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How to convince others: Monte Carlo simulations of Sznajd models

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The Sznajd model in less than a year has found several followers. An isolated person does not convince others; a group of people sharing the same opinions influences the neighbours much more easily. Thus on a square lattice, with variables $+1$ (Democrats) and -1 (Republicans) on every lattice site, a pair (or plaquette) of neighbours convinces its six (eight) nearest neighbours of its own opinion if and only if all members of the pair (plaquette) share the same opinion. The generalization to many possible states is used to explain the distribution of votes among candidates in Brazilian local elections.

Keywords: Consensus, lattices, cellular automata, elections

1 Introduction

The involvement of physics research in social affairs is quite old [1, 2, 3] but seems to become fashionable now. Even backward Germany's physical society founded a section on socioeconomic research, headed by Frank Schweitzer [4]. Ising-type models have been reviewed in [5]. The present review deals with the very modern Sznajd model [6] and its applications [7-14,17-19]. It deals with the way how opinions spread in human society.

Empirical research has shown decades ago that a single person stopping on the street and staring into the sky is mostly ignored by passers-by. If however several people together stare into the sky, they induce others to follow them. The trade union movements follow a similar strategy: By uniting, the employees have a stronger position against management than if everybody tries to negotiate alone: "United we stand, divided we fall." Thus it is not surprising that in Poland this principle was put into a computational model, the Sznajd model [6], first in one and then in two dimensions [7]. We do not deal here with all the variants but emphasize the more interesting ones.

The next section looks at the basic model, then we check complications and compare the results with the Ising model at low temperatures, and finally we bring a political application, the distribution of votes among candidates. The crucial difference of the Sznajd model compared with voter or Ising models is that information flows outward: A site does not follow what the neighbours tell the site, but instead the site tries to convince the neighbours.

2 Basic Model

Each site of a one- or two-dimensional lattice carries a spin S which can either be up (Republican) or down (Democrat) and represents one of two possible opinions on any question. The Fortran program denotes the spin by $is = \pm 1$. Two neigbouring parallel spins, i.e. two neighbouring people sharing the same opinion, convince their neighbours of this opinion. If they do not have the same opinion, then either they do not influence their neighbours, or in one dimension [6] they bring their neighbours to the opposite position. On the square lattice one may also demand that a plaquette of four sites, instead of a pair of only two sites, must have all four opinions to agree in order to convince the neighbours. We list the Fortran program for the case where a pair of two neighbouring spins convinces the six neighbours if and only if the two central spins have the same opinion (rule IIa of [7]).

Various other rules are defined and discussed in [7]. In all of them we need a random selection of the next spin pair to be selected for trying to convince its neighbours; otherwise the one-dimensional results are fixed by the few first spins in the chain. (The updating is random-sequential; with simultaneous updating instead, reaching a consensus is much more difficult.)

For these rules in one and two dimensions (and also for the majority of alternative rules [7]) the system always ends up in a fixed point: Either all spins point up, or they all point down in the square lattice. The antiferromagnetic rule in one dimension [6] in addition allows in half of the cases an antiferromagnetic ordering of up, down, up, down etc. (In ferromagnets, neighbouring magnetic dipoles want to align parallel, while in antiferromagnets, neighbours want to be opposite to each other.) The times needed to reach this fixed point increase with lattice size and are distributed log-normally or in a more complicated way, depending on the rule [7]. Thus the problem is quite time consuming and does not allow to simulate even medium-size lattices like 1000×1000.

