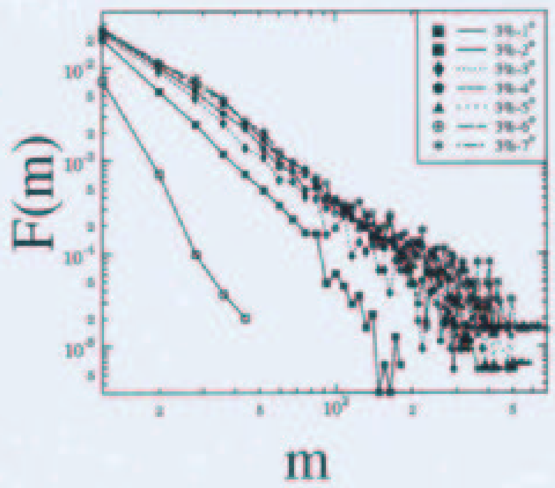
Uncorrelated crack nucleation

Crack growth and coalescence

Fragment size distribution

Varying the breaking thresholds t_ε and t_θ





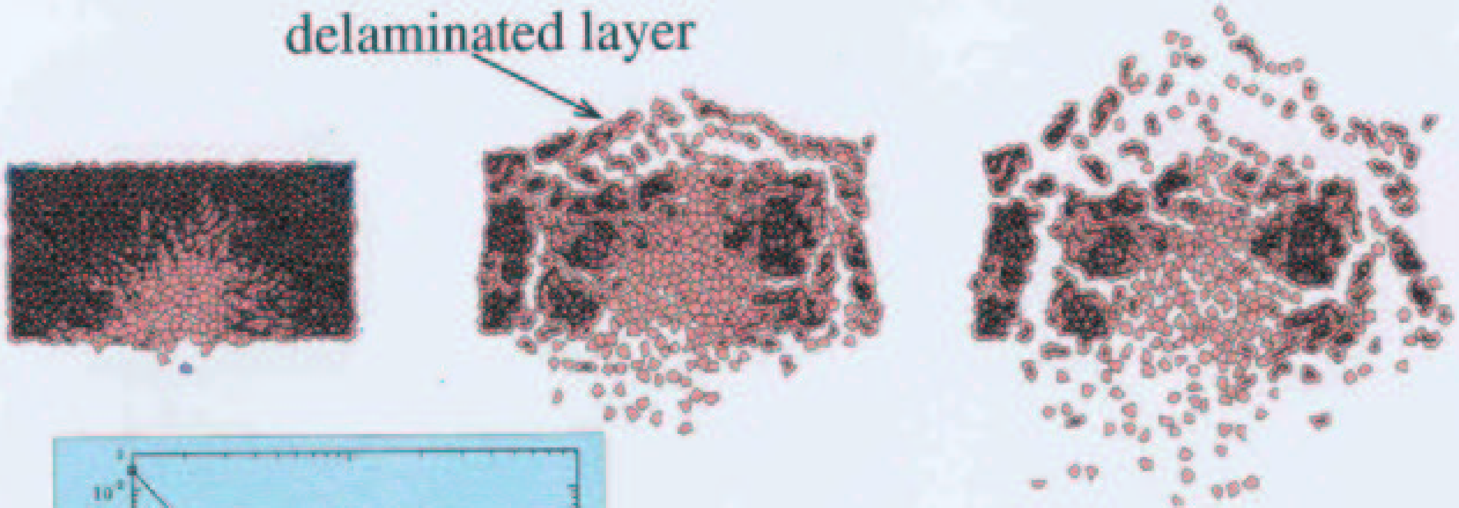
Power law size distribution

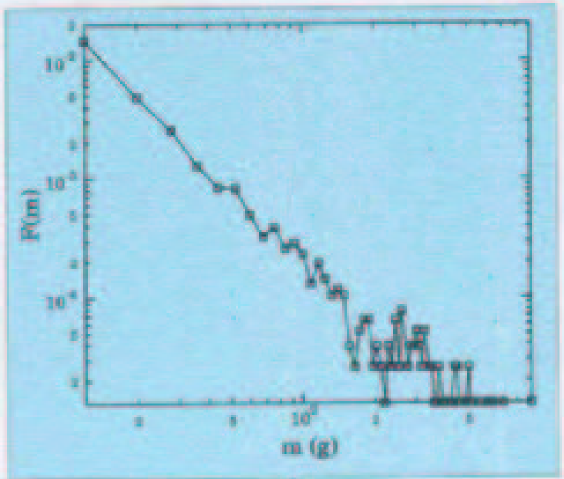
$$F(m) \sim m^{-\tau}$$

$\tau = 2.0–2.5$