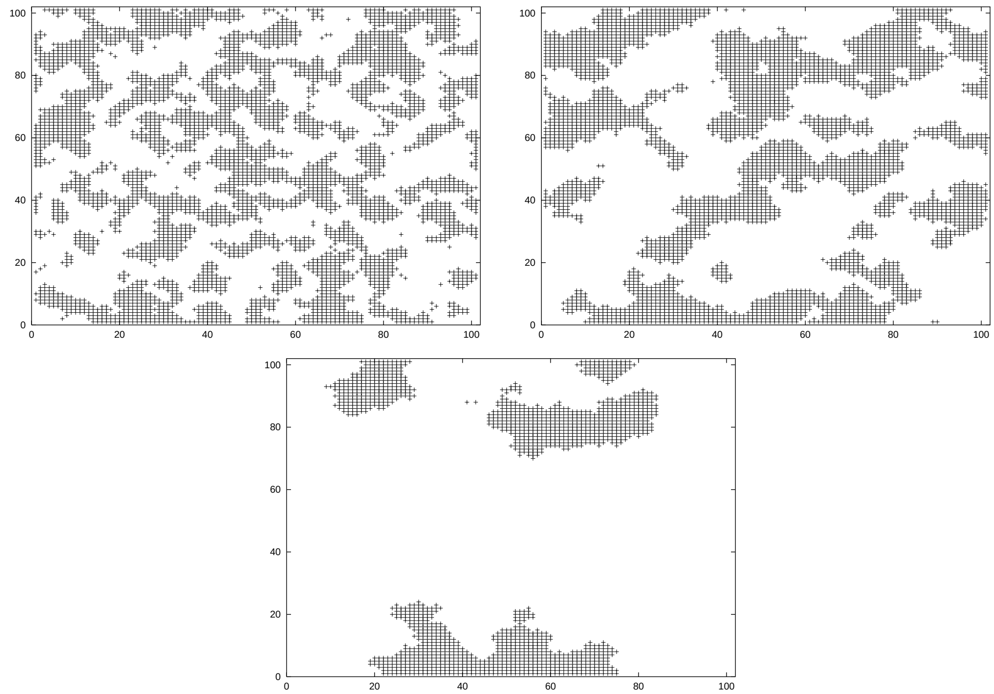

Figure 1: Sznajd square lattice, with a parallel pair convincing its six neighbours, after 1, 5 and 50 iterations. After 256 iterations the whole lattice was empty (spin down).

Usually one starts with half of the spins up and half of the spins down. If one varies the initial concentration p of up spins one sees a phase transition in the above rule for the square lattice but not for the chain: The system finally has all spins down if $p < 1/2$ and all spins up if $p > 1/2$. In a finite lattice, of course, phase transitions are never sharp, and thus the transition is indicated numerically by a slope (Fig.2) becoming steeper and steeper the larger the lattice is. In an infinite lattice one thus would get a sharp step function for the number of up fixed points versus the initial concentration p. This holds for both a regular square lattice [7] as for one where in a correlated way a fraction of sites is removed [9].

During the simulation, some people change their opinion often, some seldomly, and a small fraction never. The number of people with opinions unchanged up to time step t varies asymptotically [20] as $1/t^{\theta}$, with a per-

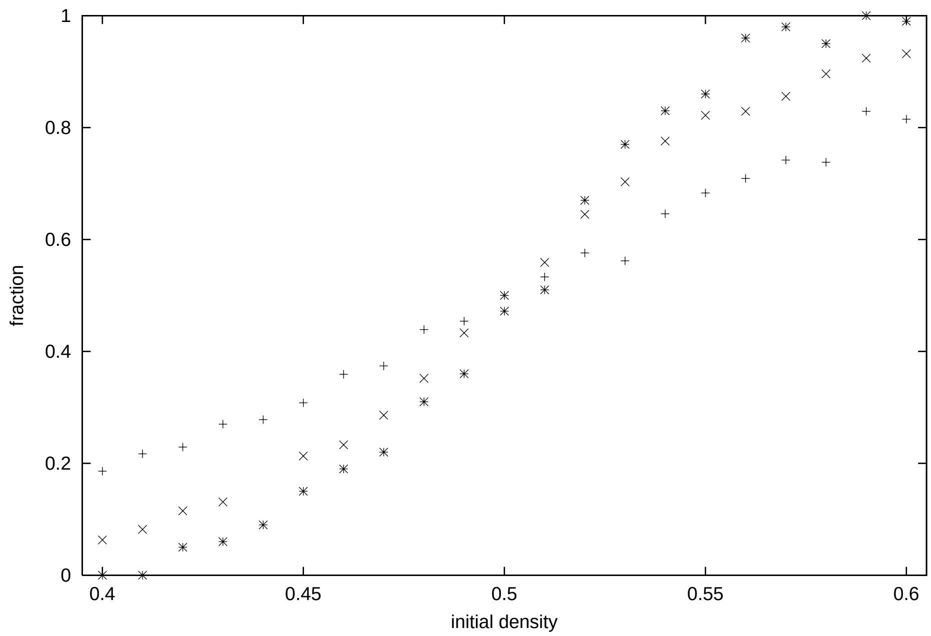

Figure 2: Fraction of samples ending with all spins up, versus initial fraction of up spins. 1000 samples L = 23 (+, 1000 samples), 53 (x, 1000 samples) and 101 (*, 100 samples), on a correlated-diluted square lattice [9].

sistence exponent $\theta(d)$ compatible for $d = 1$ with the exact 3/8 of the Ising analog; in higher dimensions it is larger.