Projectile shooting

Time evolution of the process





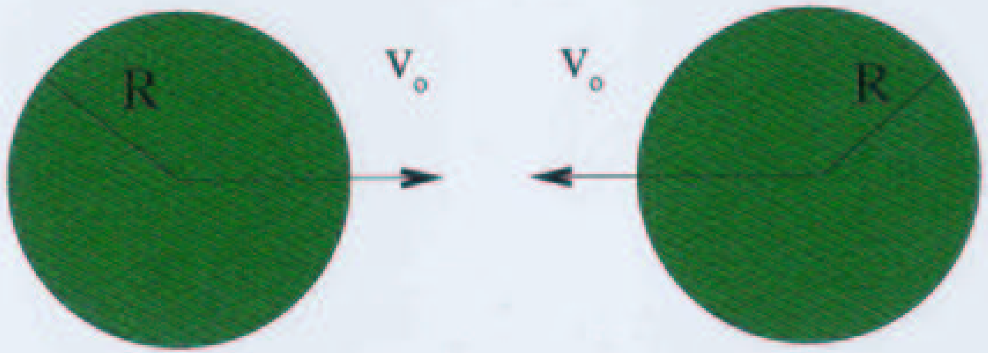
Fragment size distribution

$$F(m) \sim m^{-\tau}$$

Collision of solids

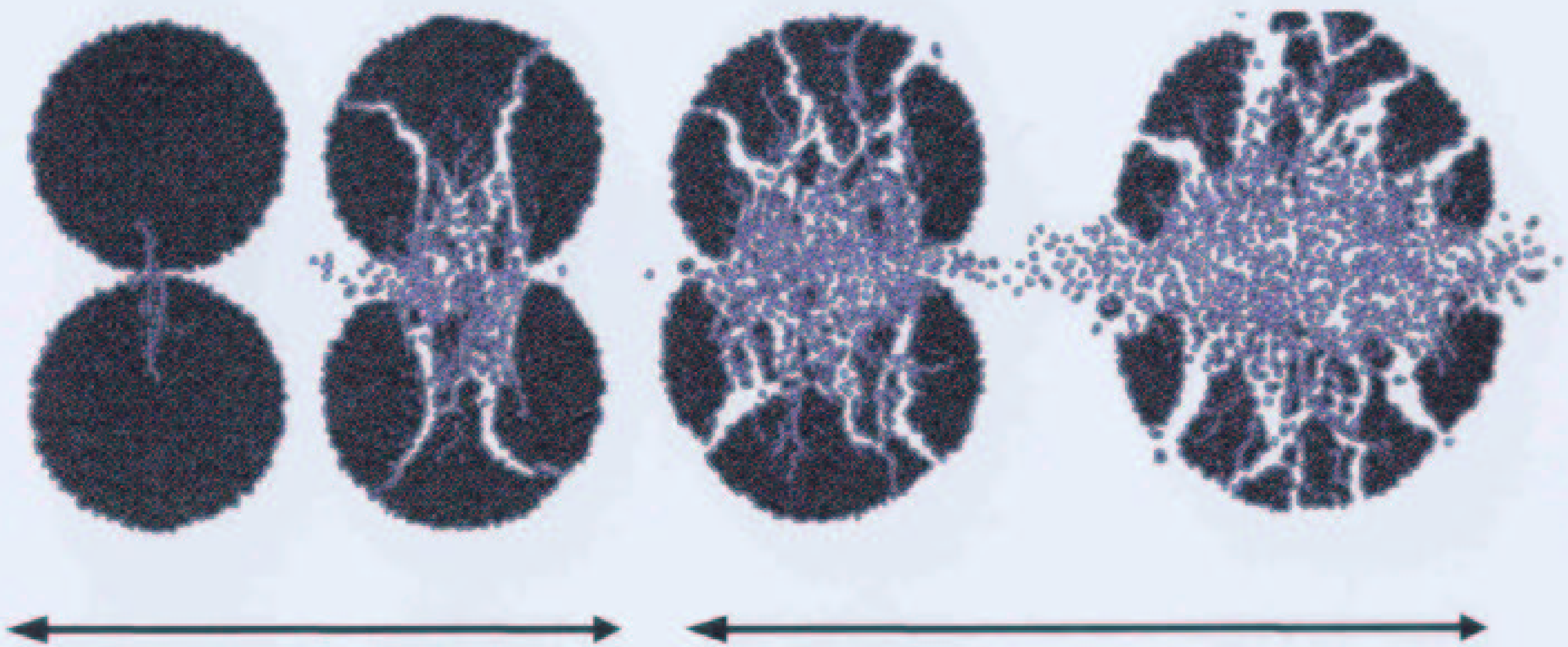
- ➔ Collision of particles in a granular flow
- ➔ Planetary rings
- ➔ Collisional evolution of asteroids, space debris

Initial condition



- Only central collisions
- Varying the initial velocity

Damage ⟶ Fragmentation

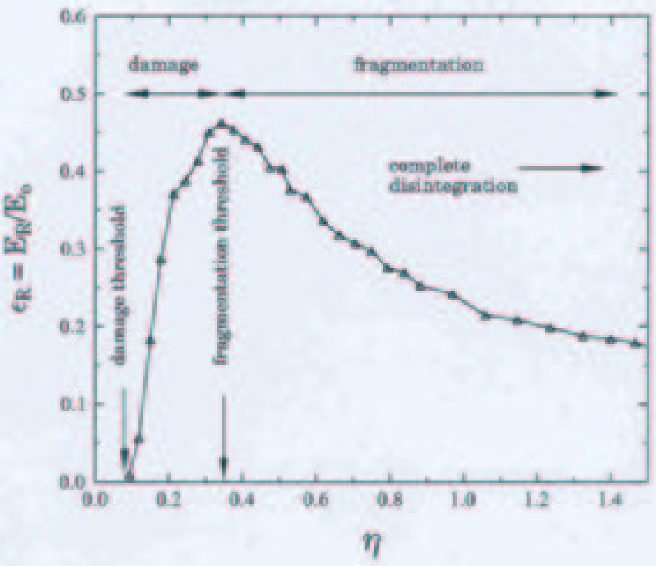


$M_{max} \sim M_o$ | $M_{max} << M_o$

Damage | **Fragmentation**

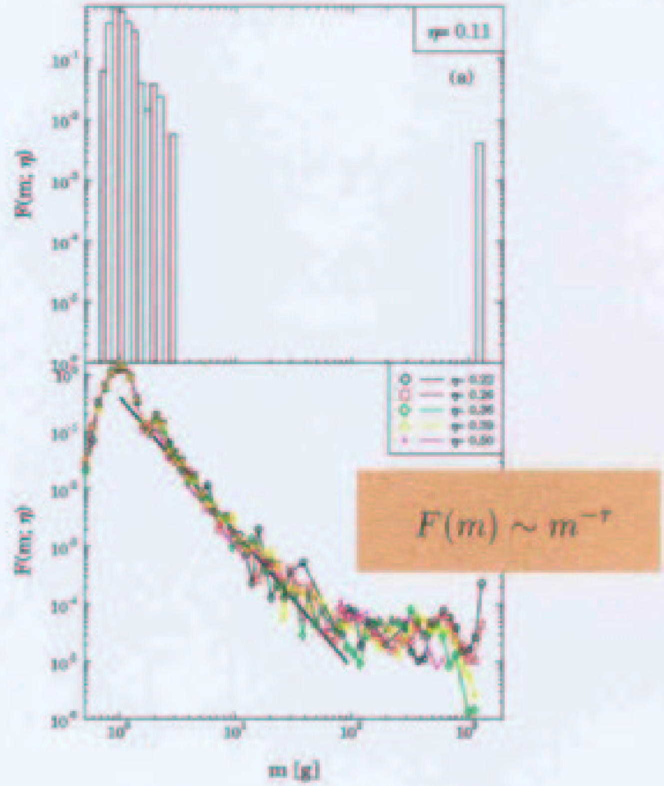
Damage ⟶ Fragmentation

Energy release



$$\eta = \frac{E_o}{E_b}$$

Mass distribution



Damage - - ▸ Critical point - - ▸ Fragmentation

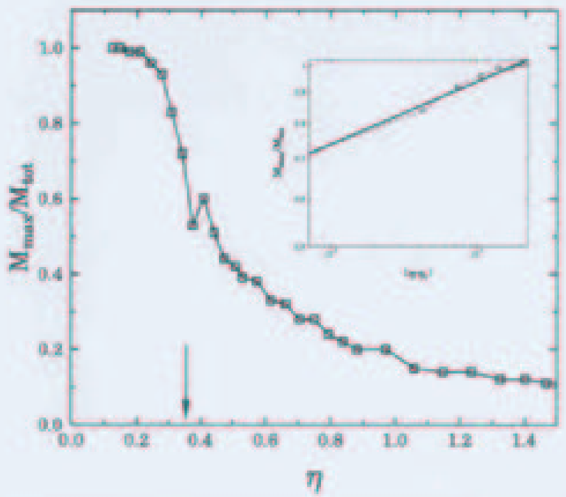
Continuous phase transition

Control parameter: **energy of collision**

Order parameter: **mass of the largest fragment**

Order parameter exponent β