The original paper was restricted to one dimension [6], where also a "disagreement function", somewhat similar to the energy in physics, describes the dynamics of the system [19].

3 Variants

To avoid dictatorship (all spins parallel) one can introduce a small number of non-conformists which are not convinced by the Sznajd rule [14]; see [6, 7] for alternatives..

Just like the Ising ferromagnet can be studied on a regular or on a diluted lattice, the Sznajd model, discussed above for regular lattioces, can also be studied in disordered systems. This is more realistic since human society does not follow a square lattice with every person having exactly four neighbours. The simplest case is a randomly diluted square lattice, where every site either is empty or carries one spin; this random occupation does not change during the simulation. More realistic are long-range correlations between the occupation of various lattice sites. Fortunately, in both cases the results were very similar to those on regular lattices from the previous section [9]. Long-range convincing strenghts, decaying with a power of the distance, are being investigated by C. Schulze.

On the triangular and simple cubic lattice, if a pair of parallel neighbours convinces its 8 (or 10, respectively) other neighbours of the pair opinion, the results are similar to the square lattice: all lattices finally have all spins parallel, after a time which is not distributed log-normally. And as a function of the initial concentration we have a phase transition at 1/2. In the triangular case, Chang [12] found at the end all spins parallel even for antiferromagnetic interactions; in the Ising model we know that triangular antiferromagnets are frustrated and thus complicated.

The Sznajd model thus is robust against geometrical disorder or rearrangement. If one convinces the neighbours only with some probability , p, and leaves them unchanged with probability $1-p$, still a consensus is reached albeit after a longer time. Advertising through mass media can be included as a small probability ϵ for a spin to point up independent of the usual rules. Then at the end all spins are up even if initially 60 percent were down and only 40 percent up; the ϵ value needed to change the opinion of the whole lattice goes to zero if the lattice size goes to infinity and if Ochrombel' simplification (see below; [11]) is used.

More threatening to the basic concept of "United we stand, divided we fall" is the idea of Ochrombel [11] to allow a single site, without any solidarity with others, to convice the four neighbours. Thus we select randomly a site and then automatically force its four neighbours to take the same orientation as that site. (Both spin up and spin down can convince; if only spin up would convince we would have the trivial infection process.) Again at the end everybody agrees. However, there is no phase transition in this simplified model: The fraction of final fixed points having all spins up, in a simulation of lattices, agrees with the initial fraction of up spins in each lattice.

The dynamics of the Sznajd model is quite similar to that of the Ising model at low temperatures: An initially random distribution of spins forms through spinodal decomposition large domains of up spins, surrounded by large domains of down spins; see the figure. The cluster growth follows a scaling law [10] known since decades for Ising models: The typical cluster radius increases as the square root of the time, and the distribution of cluster radii is determined mainly by the ratio of the cluster radius to the typical cluster radius. However, an Ising model at low but finite temperatures shows lots of isolated up spins in a sea of down spins, while the above Sznajd model is deterministic, corresponds to zero temperature, and avoids such isolated sites. (At zero temperature, the Ising model dislikes ordering [16].)

The Sznajd model was also combined [17] with the idea presented in the Hegselman talk [21], that only people who have already a similar opinion can be convinced; this requires more than two opinions and thus is similar to Bernardes's work [10]. Thus, people can convince others only if the other opinions from 1 to q differ by at most one unit from the central opinion. Thus when only opinions 1, 2, and 3 are available, those with opinions 2 can convince all others, while those with opinions 1 or 3 can only convince those with opinion 2. On the square lattice, one finds then in general a consensus for $q = 3$ but no longer for $q = 4$; on a triangular lattice this boundary shifts to $q = 5$. Combined with diffusion of agents [2] on a lattice such that only an agent which has just moved to another one can try to convince that agent, this model interpolates between random graphs and nearest-neighbour lattices. For $q = 4$ on the square lattice, usually one opinion wins, but another opinion survives as a small minority. The opinion which was on second place at half the time to reach the fixed point, at the end usually vanished completely [17], just like some athletes prefer the third place over the second place.