$$\frac{M_{max}}{M_o} \sim |\eta - \eta_c|^{\beta}, \qquad \eta < \eta_c$$



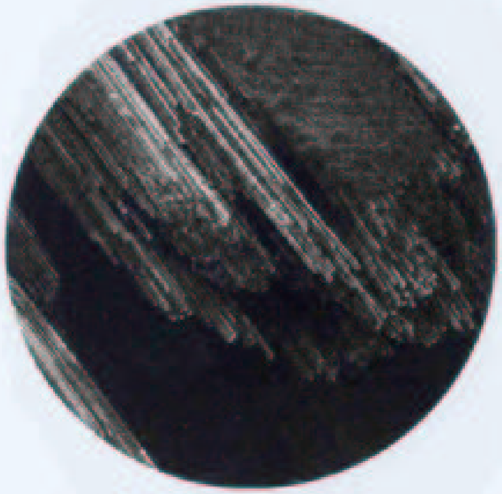
Gap scaling

$$F(m) \sim m^{-\tau} f(m^{\sigma}\epsilon)$$

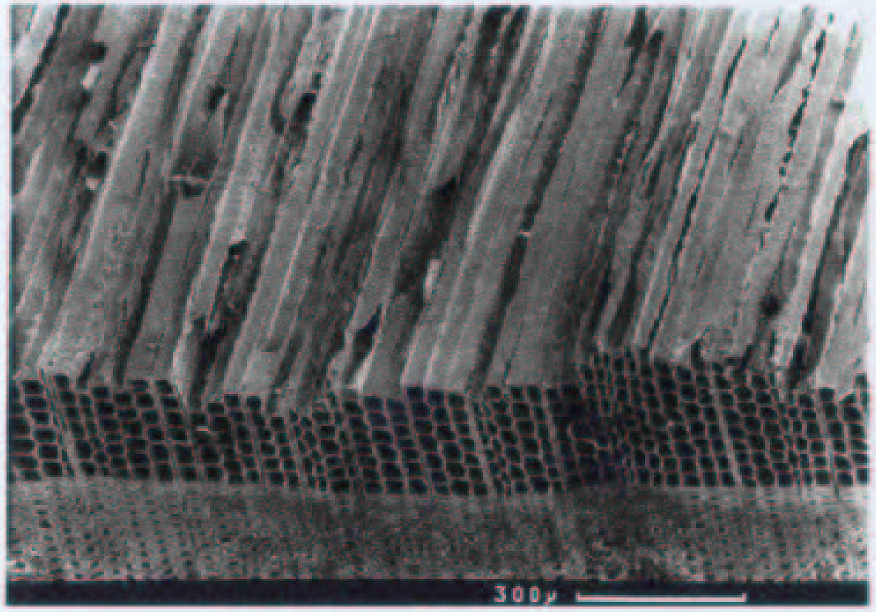
$$\tau = 2.27 \pm 0.05$$
$$\beta = 0.11 \pm 0.02$$

Fiber reinforced composites

C–SiC



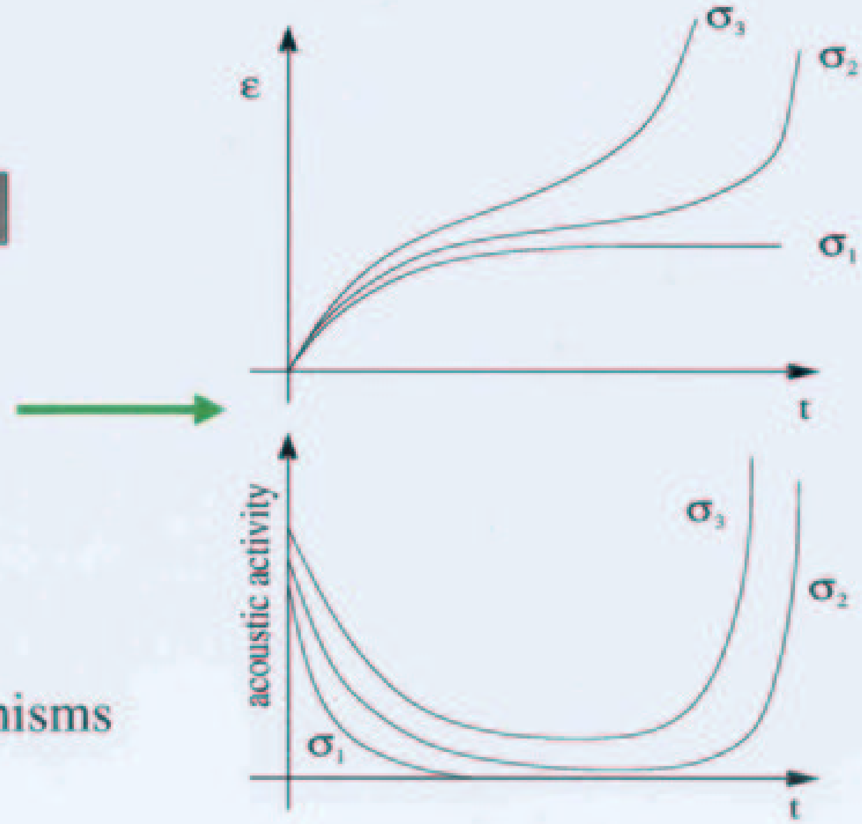
Wood



Two components: fiber + matrix

Creep rupture

Creep experiment



– Several possible mechanisms
– Material dependence

Main Goals

Models

- Analytical and numerical
- Detailed description of microstructure and stress redistribution
- Damage histories in terms of microscopic parameters

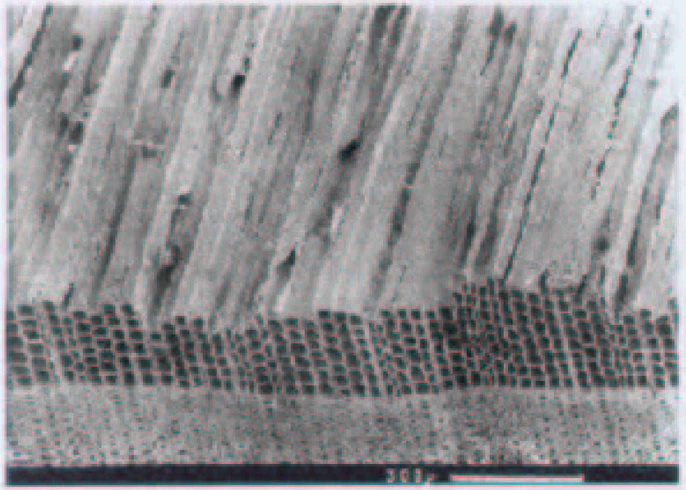
Universal aspects

- Independent of specific material properties
- Helps to evaluate experimental data and simulations
- Statistical physics of rupture

Model of Creep rupture

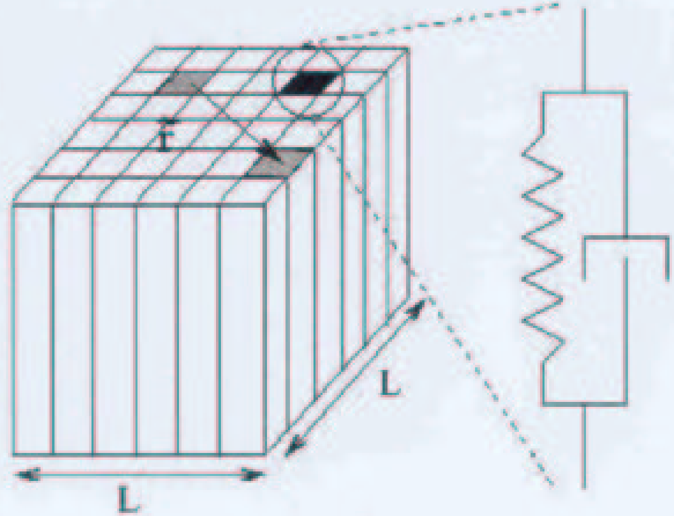
Bundle of viscoelastic fibers

Wood

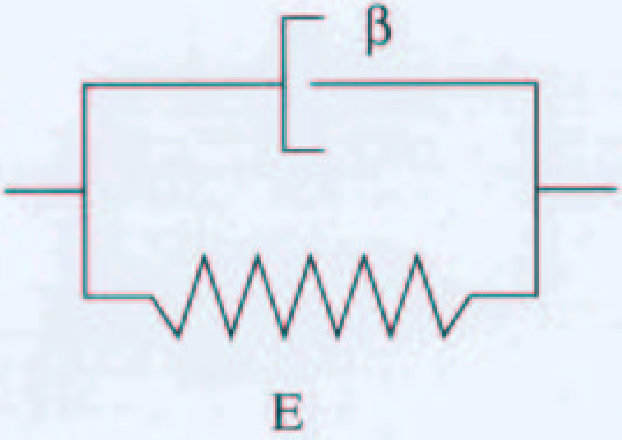


(G. Dill–Langer, S. Aicher)

Model



Viscoelastic fiber: Kelvin element

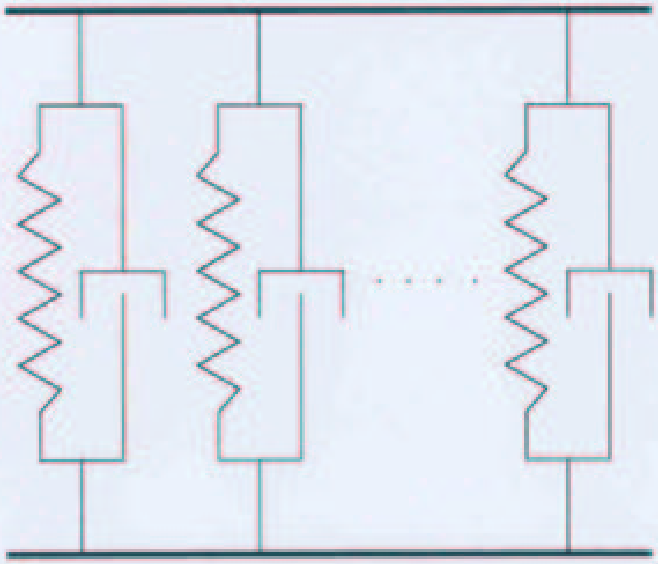


Two parameters: β, E

$$\sigma_o = \beta\dot{\varepsilon} + E\varepsilon$$

$$\varepsilon(t) = \frac{\sigma_o}{E}\left[1 - e^{-Et/\beta}\right] + \varepsilon_o e^{-Et/\beta}$$