An application of the Sznajd model to small-world networks was presented by Elgazzar [13].

4 Politics

The most interesting application, in my opinion, was to political vote distributions [10]. The number of candidates, getting a fraction v of all votes cast for city councils in Brazil, varies as $1/v$ except for downward deviations at large and small v. Ref.[10] gives real results from Minas Gerais (Brazil) and compares them with suitably normalized votes from the following simulation:

Instead of only two choices each Sznajd voter has N choices, corresponding to the N candidates. Bernardes replaces the square lattice by a Barabasi network, a structure grown on the principle that well-connected people get more easily new connections than others [15]. We start with five nodes all connected to each other. Then the network grows as follows: A newly added node makes exactly 5 connections to already existing nodes; the probability to select one such already existing node is proportional to the number of connections this node had already before. This network is known to give a wide distribution of the number k of neighbours, with the number of sites having k neighbours decaying as $1/k^3$ for intermediate k. The comparison of real and simulated votes shows them to be indistinguishable apart from normalization.

5 Conclusion

This paper summarized simulations of the recently invented Sznajd model, an alternative to the Ising model which has been applied to social phenomena since decades. While in an Ising model, each site looks at its neighbours and tries to follow them, in the Sznajd model sites which agree among each other try to convince their neighbours. The most interesting application thus far was the reproduction of the vote distribution in Brazil, Fig.3.

I thank Suzana Moss de Oliveira and Paulo Murilo C. de Oliveira for hospitality at UFF in Niterói, Brazil, where most of this work was done together with Brazilian collaborators.

6 Appendix

A short Fortran program is listed here; for questions ask stauffer@thp.uni-koeln.de

The output starts with the initial parameters: `101 90000 1 1`, then come thousands of lines with intermediate results, and when the consensus was found we stop with a histogram of relaxation times (one line only if `nrun=1`) followed by, for example, `0.500000 0 1 0` giving the initial concentration, the number of samples ending with all spins `is` down, the number of samples ending with all spins `is` up, and the number of samples ending without consensus. $-2^{31} < \texttt{ibm} < 2^{31}$ are random integers, obtained by multiplication with 16807. The $L \times L$ lattice is enlarged by *two* buffer lines on each end to allow simple helical boundary conditions.

```
      program sznajd
c     square lattice, with two neighbours convincing
      parameter(L=101, L2=L*L, Ls=L2+4*L)
      dimension nhist(0:32), neighb(0:3), is(Ls)
      data maxstep/90000/,iseed/1/,nrun/100/,nhist/33*0/,p/0.4/
      print *, L, maxstep, iseed, nrun
      ibm=2*iseed-1
      fact=L2/2147483648.0d0
      ip=(2*p-1)*2147483648.0d0
      neighb(0)= 1
      neighb(1)=-1
      neighb(2)= L
      neighb(3)=-L
      icountu=0
      icountd=0
      icount0=0
      do 4 irun=1,nrun
      do 1 i=1, Ls
        ibm=ibm*16807
        is(i)=-1
 1      if(ibm.lt.ip) is(i)=1
      do 2 itime=1,maxstep
        m=0
        do 3 k=2*L+1,L2+2*L
          m=m+is(k)
 5        ibm=ibm*16807
          if(ibm.lt.0) ibm=(ibm+2147483647)+1
          i=2*L+1+fact*ibm
          if(i.le.2*L.or.i.gt.L2+2*L) goto 5
          ibm=ibm*65539
          j=i+neighb(ishft(ibm,-30))
          ici=is(i)
          if(ici.eq.is(j)) then
            is(i-1  )=ici
            is(i-L  )=ici
            is(i+L  )=ici
            is(i+1  )=ici
            is(j-1  )=ici
```

```
            is(j-L  )=ici
            is(j+L  )=ici
            is(j+1  )=ici
          endif
3       continue
        if(iabs(m).eq.L2) goto 6
2     continue
      print *, ' error, itime =', itime
      icount0=icount0+1
6     ibin=alog(float(itime))/0.69315
      nhist(ibin)=nhist(ibin)+1
      if(m.eq.L2) icountu=icountu+1
      if(m.eq.-L2)icountd=icountd+1
4     continue
      do 7 ibin=1,32
7       if(nhist(ibin).ne.0) print *, 2**ibin, nhist(ibin)
      print *, p, icountu, icountd, icount0
      stop
      end
```

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