Rupture of bundles



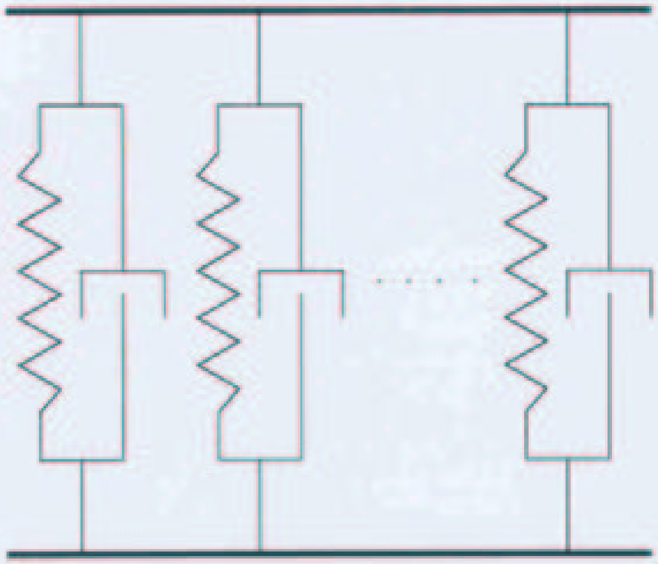
Strain controlled breaking of fibers

- P(ε) breaking threshold
- Load redistribution

$$\frac{\sigma_0}{1 - P(\varepsilon)} = \beta\dot{\varepsilon} + E\varepsilon$$

Coupling of breaking and viscoelasticity
In a global load sharing framework

Rupture of bundles



Strain controlled breaking of fibers

- P(ε) breaking threshold
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Coupling of breaking and viscoelasticity
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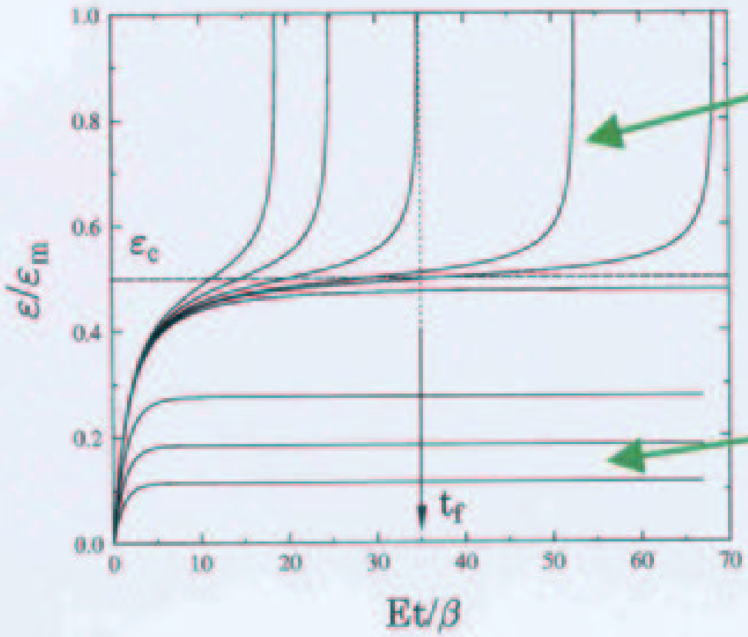
Analytic solution



Two regimes

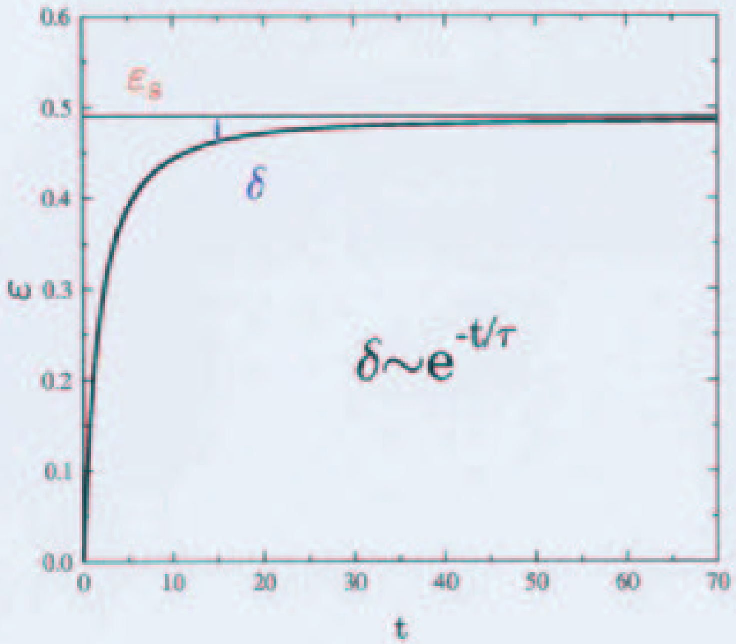
$\sigma_o > \sigma_c$:

- no stationary state
- monotonically increasing deformation
- global failre at finite time

$\sigma_o < \sigma_c$:

- macroscopic stationary state
- only partial failure
- infinite lifetime

Approaching the critical point



$\sigma_o < \sigma_c$

Relaxation by decreasing breaking activity

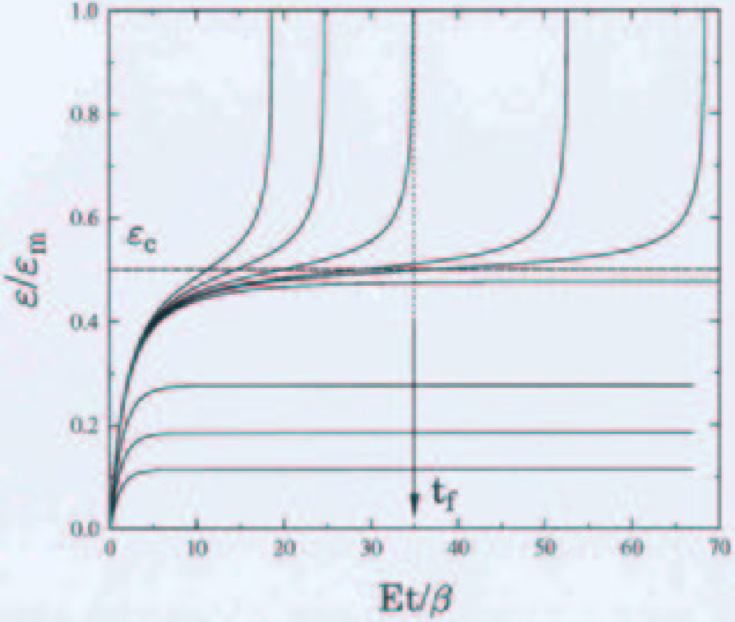
τ: relaxation time

$$\tau \sim (\sigma_c - \sigma_o)^{-1/2}$$

Universal power law divergence

Approaching the critical point

$\sigma_o > \sigma_c$



Global failure at finite time

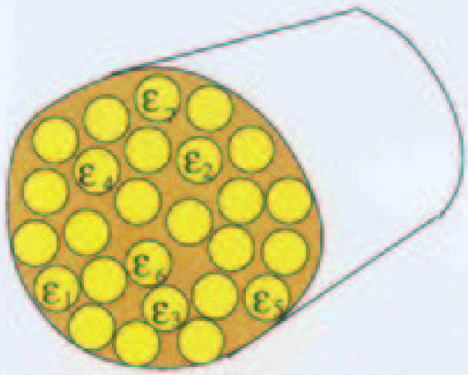
t_f: time to failure

$$t_f \sim (\sigma_o - \sigma_c)^{-1/2}$$

Universal power law divergence

→ **Continuous transition**

Process of breaking



$$\varepsilon_1 < \varepsilon_2 < \cdots < \varepsilon_N, \quad i = 1, \ldots, N$$

Time between two breakings:

– fibers break one–by–one

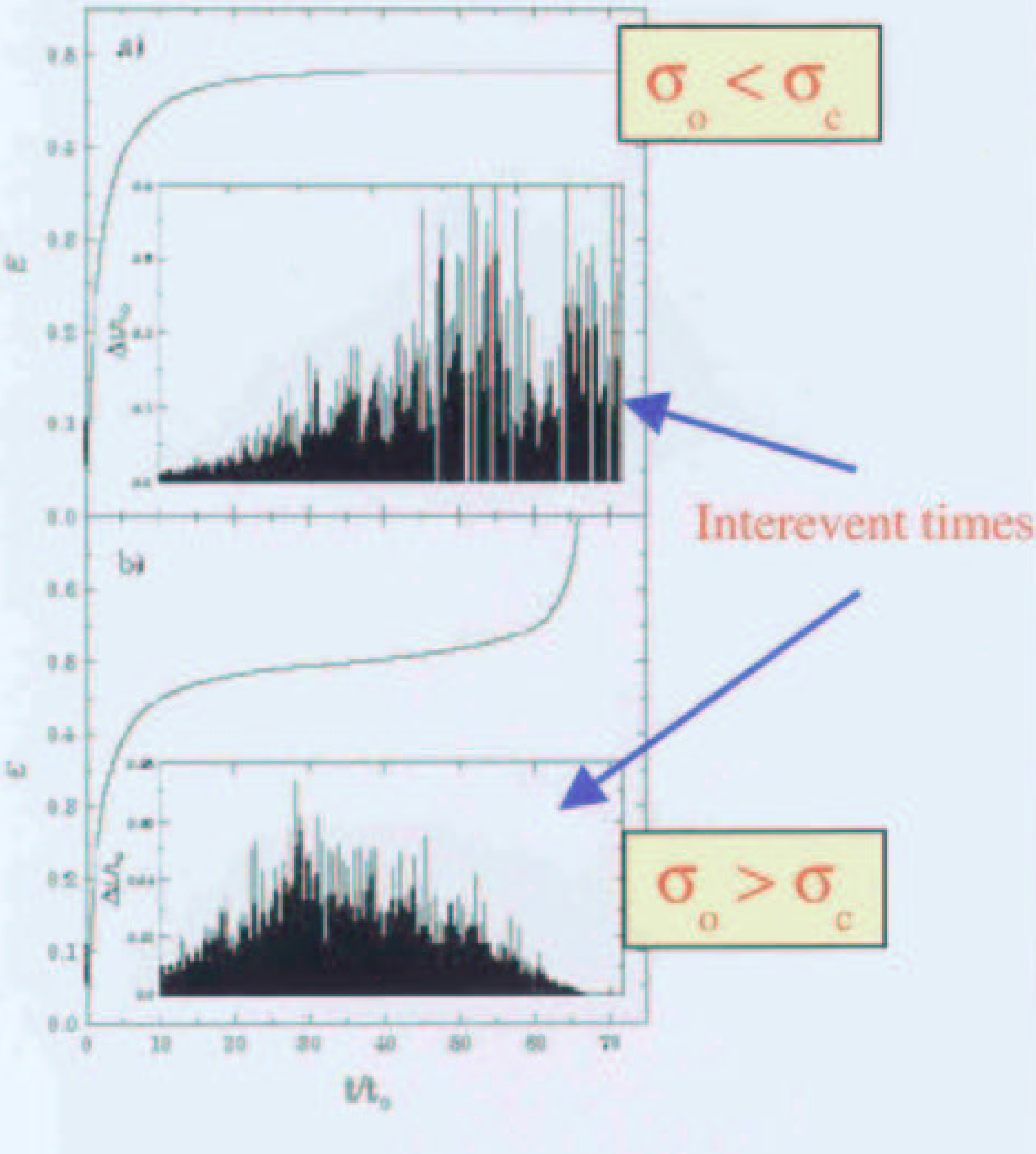
– single avalanche of breaks

$$\Delta t_i \; = \; -\frac{\beta}{E} \ln\left[\left(\varepsilon_{i+1} - \frac{\sigma_i}{E}\right) / \left(\varepsilon_i - \frac{\sigma_i}{E}\right)\right]$$

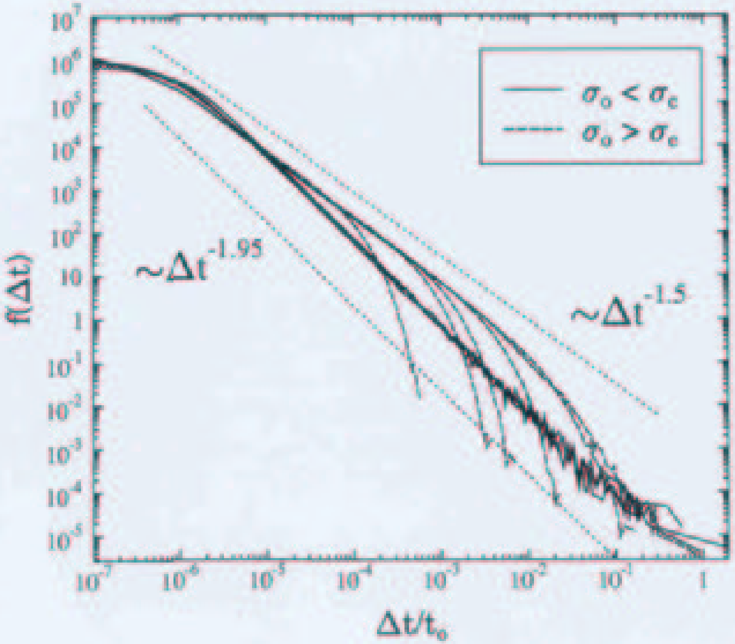
$$\sigma_i \; = \; \frac{\sigma_o N}{N - i}$$

Sensitivity to the details of load redistribution

Structure of a single avalanche



Distribution of interevent times



Power law distribution

$$f(\Delta t) \sim \Delta t^{-b}$$

Exponent

$b = 1.95 \pm 0.05$ For $\sigma_o < \sigma_c$

$b = 1.5 \pm 0.05$ For $\sigma_o > \sigma_c$

Self organization in creep

Macroscopic scale

- steady external driving
- emergence of a stationary state
- separation of time scales

Microscopic scale

- local overloads
- relaxation by breaking
- threshold dynamics

Power law distribution

$$f(\Delta t) \sim \Delta t^{-b}$$

Conclusions

→ Fragmentation of brittle solids

- Universal power law behavior
- Continuous phase transition

→ Creep rupture of fiber composites

- Scaling laws
- Self organization